Morphological layers in neural networks: how to train them and how to use them for image analysis

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1 Background

In morphological neural networks (MNNs), artificial neurons perform a non-linear operation such as a dilation, an erosion or more generally a rank filter (although less popular), instead of a dot product (see Table 1). MNNs were introduced in the late 1980s [Wil89, DR90] and have been revisited in the current deep learning era. In particular, universal representation results were stated, combining morphological and classical (linear) neurons [ZBVF+19]; morphological networks were successfully trained to perform image analysis [MDC20, FFY20, VFRA22, LBVF24] using standard gradient-descent optimization methods, with particularly promising results regarding geodesical reconstruction layers [VFRA22, LBVF24]; insights in alternative optimization methods were proposed [CM17, GDG23, Blu24]; approximations of morphological operators were explored [HTP+22]; code to implement morphological architectures within deep learning frameworks has been developed and shared [PCC+19, VF20].

Classical neuron	Dilation neuron	Erosion neuron	Weighted r-rank neuron
$y = \sum_{1\leq i\leq n} x_i \cdot w_i$	$y = \max_{1\leq i\leq n} x_i + w_i$	$y = \min_{1\leq i\leq n} x_i + w_i$	$y = R^{(r)}\big(\{x_i + w_i, 1 \leq i \leq n\}\big)$

Table 1: Comparison between classical and morphological neurons, for an input $x \in \mathbb{R}^n$ and a parameter $w \in \mathbb{R}^n$ (activation functions are not considered).

Yet, compared to classical neural networks, the field of morphological ones is still immature. It is remarkable for example that all architectures implemented in the literature are rather shallow (few layers, few filters per layer), and rarely outperform the state-of-the-art CNNs or transformers on usual benchmark datasets. This is because MNNs are still hard and slow to train, and often yield disappointing results when the architecture complexity grows. Therefore, several research tracks are now open to make significant progress in the understanding and use of MNNs, as suggested next.

2 Research tracks for the thesis

Exploiting topological and geometrical *a priori* knowledge Mathematical Morphology (MM) has proved to be very powerful in image filtering based on shape, size, contrast and connectivity properties. Since its theory is well established, these properties can be provably guaranteed at the output of a morphological pipeline. Therefore, including morphological operators within a neural architecture allows its optimization for a given task, while ensuring some desired properties like: the number of holes or connected components, the size, contrast, orientations of the segmented or classified objects. Recently, in a microscopy image context, geodesical reconstruction layers [VFRA22] were successfuly used to (1) characterize melanocytes by their contrast and (2) count them in an end-to-end supervised training [LBVF24]. This is only the first attempt of this kind, and many others need to be explored.

Learning morphological representations The morphological representation theory pioneered by Matheron [Mat75] and subsequently extended [Mat75, Mar89, BB93] tells much about the potential of MNNs. Indeed, as shown by Maragos [Mar89], any increasing, translation invariant and upper semi-continuous lattice operator (typically a mapping between images) can be represented by the supremum of a minimal, and possibly finite, family of erosions. Banon and Barrera managed to relax the increasingness [BB91] and the translation invariance hypothesis [BB93] by including anti-erosions and anti-dilations in the decomposition. Since these operators can be implemented as morphological layers, the results above offer a large playground to study MNNs. The goal of this track is to address simultaneously two fundamental questions: 1. Can we go beyond theory and learn to approximate these morphological representations for real image operators? 2. Can these results guide the choices of architecture in practical cases, and improve optimization by searching in the right family of functions?

Optimizing morphological layers The first two tracks above formulate original and crucial optimization problems, relevant to the application of morphological networks to image analysis. This implies a third track, which is to address optimization issues and find the best reachable optimums. Although morphological layers are known to be hard to train, the reasons for this are not clear, since they have not been systematically studied. It is often hypothesized that their non differentiability is not compatible with gradient-descent algorithms, despite the fact that these layers are actually almost everywhere differentiable, like other commonly used ones (ReLU, max/min-poolings); and that smooth approximations of min and max functions [HTP+22] have not shown to be sufficient to solve these issues. If non-differentiability does matter, we may still derive update rules for the parameters, as recent work has started to explore [Blu24]. Other hypothesis regard the sparsity of these layers' gradient, when it exists, and the importance of the initialization of morphological weights. All these hypothesis shall be tested in an appropriate experimental setting, and the conclusions should lead to design optimization algorithms best suited to morphological layers.

3 Required skills

The successful candidate should have a strong mathematical background, experience with deep learning tools such as Tensorflow, Jax and/or PyTorch, and motivation for research work in mathematics and image processing.

4 Contact and supervision

The Center for Mathematical Morphology (CMM) is part of Mines Paris, PSL University, and is located in Fontainebleau, France (40 minutes from Paris by train), near the castle and the forest. Flexibility in the workplace is also possible. Please send your application (CV and short motivation letter) to the team who will supervise the thesis: Santiago Velasco-Forero (santiago.velasco@minesparis.psl.eu) and Samy Blusseau (samy.blusseau@minesparis.psl.eu).

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