RANDOM PROCESSES SIMULATIONS
ON THE TEXTURE ANALYSER

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The prototype Texture Analyser at Fontainebleau permits very swift simulations of many random processes, be they point processes or random set processes.We shall illustrate, with some examples of general processes, the possibilities of this apparatus. Three kinds of processes are presented here :

- The Poisson point process
- Cluster Processes ("shooting" scheme)
- Hard-Core processes (repulsion models)

With various combinations of these three fundamental processes, one may induce more complex realizations of random sets containing two or three phases.

Poisson point process

A Poisson point process, with intensity θ, is very easy to simulate on a bounded set A. The simulation is divided in two stages :

One chooses n, the number of points of the set A from a Poisson distribution with mean $\theta \cdot x\ |A|$. ($|A|$, Lebesgue measure of A).

One distributes the n points at random over the set A, which in the present case, is the rectangular field of the texture analyser.

It is possible, starting from a Poisson realization, to simulate random sets processes by centering in every point of the Poisson process one realization from a non-stationary random set, called the primary grain X'. Two particular realizations of such processes, the Boolean Schemes, are shown in figure 1.

Cluster Processes

When the primary grain of a Boolean Scheme is a cluster of N points distributed independantly around each of the Poisson process points according to a distribution law F, one is simulating a cluster process, also called a "schooting scheme", ("Schema du Tireur" - Matheron G.) or a Neymann-Scoot process (fig. 2).

Once such a point process is achieved, the subsequent simulation of random sets is a simple task (fig. 3).

Hard - Core Models

These models were defined by Matern B. (1960). One samples a Poisson process of intensity θ , and deletes any point which is within R of any other point. Figure 4 illustrates this kind of process.

Mixtures of models

These models are obtained by mixing simpler processes as shown in Figure 5.

Analyser program and applications

The afore-mentioned simulations do not consume large amounts of computer time (two or three minutes for the longest ones). Biases introduced by edge effects are eliminated by suppressing a strip around the sampling field, whose width depends on the parameters of the processes. Balls are approximated by hexagons (one could use dodecagons to increase the accuracy) ; Hence, only three needle directions can be defined on the hexagonal grid of the analyser, o, $\pi/3$, 2 $\pi/3$. Thus the orientation of needles is chosen at random among these three values.

Because of the rapidity of the simulations, the production of a large number of runs for these models for various parametric values is easy. In figure 6, an example of a covariance function calculated from the simulation of a hard-core model is schown.

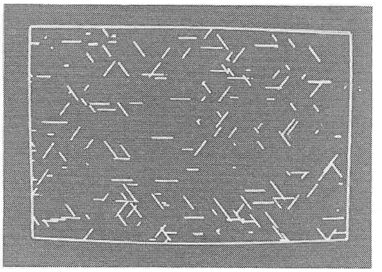

Figure 1 : Boolean Scheme

The primary grain is a needle, whose length is a random variable chosen from a uniform probability distribution, and oriented at random on the plane

$$\theta \times |A| = 250$$

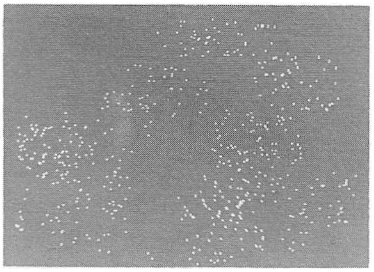

Figure 2 : Cluster Process (Schéma du tireur)

This realization is obtained by sampling in a circle of radius R centered on each point of the parent Poisson process of intensity θ , a daughter Poisson process of intensity θ . Then, the number N of points in the primary grain is a random variable following a Poisson law of mean $\theta' \pi R^2$, and the distribution function of these points is uniform over the R-Cercle.

R = 30

Mean of the Parent process : $\theta |A| = 20$

Average number of points in a cluster : 50

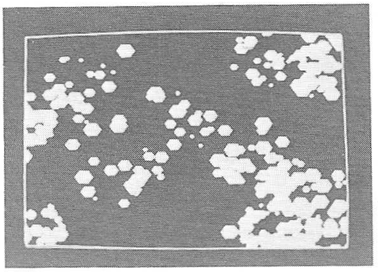

Figure 3 : Cluster Random Sets
R = 30 ; $\theta \; |A| = 15$; Ave(N) = 30

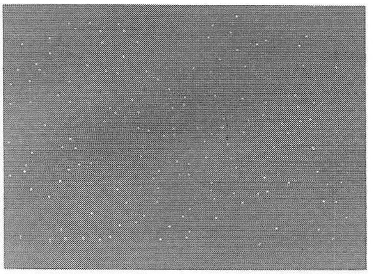

Figure 4 - Hard-Core Model

This sample corresponds to the first model defined by Matern. One takes into account every point of the parent Poisson process, whether or not it has been already deleted.

Average number of Parent process : $\theta \ |A| = 200$

Repulsion distance $R = 5$.

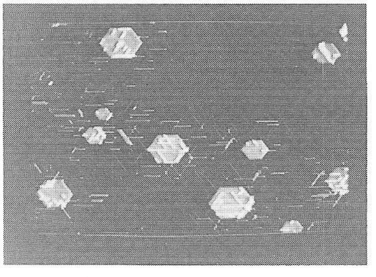

Figure 5 : Three-phase boolean scheme

Boolean scheme (the primary grain is a ball) and cluster process (the needles are centered on the points of this second process)

$$\theta . |A| = 20$$

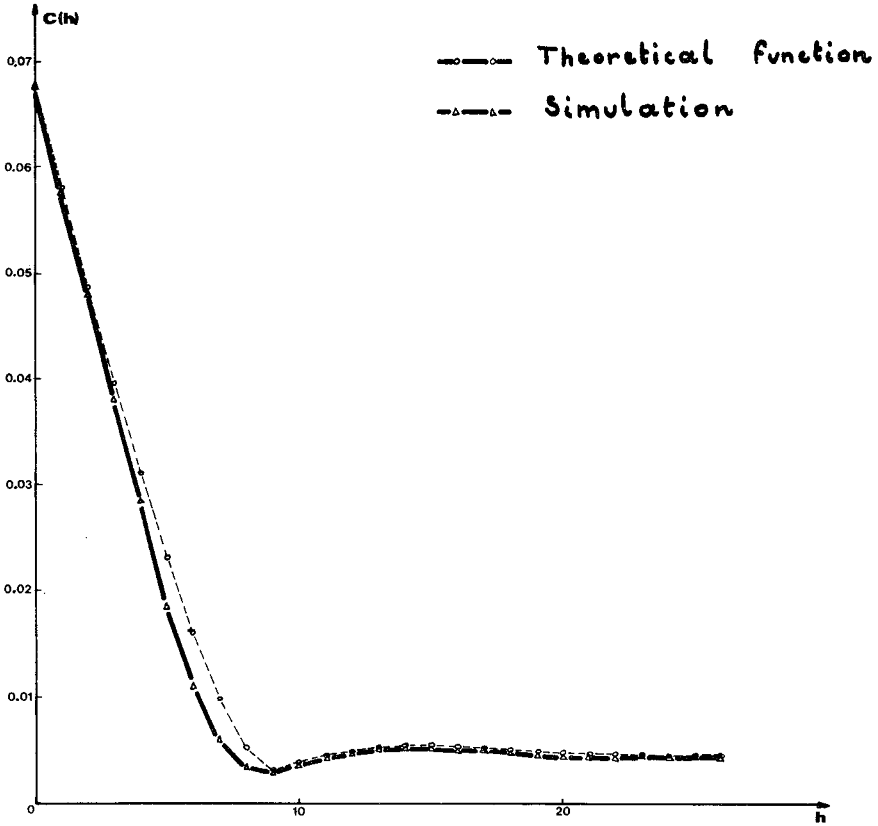

Figure 6 : Covariance function for the Hard-Core model

On each point of the Hard-core model of intensity λ and Repulsion distance R, we center a ball of fixed radius r with $2r < R$ (non-overlapping balls). The theoretical formula is given by :

$$C(h) = \alpha\lambda \; K_r(h) + \lambda^2 \int_{R^n} K_r(z-h)k(|z|)dz$$

with $\alpha = \exp[-\lambda C_n R^n]$, where C_n is the volume of the unit ball in R^n

$$K(z) = \begin{cases} 0, \text{ if } |z| < R \\ \exp\left[-\lambda(2 K_R(0) - K_R(z))\right) \quad \text{if } |z| \geq R \end{cases}$$

K_r and K_R, are the geometric covariograms of balls with radii r and R (resp.) in R^n.

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CONDITIONAL PROBABILITIES SIMULATION

Problem introduction

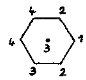
Let us consider the picture on figure 1.This image presents a great variety of grey tones. One digitalizes this image by partitioning the grey scale into n classes. Here four grey levels were chosen, so every picture point belongs to one class level, denoted (1), (2), (3) and (4). One wants to estimate the conditional probability that a picture point belongs to level (i), when its neighbowring points belong to given levels. In this case, only the six vertex points of an hexagon centered on the point will be considered. These points are at distance 1 from the central point (fig. 2).

Probleme solution

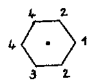
Let us take a numerical example. Suppose one wants to know the probability for a picture point to be at level (3), given that the six vertices of the size 5 hexagon belong, respectively, to levels (2), (3), (4), (4), (2) and (1). We estimate this probability by the formula :

$$p^{*} = \frac{n}{m}$$

n, number of configurations in the image



m, number of configurations



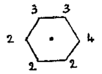
(central point not taken into account).

This estimator p^{*} will be meaningful only when the number m of configurations is great enough. This would be expected, even in a first approach, since there are at least 45.000 patterns in the image.

Using the texture analyser, one can easily look for such configurations as (2) - (3) - (4) - (4) - (2) - (1). The results are shown in figures 3 et 4.

As can be judjed from the figure, the value of m is very low. Hence, the calculated estimator p^{*} will not be meaningful.

The result obtained from other configurations are the same. For example, with :



, we obtain $m = 10$.

In the fact, with six conditioning points and four grey levels, there are $4^6 = 4096$ different possible configurations, that is, an average value of ten possible events for a given configuration. Actually, one notices that, in many cases, the actual number of configurations is lower than this average value.

When the number of conditioning factors is greater than a very low value (here six points, four grey classes), the number of events for a given configuration is too low to give a meaningful statistical inference for the conditional law.

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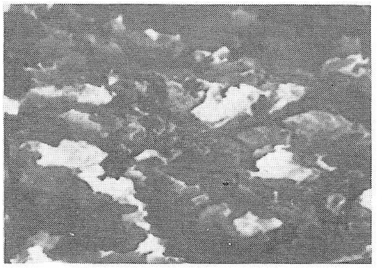

Figure 1 : Scanning electron microscope image of a clay sample (x 3000)

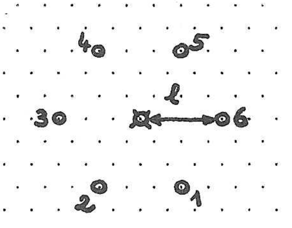

Figure 2 : Sampling grid of the image points

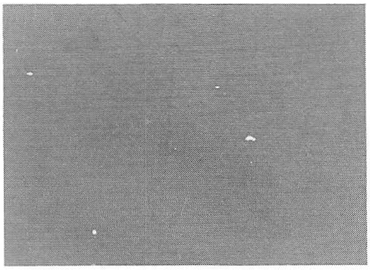

Figure 3 : Picture points surrounded by the outline (2) - (3) - (4) - (4) - (2) - (1)
at distance 5
four points with proper configurations

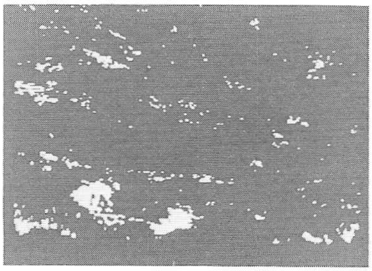

Figure 4 : intermediate result

The six vertices of the hexagon are tested successively.

Image is shown after the three first vertices have been processed.