ON THE CHANGE OF SPACE IN IMAGE ANALYSIS

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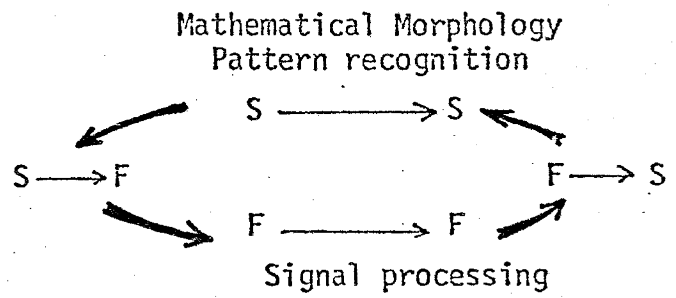
ABSTRACT

In mathematical morphology, the transformations commonly used are set-set transformations (e.g. dilation, erosion, skeletonization, etc...). These transformations provide shape and size descriptors. Unfortunality, most of the real-world images cannot be modelled with sets, without losing a large part of their significance. Generally, an image is represented by a function, say f, defined on a certain field X. If $x \in X$, $f(x)$ is the grey tone of the image at point x.
In signal processing, the transformations commonly used are function-function transformations (e.g. gradient modulus, anamorphosis, etc...). These transformations are effective in image-enhancement, but are not adapted to pattern-recognition.
Most of the time, image analysis consists of two steps : first the image-enhancement, and second the quantification. Using both domains requires a change of work-space.
This paper deals with two classes of transformation : the function-set and the set-function transformations



The function-set transformations : some of these are in common use (e.g. thresholding). We present here some finer transformations, which lead to image-segmentation processes using the local extrema of a function.
The set-function transformations : the use of these transformations provides illuminating descriptions of set-segmentation problems.
All these classes of transformations may be used in sequential or iterative processes. Several examples are given.

ON THE CHANGE OF SPACE IN IMAGE ANALYSIS

Until recently, the transformations most commonly used in Mathematical Morphology were set-set transformations. Starting from a set X, we defined a transformation T ; a new set T(X) was produced. Such transformations provide shape and size descriptors. For instance, the erosion by a ball B is a good descriptor of the size of the connected components of the set X :

$$X \ominus B = \{x \in R^2 | \; B_x \subset X\}$$

These transformations are very effective, provided that it is possible to easily define the set X which we want to analyse. Unfortunately, most of the real-world images cannot be modelled with sets. For example, an image analyser will not "see" a cytological sample as a set of cells on a more-or-less homogeneous background, but as a function f defined in the field of study : f(x) is the grey-tone of the image at point x.

On the other hand, there exist function - function transformations. These transformations are very powerful in image-enhancement, noise-cleaning etc..., but are not adapted to pattern recognition. As an example : how does one distinguish between cells without nuclei and cells with nuclei if one knows only the grey-tone function of the image. If we use only function-function transformations, the solution does not seem obvious. On the contrary, the set-set transformations provide a very simple solution : if we denote by X the set of the cells and by Y the set of the nuclei, one can use Y as a marker for the reconstruction of X by successive dilations. (For this algorithm, see [1]).

Most of the time, image analysis consists of two different steps : first the image-enhancement, using function-function transformations, and then the pattern recognition and quantification of the features detected in the image (Set-set transformations domain). Using both domains requires a change of work-space : This change of work-space is performed by two new kinds of transformations : the function-set transformations and the set-function transformations. The set-function transformations are very helpful in many problems of set-segmentation.

We shall describe these various transformations and we shall see their use in sequential and iterative processes.

THE FUNCTION-SET TRANSFORMATIONS

1) - An illuminating example : thresholding

The thresholding operation is in common use. The philosophy of this transformation is the following : the objects of interest are supposed to have an almost constant grey level which is different from that of the background. The image can be modelled by a positive function f, defined in a subset D of R^2. To threshold the image f at level a consists in replacing f by a subset X of D defined as follows :

$$X = \{x \in D \mid 0 \leq f(x) < a\}$$

By this transformation, we hope to separate the objects of interest from the background. Every point belonging to the phase of study is supposed to have a grey level lower than a, and every point of the background a grey level greater than a. This simple operation raises two very important problems :

- The first one is a problem of choice of the parameter a. What criterion can be used to select the optimum value of a ? In many cases, this criterion is visual. The operator selects a value of a which corresponds to the best result for his eyes. This method is indeed very simple, but also very dangerous : we are not sure of having reproduceable measurements. (In fact, we do not ; in area measurement, the result may vary by 30 % from one operator to another !).

- The second one is the problem of the validity of the hypothesis concerning the grey-tone distribution of the different phases of the image. The assumption that the grey levels of the different phases of the picture are well separated and almost constant is a very severe hypothesis. In practice, we often deal with inhomogeneous backgrounds, variations in grey values from one point to another, etc... Furthermore, this approach to the problem is unrealistic : before we transform a grey-tone picture into a binary one, we have already assumed that the picture is binary !

Many solutions have been proposed in recent years to solve the first problem. The simplest one is the following : let us consider a two-phase image (objects and background). We can compute the grey-levels densitogram of this picture. If the picture was perfectly two-valued, the grey levels densitogram would be that of Figure 1, with g_0 and g_1 being the grey values of the background and the objects resp., and p_0 and p_1 the amounts of background and particles.

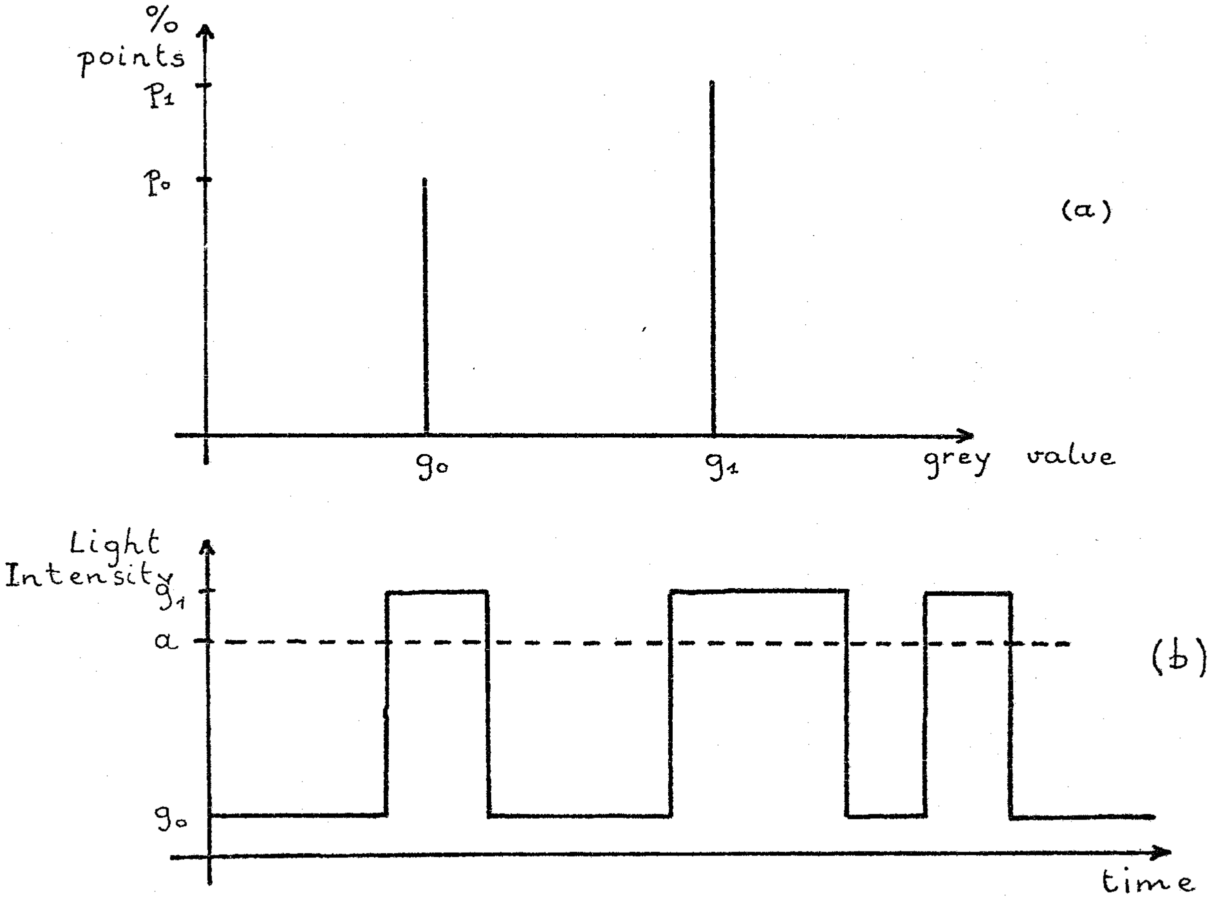


Figure 1 : Thresholding a perfectly two-valued function : (a) grey level densitogram (b) video signal

In this case, every value a of the thresholding falling between g_0 and g_1 would perfectly separate the two phases. In fact, the boundaries are not that sharp because of convolution phenomenae introduced by the optical and electronic devices ; the real densitogram is given in Figure 2.

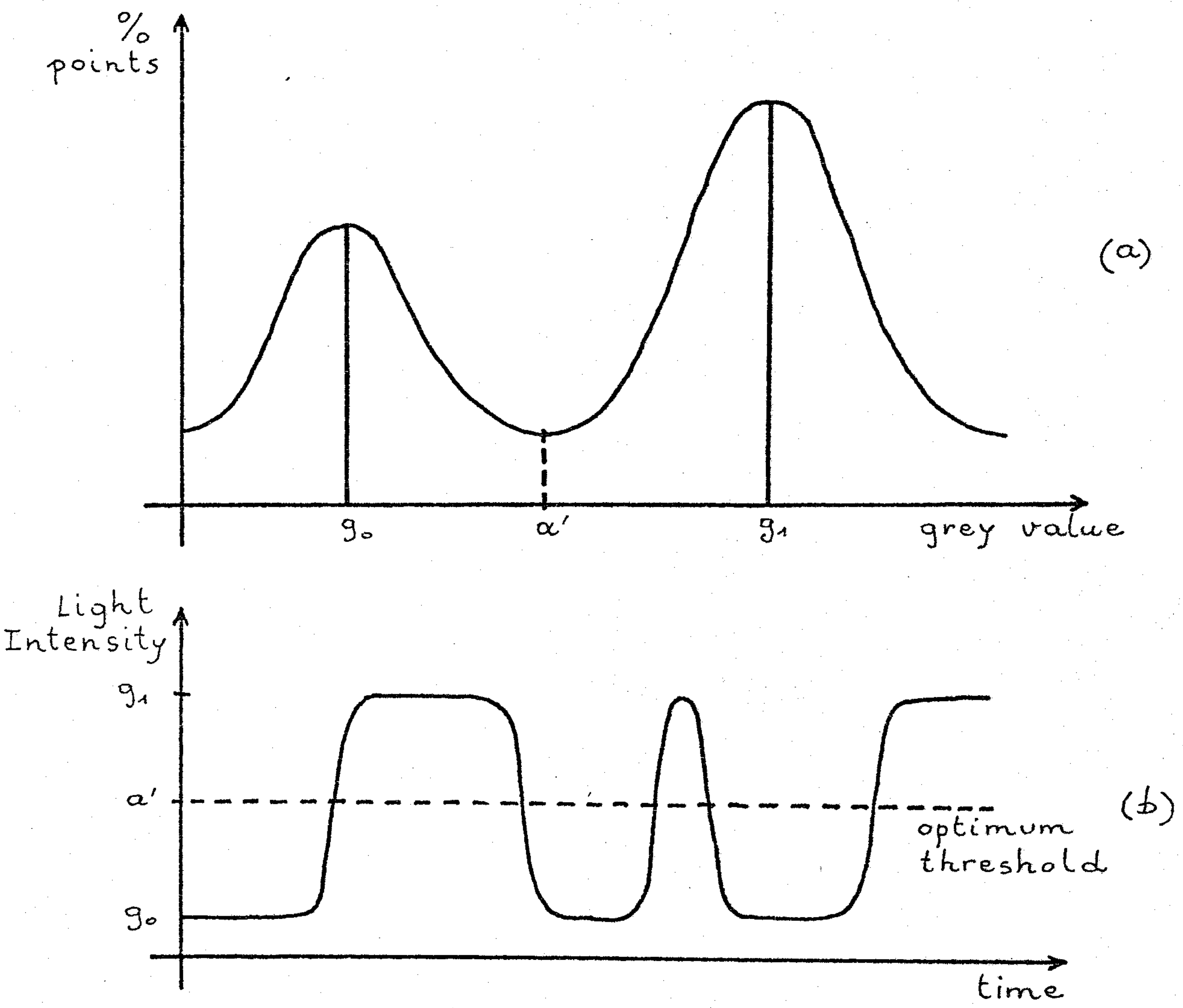


Figure 2 : Thresholding in real-world images

The optimum value of the threshold is now the value a' which corresponds to a minimum of the densitogram function.

This method is effective provided that the percentages of the two phases are not too different. If such is not the case, finding a minimum is not very easy (Figure 3).

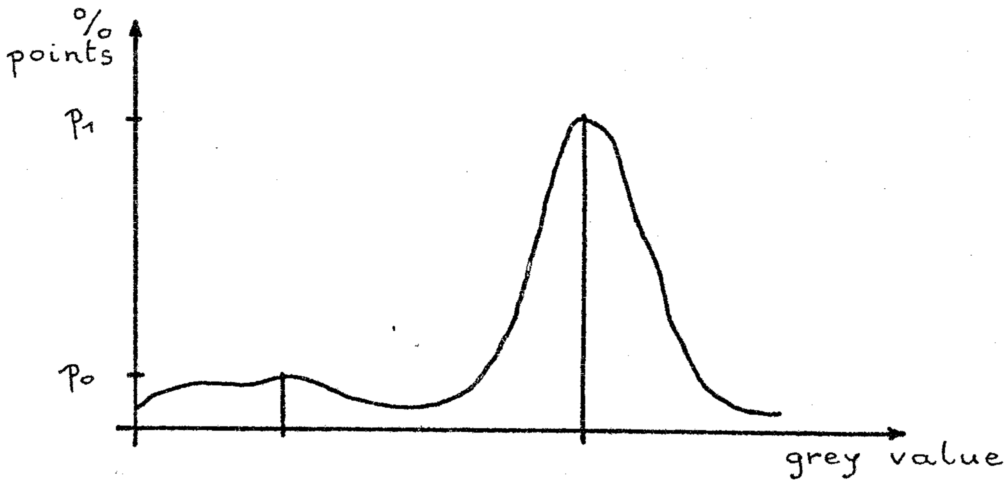
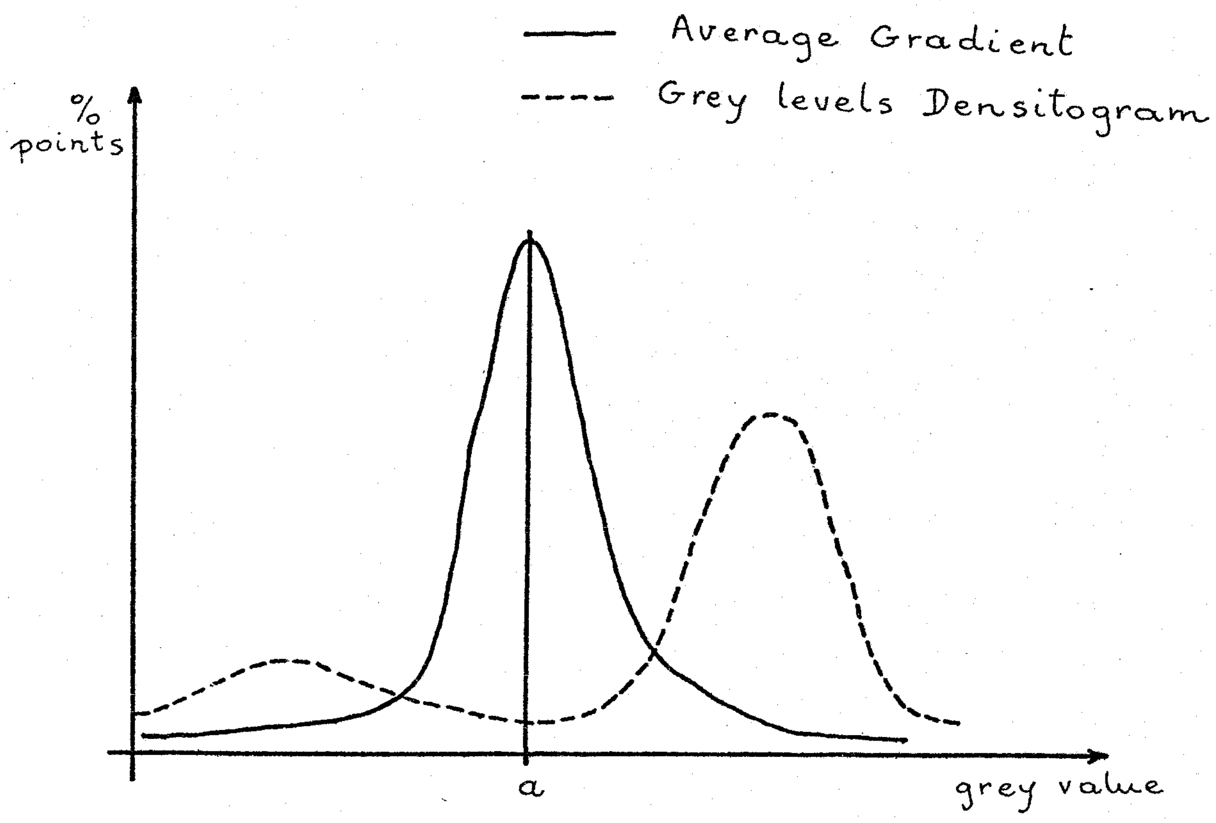


Figure 3 : p_0 and p_1 are too different

A more interesting method consists in taking into account the gradient-modulus of the grey-tone function f, which is supposed to be differentiable. We define a new function g :

$$g(x) = |\text{grad } f(x)|$$

The maximum values of this function are obtained for the picture-points belonging to the boundaries of objects. The grey-value of these points gives the optimum threshold. (figure 4).



Average gradient modulus :

g, grey level :

$$\text{Ave}(g) = \text{Mean value } (|\vec{\text{grad}}\ f(x)|)$$
$$(\forall x : f(x) = g)$$

Figure 4 : Optimum threshold detection using the gradient function.

Let us now consider a multilevel image : we call multilevel image an image in which we can distinguish n different phases. It seems easy, a priori, to threshold such an image, using n-1 values of threshold a_i. Actually, in most cases, this solution introduces very important measurement biases, as can be seen in the following example :

Imagine a three-phase image of inclusions in steel (Figure 5). We find steel, inclusions, and sulfide deposits around the inclusions.

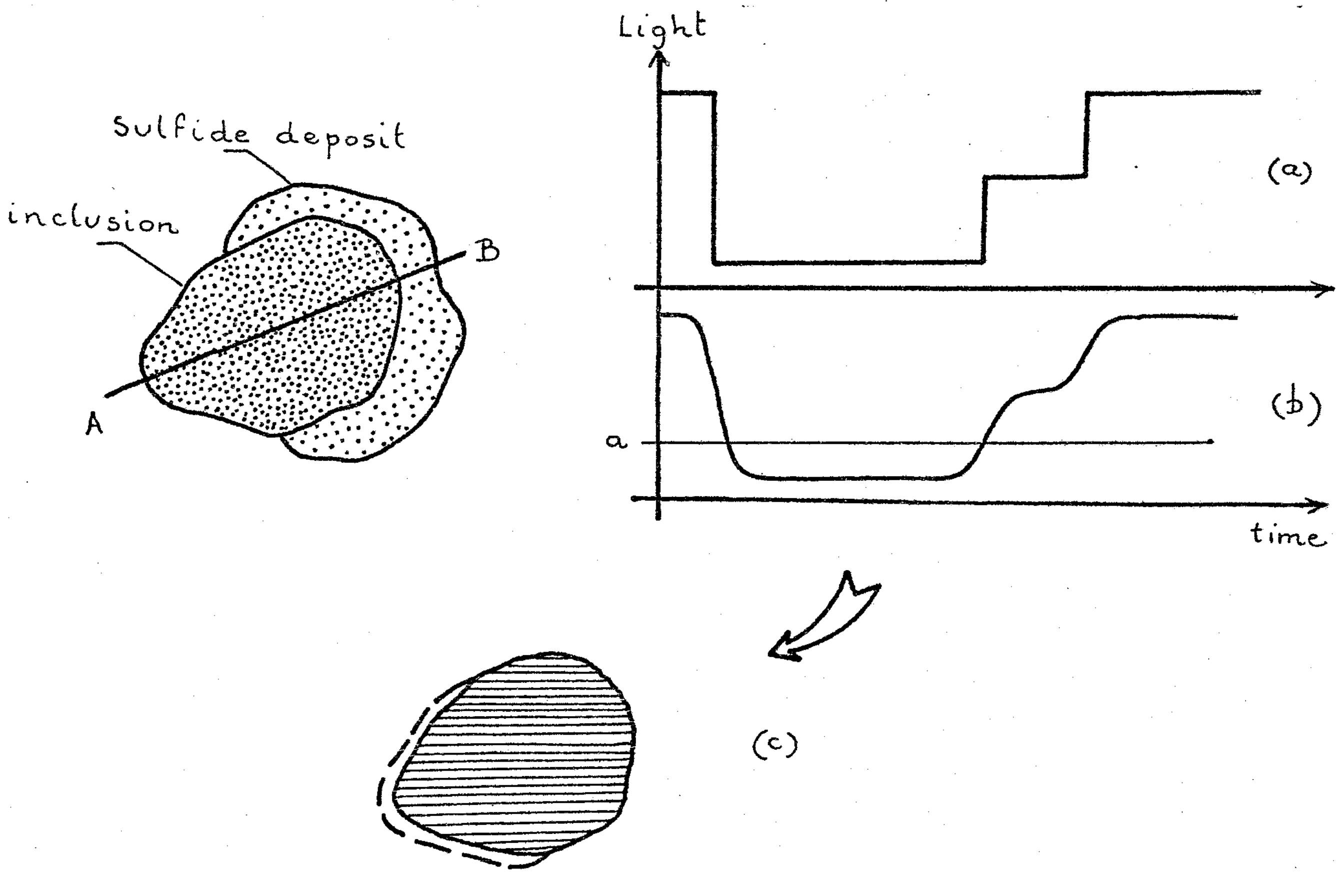


Figure 5 : Three-phase image -(a)ideal video signal ; (b) real signal ; (c) Bias in thresholded picture.

Along the line AB, the ideal video signal should be the signal (a). In fact, we find the signal (b). If we want to separate the inclusions and the sulfide deposits using the threshold a, we introduce a bias in the area of the inclusions. To eliminate this error, it would be better to use not a constant threshold value, but rather one depending upon the coordinates of the points in the picture.

So, the class of "thresholdable" images is very poor. In many cases, simple thresholding is a very coarse operation, simply because the assumptions made concerning the grey-tone distributions of the pictures are not verified.

2) - Towards a solution to the thresholding problem

What can we possibly do then, when thresholding does not work ? Given a picture, we want to separate the various phases (particles from background, for instance). The grey-tone function f of the image is supposed to be differentiable. This function is positive, bounded and, for simplicity's sake, normalized to 1 :

$$\forall\, x \in D,\ 0 \leq f(x) \leq 1$$

The question is : Can we give a mathematical definition of what we call "an individualized object"? For that, it is helpful to study the intuitive notion of an object : an object is a connected component of the image. This connected component does not necessarily have a constant grey value. Imagine a bounded particle X in a field ; let x be a point inside X. Starting from x and following a straight line in any direction, we cross the boundary of X. What criterion do we use to detect this boundary ? We rely, indeed, upon the most rapid variation in grey levels along the line. This variation can be quantified with the gradient modulus of f :

g, gradient-modulus of f :

$$g(x) = |\overrightarrow{\mathrm{grad}}\ f(x)|$$

So, it is possible to mark every object in the field by detecting the minima of the gradient-modulus (Figure 6).

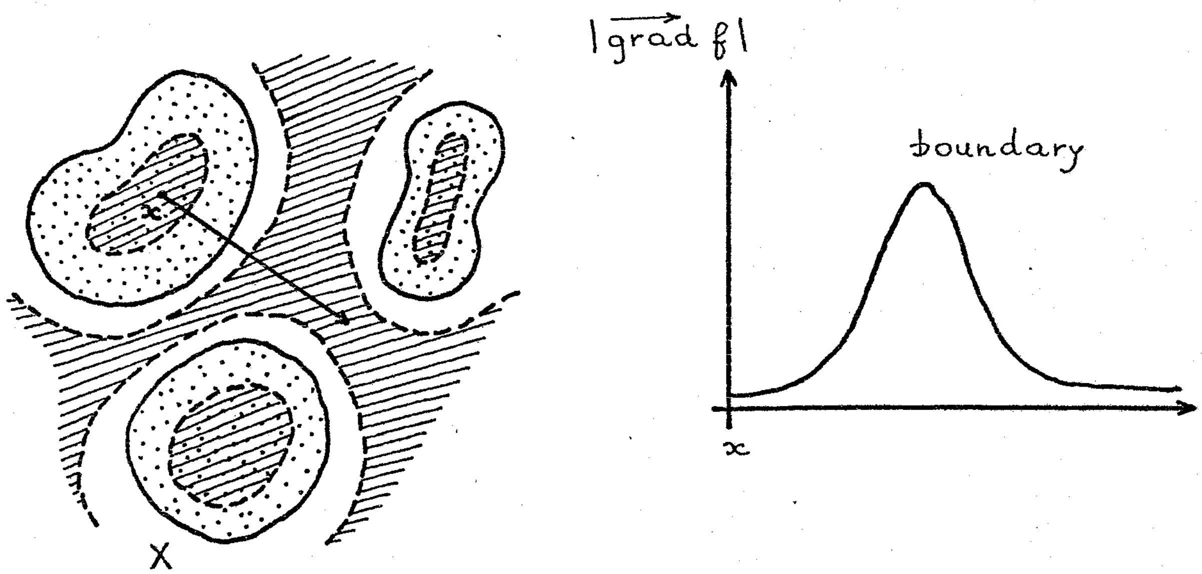


Figure 6 : Minima of gradient in a two-phase image (striped area)

3) - Minima of a function

Let h be the grey value function of an image. As before, we have $0 \leq h(x) \leq 1$ for any point $x \in D$. Using the vocabulary of geography, we can interpret h as a relief function. The surface of the relief is the set G of all points $[x, h(x)]$. Suppose we can wander over this surface. We shall say that a point $p[x,h(x)]$ of G is minimum iff it is impossible, starting from p, to reach anypoint $q[y,h(y)]$ of G, with $h(y) < h(x)$, using an a continuously descending path.

Let Z be the set of minimum points of G. By extension, each connected component of Z is said to be a minimum. It is easy to prove that if $p[x,h(x)]$ and $q[y,h(y)]$ of G belong to the same minimum, we will have :

$$h(x) = h(y)$$

Let $r[z,h(z)]$ be a point of G and K a connected component of Z. The point r is said to be linked to K, if there exists a descending path starting at r and ending at a point of K.

We can see that a point r of G may be linked to several components K_i of Z. But on the contrary, there exists no point of G which is isolated (that is to say, not linked to at least one K_i). We define as zone of influence of K_i, the set of points r of G linked to K_i only.

If we now apply these definitions to the gradient function g of the image, we can separate objects by computing the zones of influence of the minima of the gradient. The following pictures (Fig. 7 to 9) illustrate this algorithm. The sample used is an X-ray photograph of pores in a radioactive material. As can be seen, the final picture (figure 8) is over-segmented ; but it is easy to eliminate false boundaries by taking into account the maxima of the grey-tone function (we can define a maximum of a function as we did for the minimum).

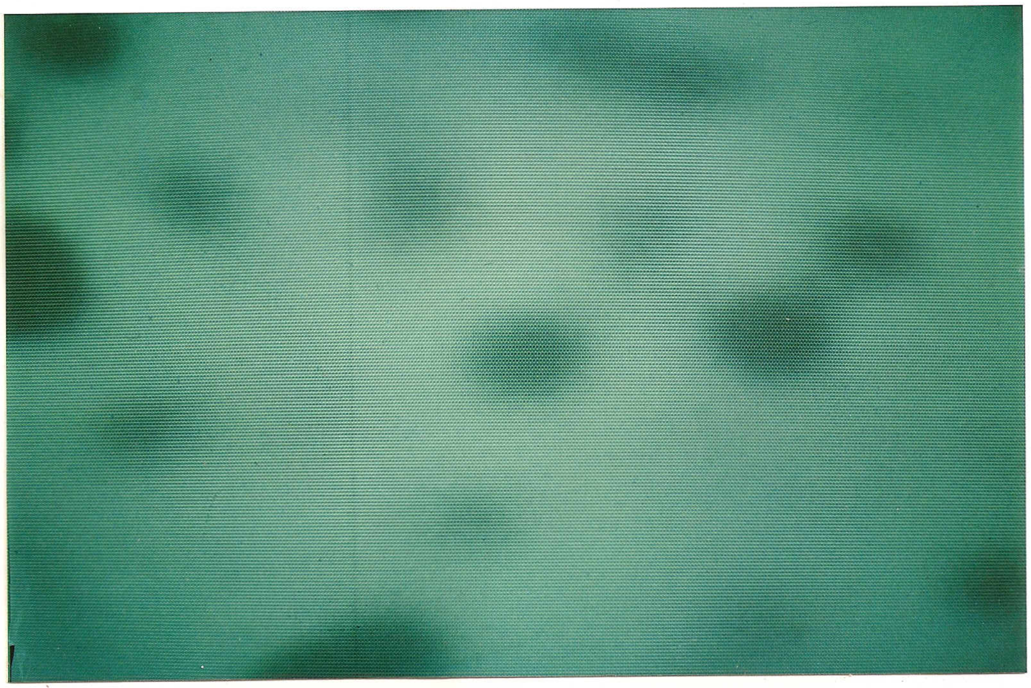


Figure 7 : Radioactive material - Initial picture.

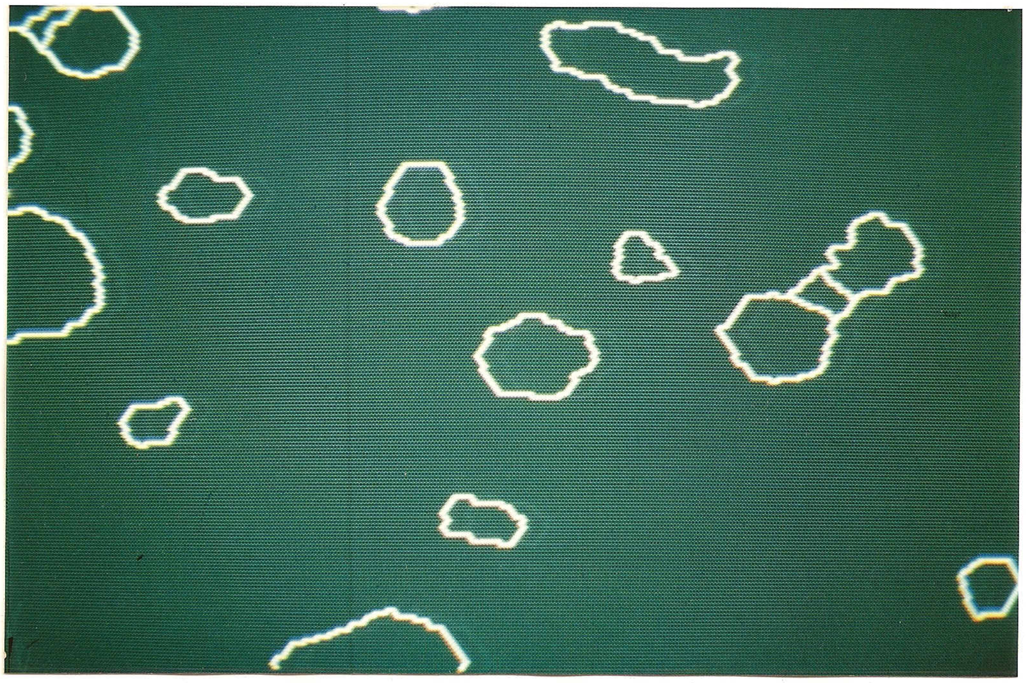


Figure 8 : Result of segmentation. Note the over-segmentation.

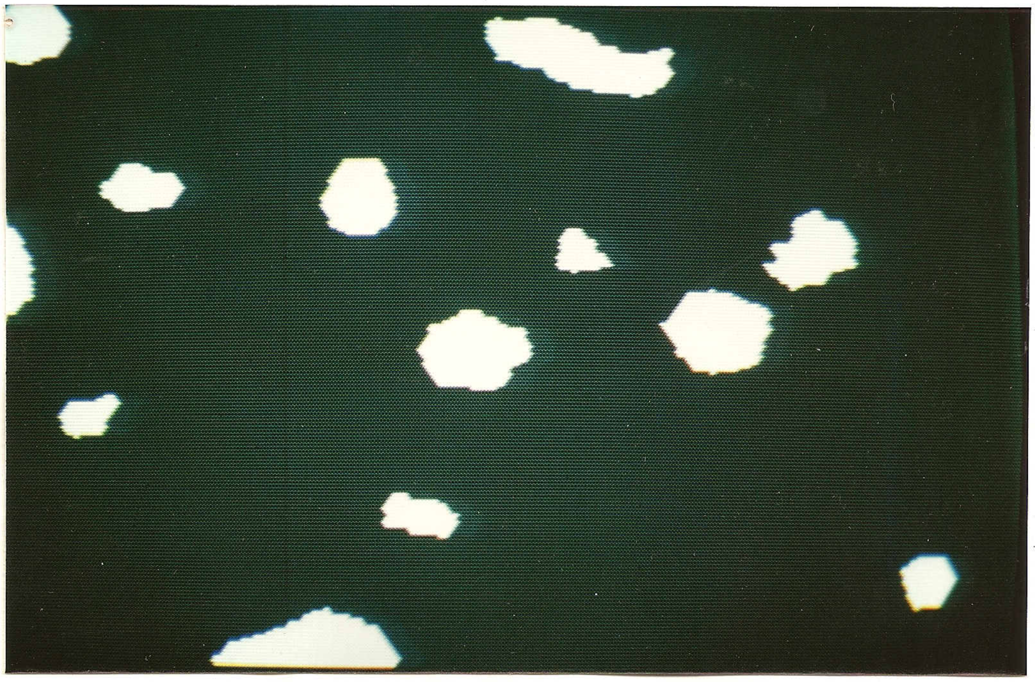


Figure 9 : Thresholding of the pores, after elimination of false boundaries.

The same algorithm can be used for more sophisticated pictures. As an example, Figures 10 and 11 show the result obtained in the segmentation of a S.E.M. micrograph of cleavage fractures in steel.

It is important to note that no parameter is introduced in the process. It is interesting to find that the algorithm when used with the picture of pores, is quite similar to a variable thresholding of the grey-tone function, the value of the parameter a depending on the coordinates of the picture-points.

Figure 10 : Scanning Electron Microscope micrograph of a fracture in steel.

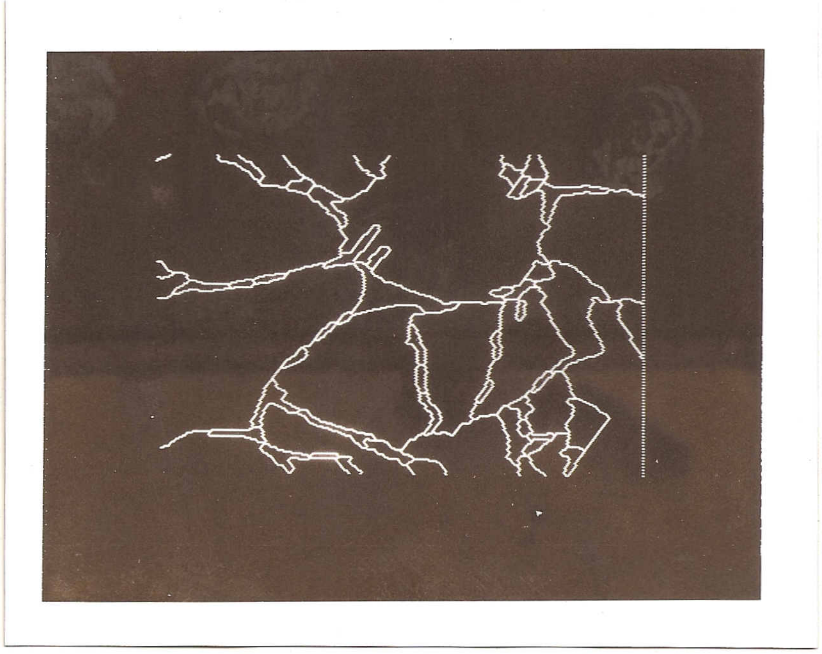


Figure 11 : Same picture after segmentation.

In brief, the method used, which is quite general, is the following :

- Given a function (chosen by the operator depending upon the problem to be solved), find the extrema (Minima or Maxima)

- Compute the zones of influence of these extrema.

- If necessary, suppress false signals by comparisons between extrema of different functions.

It is also possible to use both minima and maxima of a function to solve more refined problems.

THE SET-FUNCTION TRANSFORMATIONS

These transformations are not yet commonly used. But they seem to be very powerful for many problems of image analysis.

Let $\{X_\lambda\}_{\lambda \in R^+}$ be a sequence of decreasing sets :

$$0 \leq \lambda \leq \mu \Rightarrow X_\mu \subset X_\lambda$$

We can define a function h by

$$h(x) = \sup \{\lambda \in R^+ | \; x \in X_\lambda\}$$

and apply the previous algorithms to this new function.

In mathematical morphology, many set-set transformations are parametric : $\{T_\lambda\}_{\lambda \in R^+}$ (the parameter λ is generally the size of the structuring element). Many of them are also decreasing :

$$\forall X \in R^2 \quad 0 \leq \lambda \leq \mu \quad T_\mu(X) \subset T_\lambda(X)$$

Starting from a subset X of R^2, we can define, as before a function h by :

$$h(x) = \sup \{\lambda \in R^+ \;|\; x \in T_\lambda(X)\}$$

What is the utility of this operation ? The main advantage is to be able to look at some problems from a different angle. As an example, let us consider the problem of the segmentation of balls (for this problem, see [2]). This problem can be seen as a pure set-set transformation problem. But it is also possible to solve it by set-function and function-set transformations. (The final result is the same in each case !).

Let X, subset of R^2, be a finite union of disks :

$$X = \bigcup_{p=1}^{n} B(x_p, \lambda_p)$$

The function h introduced here is the euclidian distance function of the points $x \in X$ to the boundary of X :

$$h(x) = \begin{cases} d(x \, , \partial X) & \text{if } x \in X \\ 0 & \text{if not} \end{cases}$$

Note that $h(x) = \sup\{\lambda \in R^+ | x \in X \ominus \lambda B\}$

Let us now compute the maxima of this function, and build their zones of influence. We obtain a new set which is the union of the segmented disks and the background. (Figure 12).

In the case where the disks of the population can be separated, the maxima of h are none other than the centers of the disks. Their zones of influence have for their boundaries either the boundary ∂X of the population, or the lines of contact of the disks. When assimilating h to a relief function, these later lines are the pass lines of the surface of the relief.

Notice the main difference between the two methods : the first one (set-set transformation) defines the contact

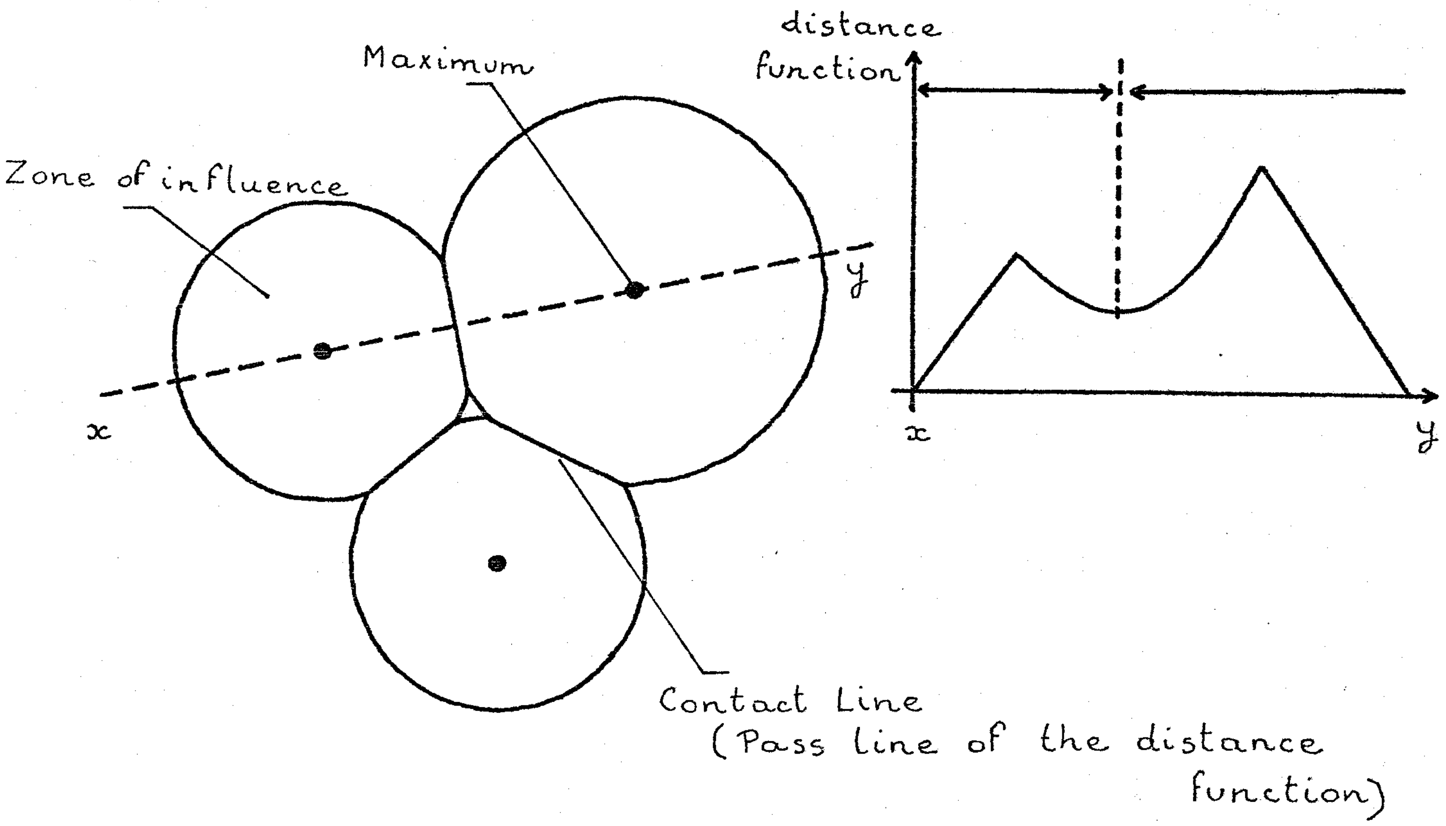


Figure 12 : Balls segmentation - Principle of the method.

lines, while the second one finds every point of X which is not a contact line.

There exist other applications of this algorithm. They have not all been explored, because the family of decreasing transformations is very large.

All these methods are operational. The algorithms can be performed with a texture analyser. When combined with a numeric fast-access memory, the analysis of grey-tone images could be every fast, even when sophisticated algorithms are used.

In this paper, we tried to fill in the gap between the two great domains of image analysis : the space of functions and the space of sets. This note is the first approach to problems of changing from one space to the other. It is for this reason that we did not present many algorithms and applications, but on the contrary, tried to lay out a methodology and to explain the philosophy of such transforms.

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