RESIDUES

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Introduction: properties of morphological transformations

- Increase

$$X \subset Y \Rightarrow \psi(X) \subset \psi(Y)$$

- Extensivity/anti-extensivity

$$X \subset \psi(X)$$

$$\psi(X) \subset X$$

- Idempotence

$$\psi(\psi(X)) = \psi \circ \psi(X) = \psi(X)$$

Residual operators are not increasing. Many of them are idempotent and anti-extensive. Some others fulfil another property: homotopy

General definition of a residue

An elementary residual operator is defined by the difference of two operators

- Set difference in binary morphology

$$r = \psi \backslash \zeta \quad \zeta \subset \psi$$

- Algebric difference for functions

$$r = \psi - \zeta \quad \psi \geq \zeta$$

r is called <u>residue</u>, ψ and ζ <u>primitives</u>

There exists differents ways to use these residues, in particular when they are generated by families $\{\psi_i\}$ and $\{\zeta_i\}$ of primitives

Lecture contents

- Set residues
  - Ultimate erosion
  - Skeleton by maximal balls
  - conditional bissector
  - Geodesic residues
- HMT, thinnings, thickenings
  - Definitions
  - Homotopic thinnings
  - Skeletons
- Numerical residues
  - Extension of the concept to functions
  - Ultimate opening
  - Critical balls
  - Quasi-distance

Part one

Residues in binary morphology

Operators based on the difference of two families of operators depending on a parameter i:

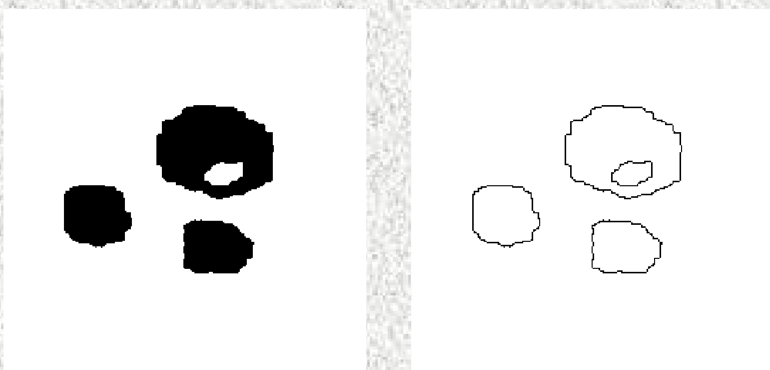
$$r_i = \psi_i \setminus \zeta_i \quad , \psi_i \geq \zeta_i \qquad\qquad \theta = \bigcup_{i \in I} r_i$$

<u>Trivial example</u>:

• If I = {1} (the family of operators is reduced to a single pair), taking ψ = I and $\zeta = \varepsilon$ (elementary erosion), we have:

r = I \ ε, interior contour of the set

• If $\psi_i = \varepsilon_i$ and $\zeta_i = \varepsilon_{i+1}$, $\theta = I$



(The intersection of residues gives also a residual transform, but this transform is elementary, $\psi \setminus \zeta$)

Residues in set morphology

<u>More refined examples:</u>

• Ultimate erosion

$$\psi_i = \varepsilon_i \,;\, \zeta_i = \gamma_{rec}\left(\varepsilon_i\right)$$

• Skeleton by maximal balls

$$\psi_i = \varepsilon_i \,;\, \zeta_i = \gamma\left(\varepsilon_i\right)$$

• Conditional bissector

$$\psi_i = \varepsilon_i \,;\, \zeta_i = \delta^l_{\varepsilon_i} \circ \varepsilon_k\left(\varepsilon_i\right)$$

These operators are composed of a doublet:

• The set transformation θ

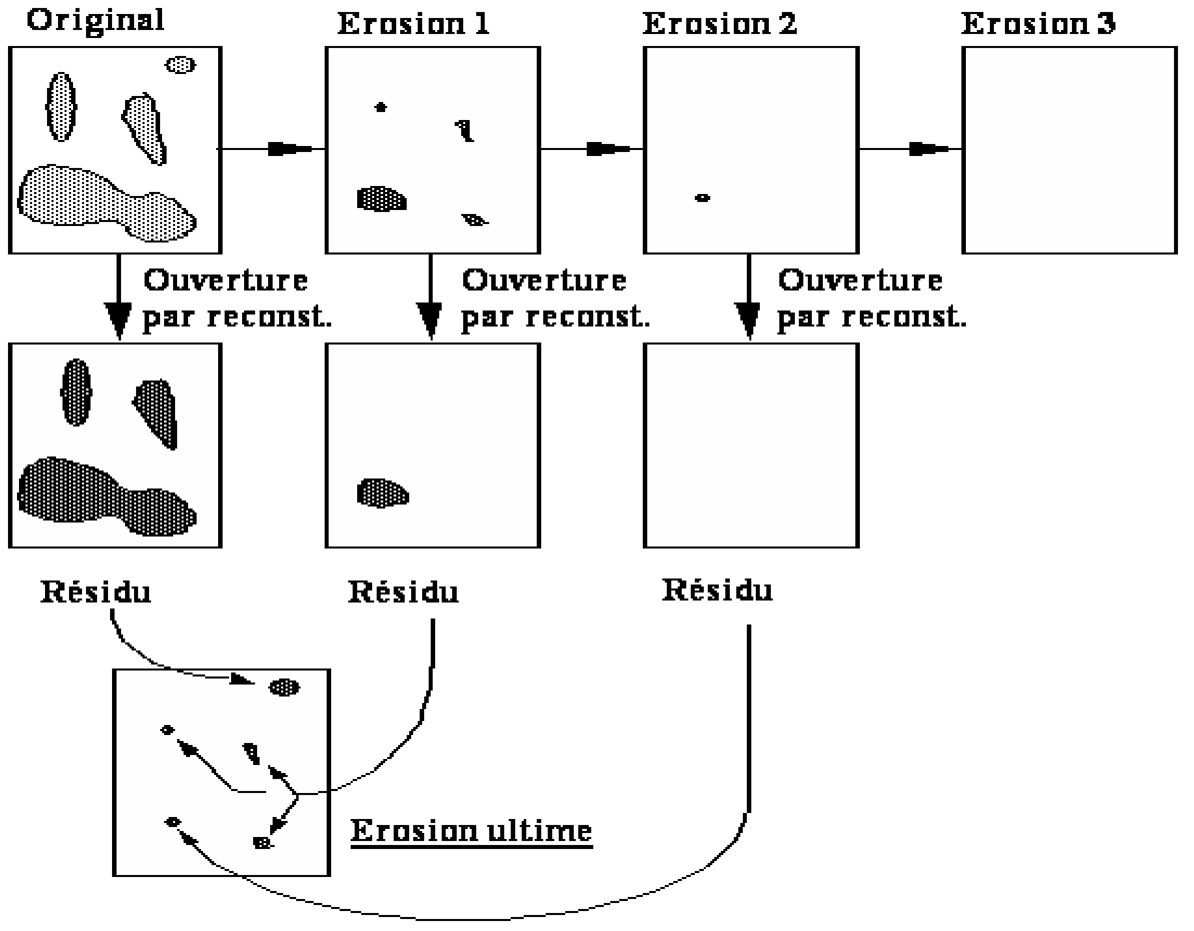
• An associated function q: q(x) = i + 1 if and only if $x \in r_i$

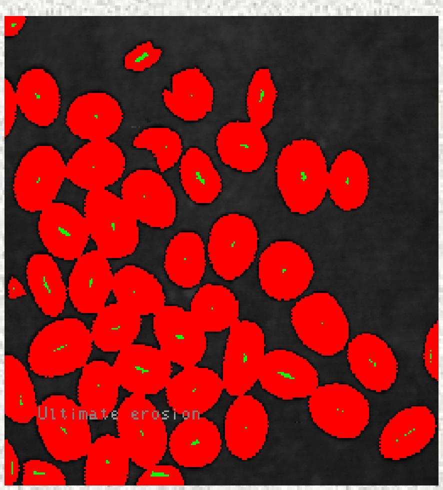
(We add 1 to q (x) in order to obtain a strictly positive value on the function support)

Ultimate erosion

The two families of operators are made of the successive erosions of the set and of the opening by reconstruction of each eroded set

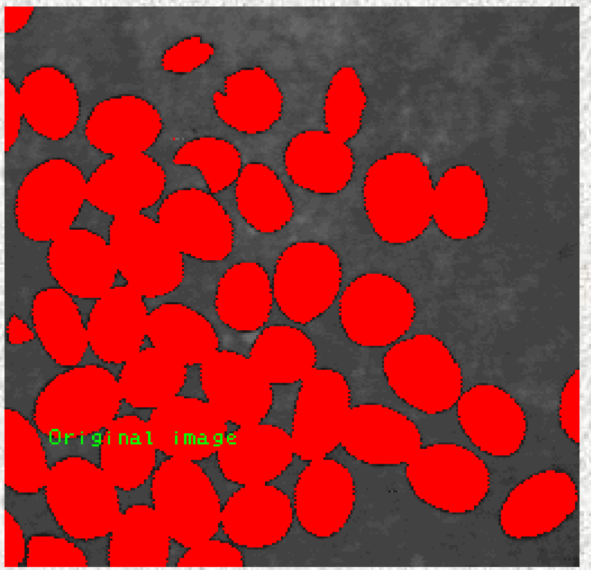
$$\psi_i = \varepsilon_i \,;\, \zeta_i = \gamma_{rec}(\varepsilon_i)$$

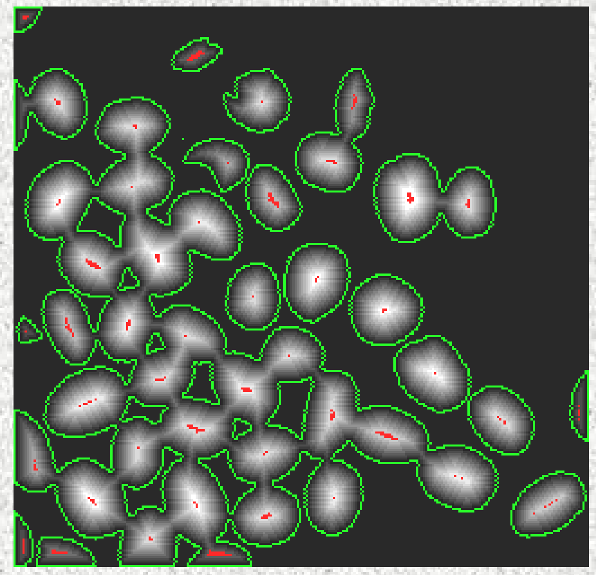




Ultimate erosion and distance function

The distance function is built by piling up the successive erosions of X



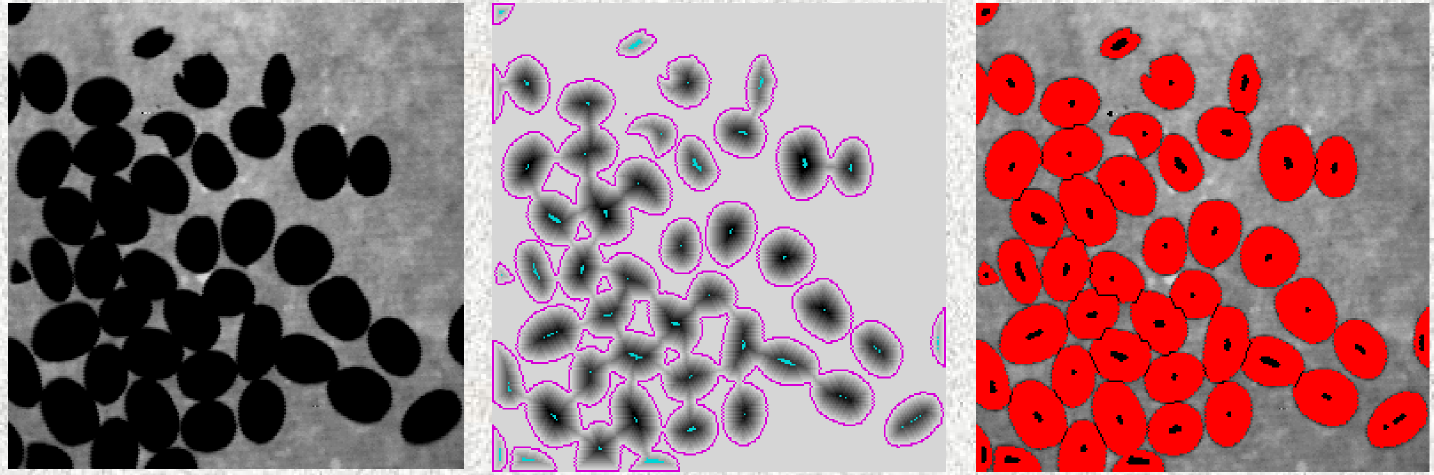


The ultimate erosion corresponds to the maxima of the distance function

The associated function q gives (up to a unit value) the size of the erosion corresponding to the appearance of each connected component of the ultimate erosion

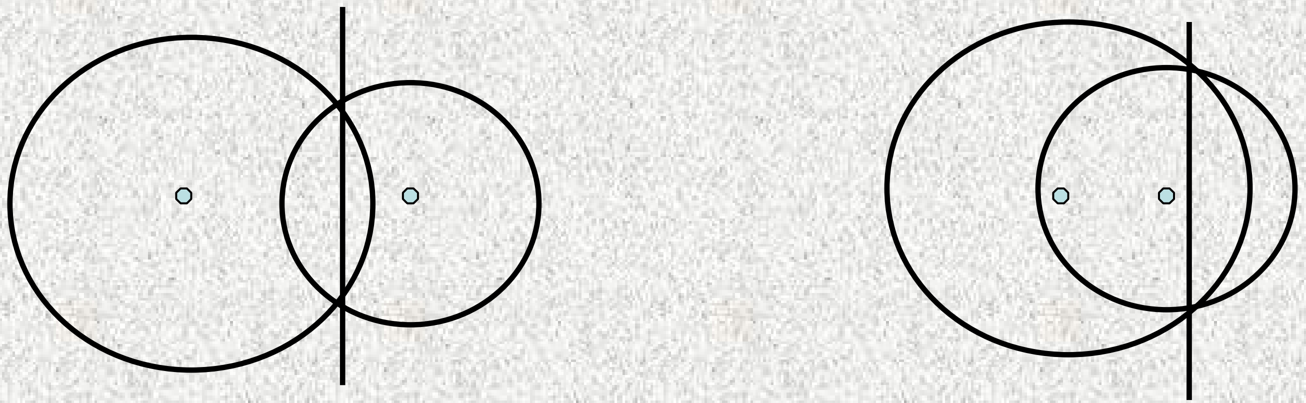
Use of the ultimate erosion

- Generation of markers for segmentation



The watershed transformation of the inverted distance function is computed. The marker set is made of the maxima of the distance function.

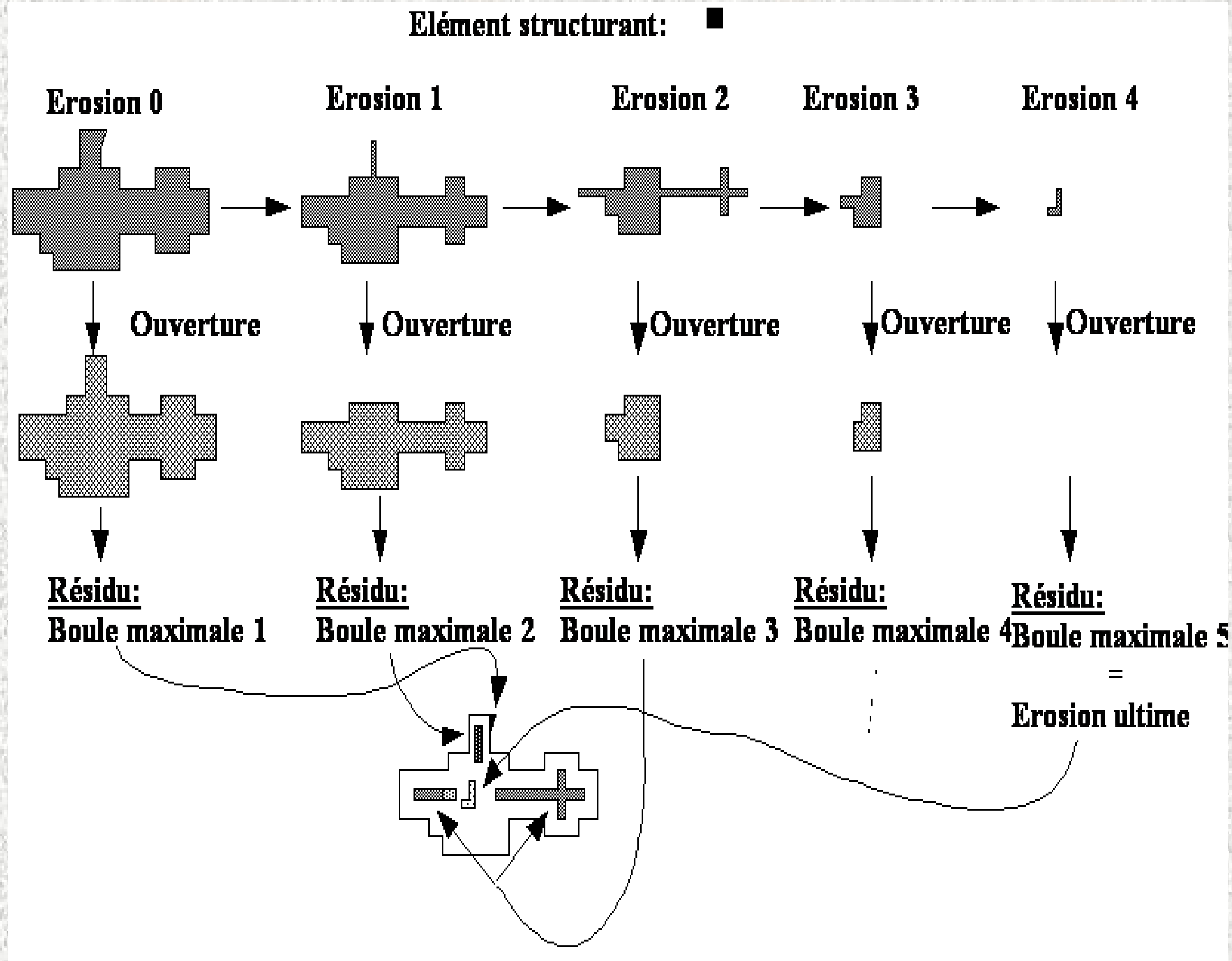
- Problem of the separation of two overlapping disks

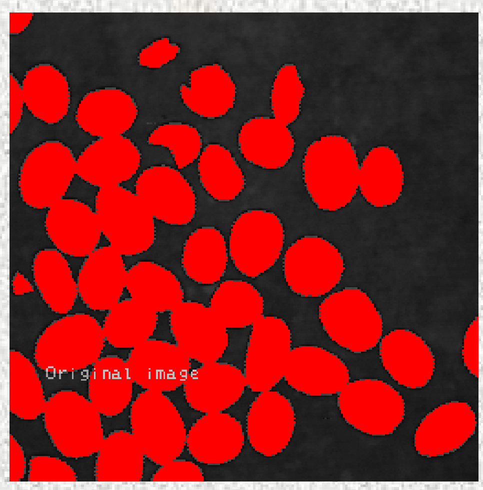


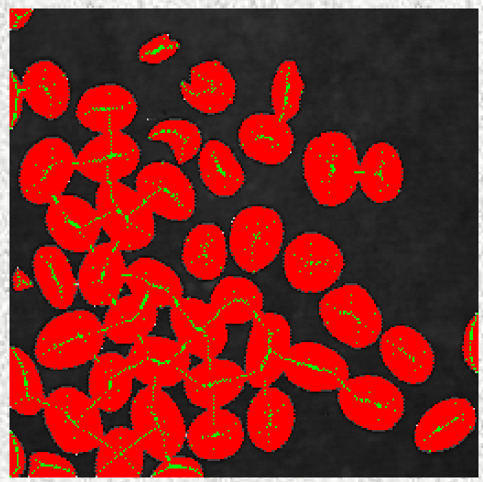
Skeletons by openings

(or skeletons by maxima balls)

$$\psi_i = \varepsilon_i \, ; \zeta_i = \gamma\left(\varepsilon_i\right)$$

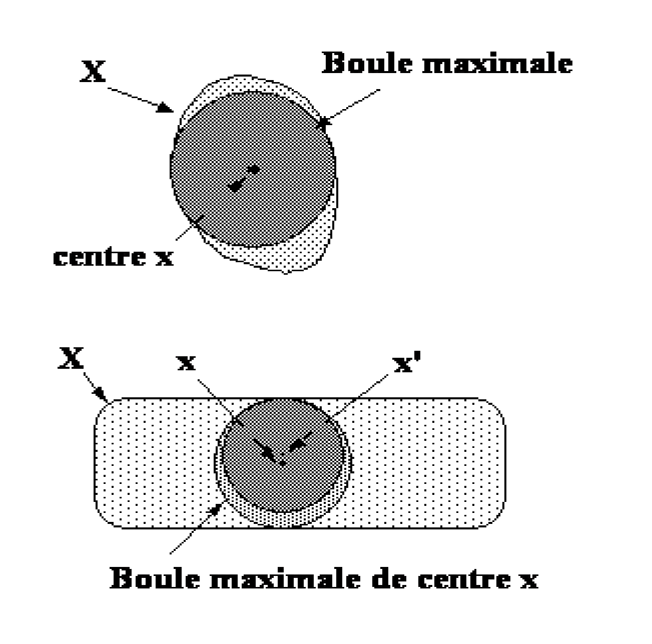






Maxima balls

A ball $B_n(x)$ of size n and centered in x is maximal with repect to the set X, if there exists no other index k and no other center y such that:

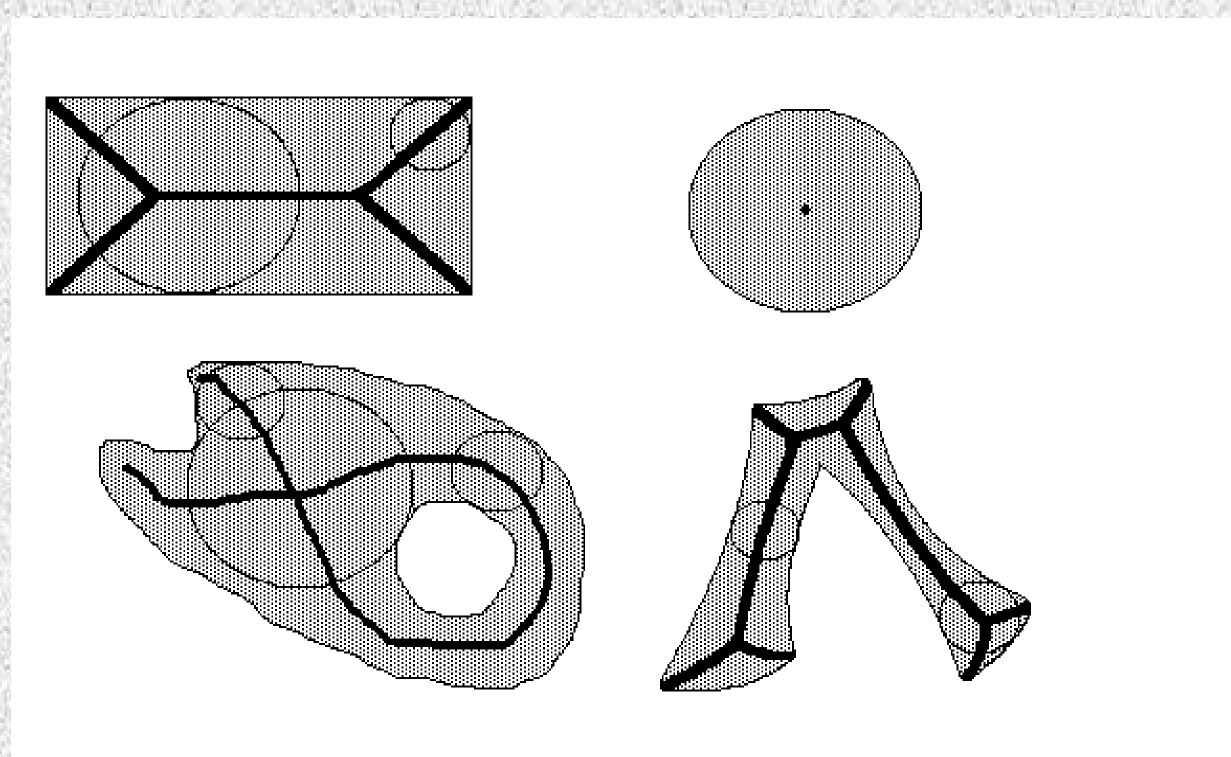


$$B_n(x) \subset B_k(y) \subset X \quad n \leq k$$

The skeleton of a set X according to a family of balls $\{B_n\}$ is the locus of the centers of all its maximal balls:

$$S(X) = \{x \in X : \exists B_n(x) \text{ maximal}\}$$

Maximal balls skeleton

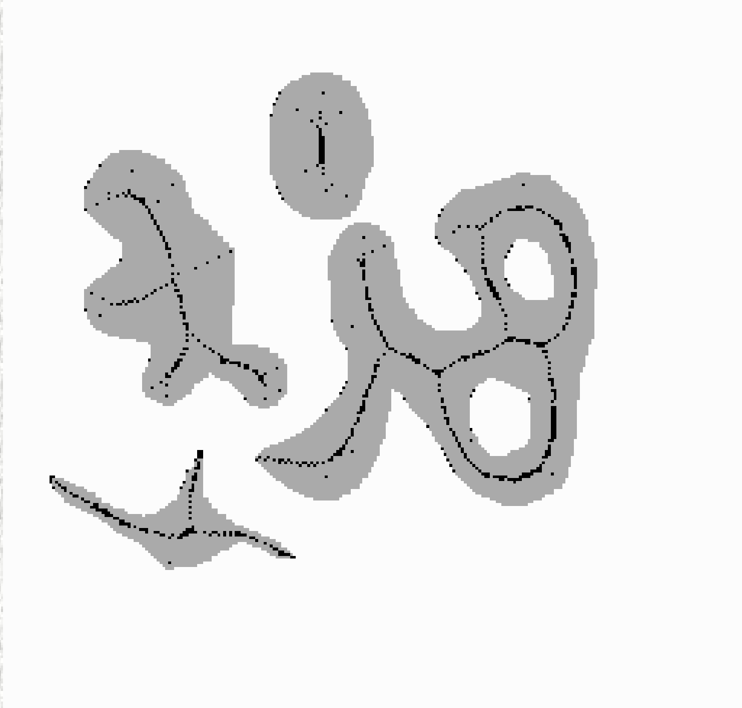


At each point x of the skeleton, we can associate a function q(x) taking the size of the maximal ball centered in x. This function is called extinction function or quench function

$$q(x) = n : x \in S(X), B_n(x) \text{maximal}$$

Skeleton by maximal balls and by openings

It can be proved that both skeletons are identical.



$$S(X) = \bigcup_{i \in N} \left[\varepsilon_i(X) \setminus \left(\gamma \circ \varepsilon_i(X) \right) \right]$$

- Each residue r_i (also denoted S_i) is the locus of the centers of the size i maximal balls
- The maximal balls are defined on homogeneous families of balls obtained par successive dilations of the elementary ball B_0

Properties of the maximal balls skeleton

- The maximal balls skeleton of a connected set is not connected (generally speaking, it is not proven)
- The skeleton is anti-extensive and idempotent. It is not increasing:

$$S(X) \subset X$$

$$S(S(X)) = S(X)$$

$$X \subset Y \textit{ does not imply } S(X) \subset S(Y)$$

However the following property is true:

$$S(\varepsilon^n(X)) \subset S(X), \forall n \geq 0$$

Properties of the maximal balls skeleton

Contrary to most of the morphological operators, skeletonization is a reversal operation.

The set X, together with its erosions, its dilations and its openings can be built from the skeleton and the quench function:

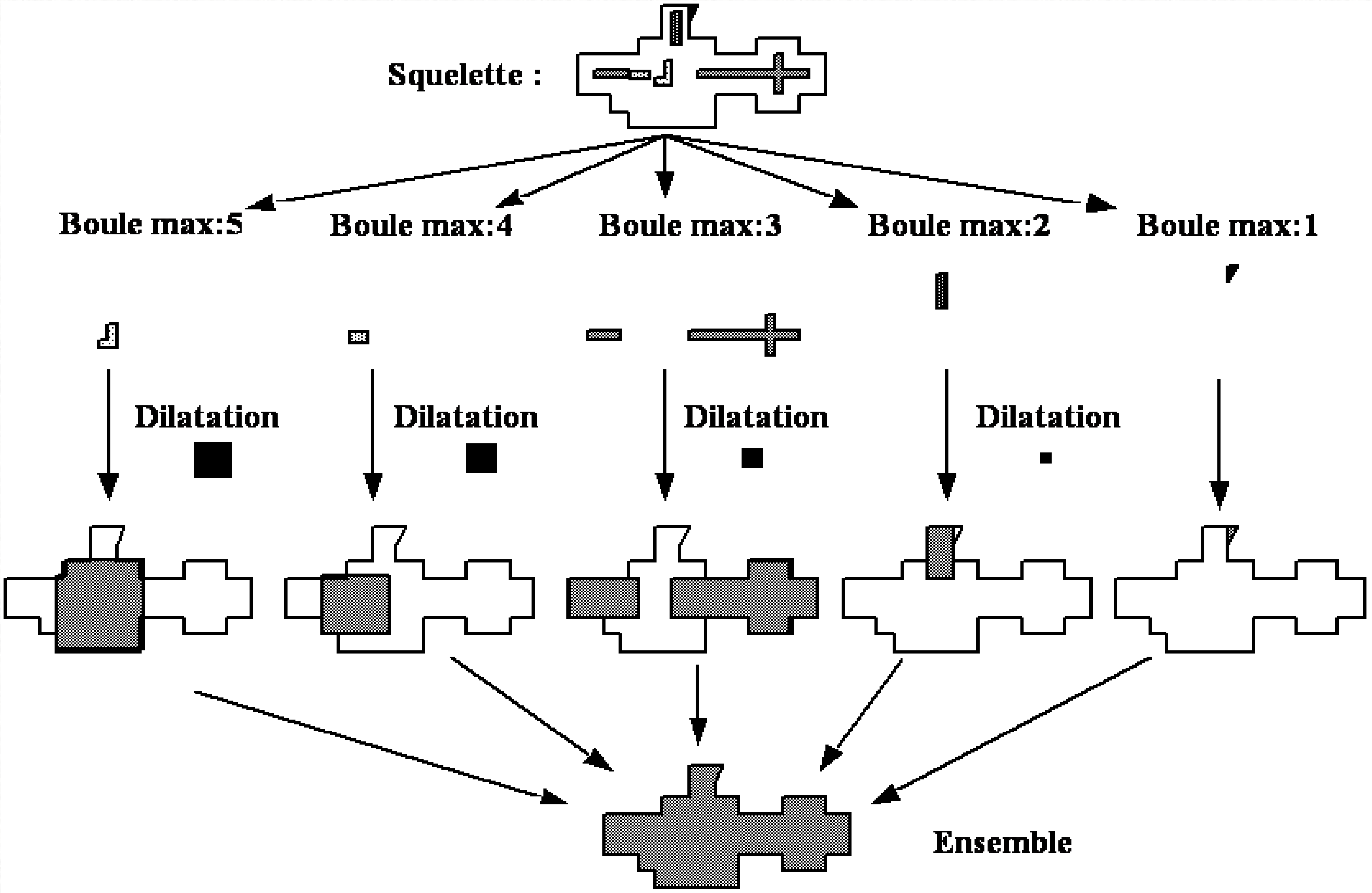
$$X = [X \setminus \gamma(X)] \cup \gamma(X) = S_0(X) \cup \delta(S_1(X)) \cup \delta^2(S_2(X)) \cup \ldots$$

And finally:

$$X = \bigcup_{i \in N} \delta^i(S_i(X))$$

This reversible transformation produces another representation of X: X is equivalent to the doublet [S(X),q]

Reversibility of the skeleton



Properties (continued)

The datum of S(X) and q allows also to build:

- The erosions of X

$$\varepsilon^n(X) = \bigcup_{i \geq n} \delta^{i-n}(S_i(X))$$

- The dilations of X

$$\delta^n(X) = \bigcup_{i \in I} \delta^{i+n}(S_i(X))$$

- The openings of X

$$\gamma^n(X) = \bigcup_{i \geq n} \delta^{i}(S_i(X))$$

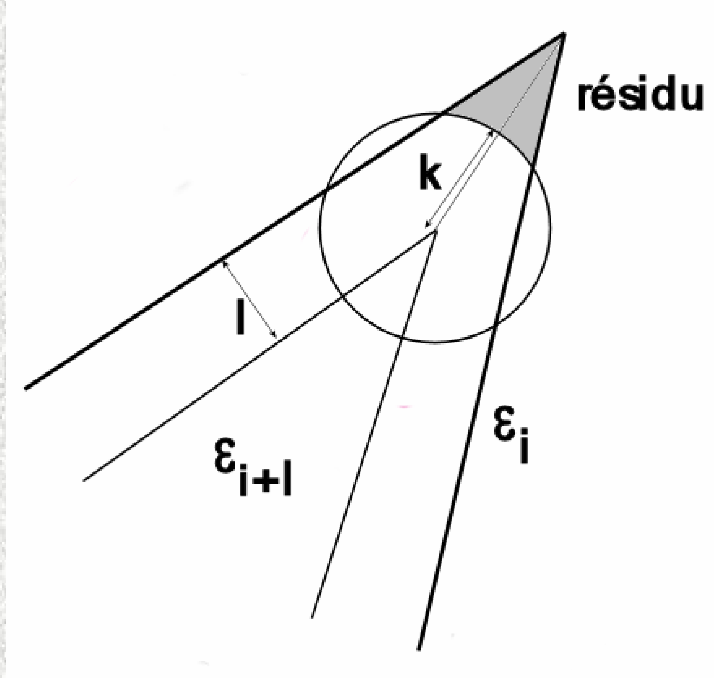
(closings cannot be built – *why?*)

The ultimate erosion is always a subset of the maximal balls skeleton. The ultimate erosion corresponds to the centers of the ultimate maximal balls.

Conditional bissector

The conditional bissector is the residue between the family of erosions of size l and the geodesic dilation of size k (k>l) of these erosions:

$$\psi_i = \varepsilon_i ; \zeta_i = \delta^k_{\varepsilon_i} \circ \varepsilon_l \left(\varepsilon_i \right)$$



Apperance of a residue when k < l/sin(α)
l/k > sin(α) = q'(x)

The conditional bissector is a threshold on the derivate of the quench function

The conditional bissector allows a more accurate identification of the set components than that obtained with the ultimate erosion

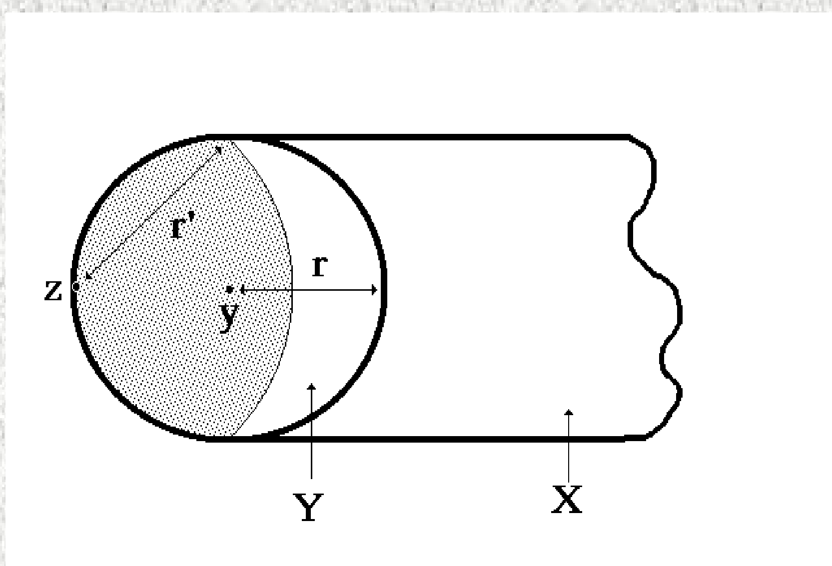
Geodesic residues

All the residual operators defined in an euclidean context can be transposed in geodesic spaces:

- The geodesic balls can be defined from the geodesic distance

$$B_X(x,r) = \{y \in X : d_X(x,y) \leq r\}$$

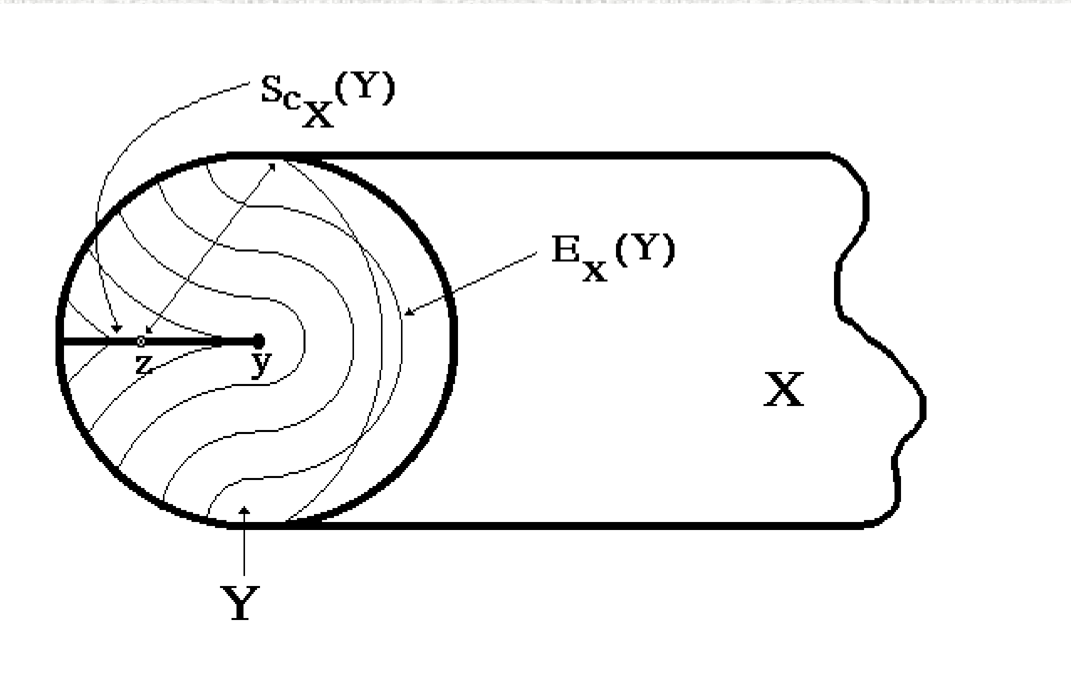
- The geodesic maximal balls have the same definition as the maximal euclidean balls (mutatis mutandis). Nevertheless, some traps must be avoided…



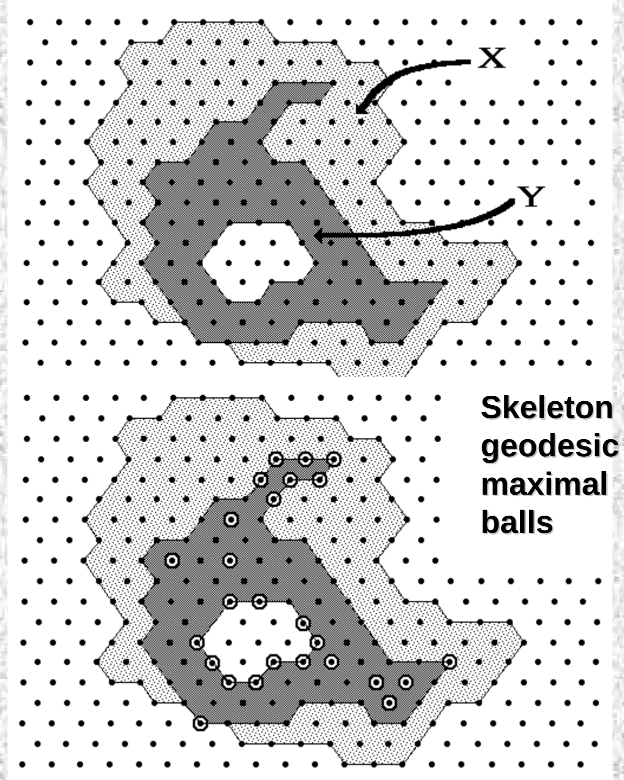
Geodesic skeleton

The skeleton by geodesic maximal balls $S_X(Y)$ of a set Y included in the geodesic space X is defined as:

$$S_X(Y) = \bigcup_{i \in N} \left[\varepsilon_X^i(Y) \setminus \left(\gamma_X \circ \varepsilon_X^i(Y) \right) \right]$$



Construction of the geodesic skeleton



Skeleton by geodesic maximal balls

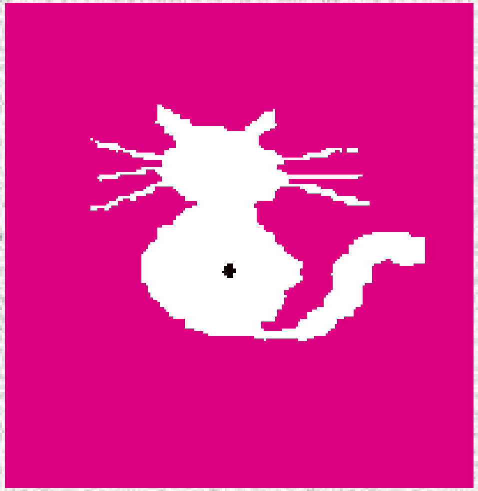
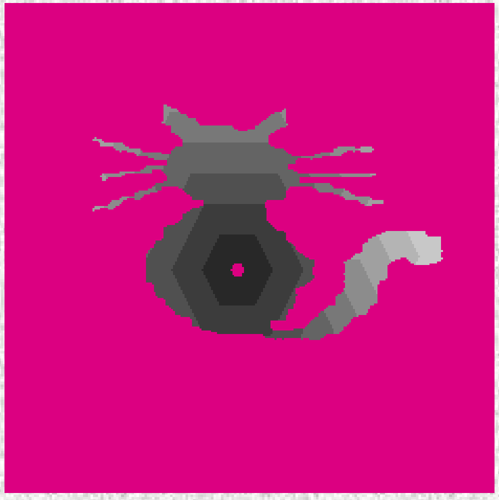
Extremities of a particle

The geodesic ultimate erosion can be used to extract the extremities of a simply connected particle

• It is used with a *centroïd C* (obtained by a thinning Dthin for instance, see below)

• The extremities of the particle are then defined as the geodesic ultimate erosion, in X, of the set Y = X\C

$$Extr(X) = \bigcup_{i \in N} \left[\varepsilon_X^i(Y) \setminus \left(\gamma^{rec}\left(\varepsilon_X^i(Y); \varepsilon_X^{i+1}(X) \right) \right) \right]$$

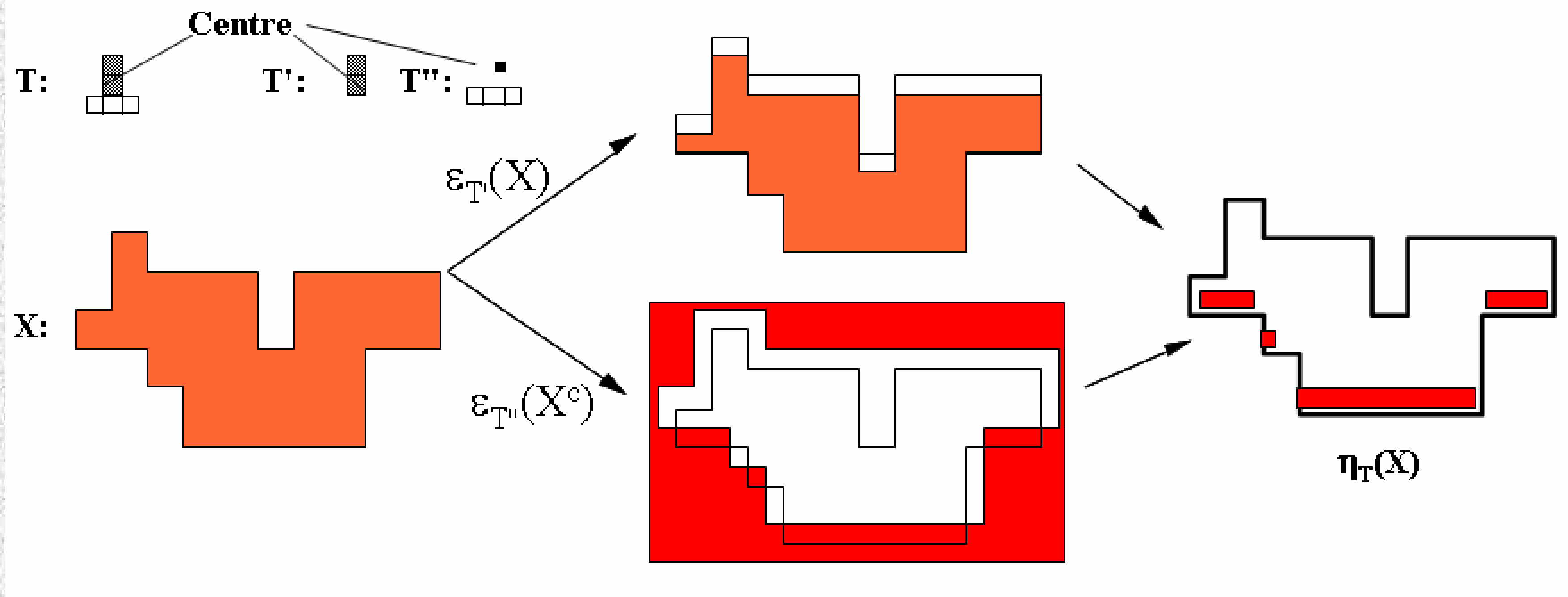




Part two

The Hit-or-Miss transform

The Hit-or-Miss transform (HMT) η_T extends at the same time the erosion and the dilation, by using the doublet of disjoint structuring elements T= {T',T"}

$$\eta_T(X) = \{x : T''(x) \subset X^c \text{ and } T'(x) \subset X\} = \varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)$$

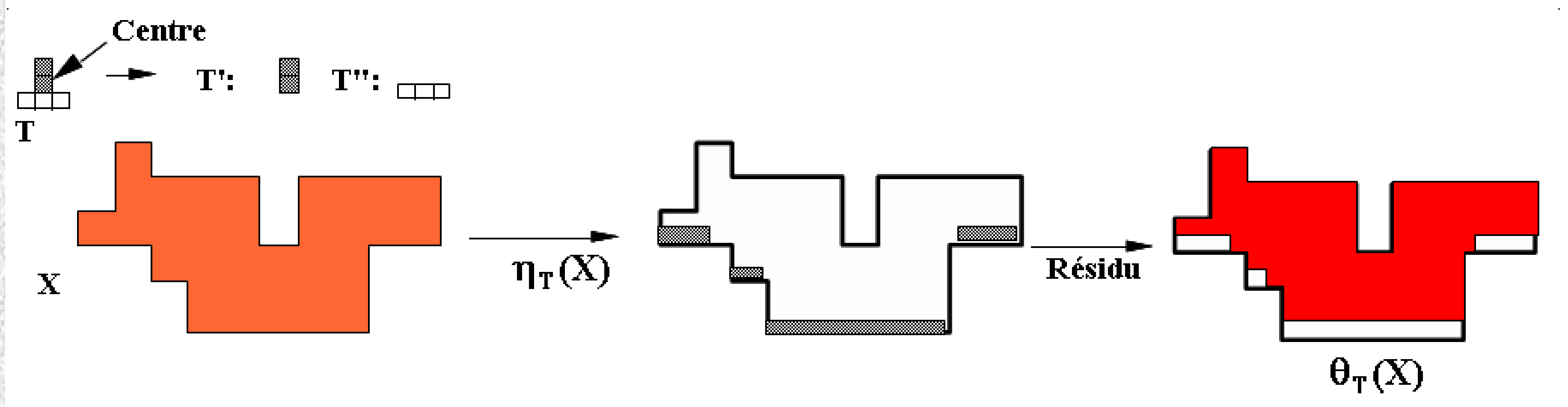


The HMT is a residue: $\eta_T(X) = \varepsilon_{T'}(X) \setminus \delta_{T''}(X)$

Thinning, thickening

The thinning θ_T is the residue between the initial set and its HMT transform :

$$\theta_T(X) = X \setminus \eta_T(X) = X \setminus \left[\varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)\right]$$



The thickening ξ_T is defined by duality with the complementary set:

$$\xi_T(X) = X \cup \eta_T(T) = X \cup \left[\varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)\right]$$

Properties

- The thinning with T=(T',T'') is dual for the compementation of the thickening with T*=(T'',T') :

$$\theta_T(X) = \left[\xi_{T*}(X^c)\right]^c$$

- Any thinning is anti-extensive, and any thickening extensive. In order that these operations be different from the identity, the origin of T must belong to T' for the thinnings or to T'' for the thickenings.

- Simple thinnings do exist. However, the most interesting ones fulfil a specific topological property, the homotopy.

Combination of thinnings

Thinnings can be combined in two different ways:

- Sequentially:

Sequence $\{T_i\}$ of structuring elements

$$\theta_{T_n} \circ .. \circ \theta_{T_i} \circ .. \circ \theta_{T_1}$$

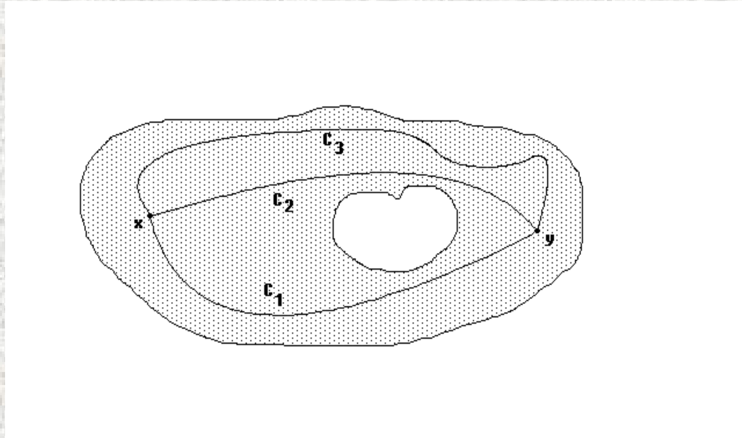
- By unions:

Family $\{T_i\}$ of structuring elements

$$I \setminus \left(\bigcup_i \eta_{T_i} \right) = I \cap \left(\bigcap_i \eta_{T_i}^c \right) = \bigcap_i \left(I \setminus \eta_{T_i} \right) = \bigcap_i \theta_{T_i}$$

Homotopy

Property linked to the deformation of paths and loops



Intuitive definition:

• Two paths (or loops) of an arc-connected space X are homotopic if it is possible to continuously apply one on the other

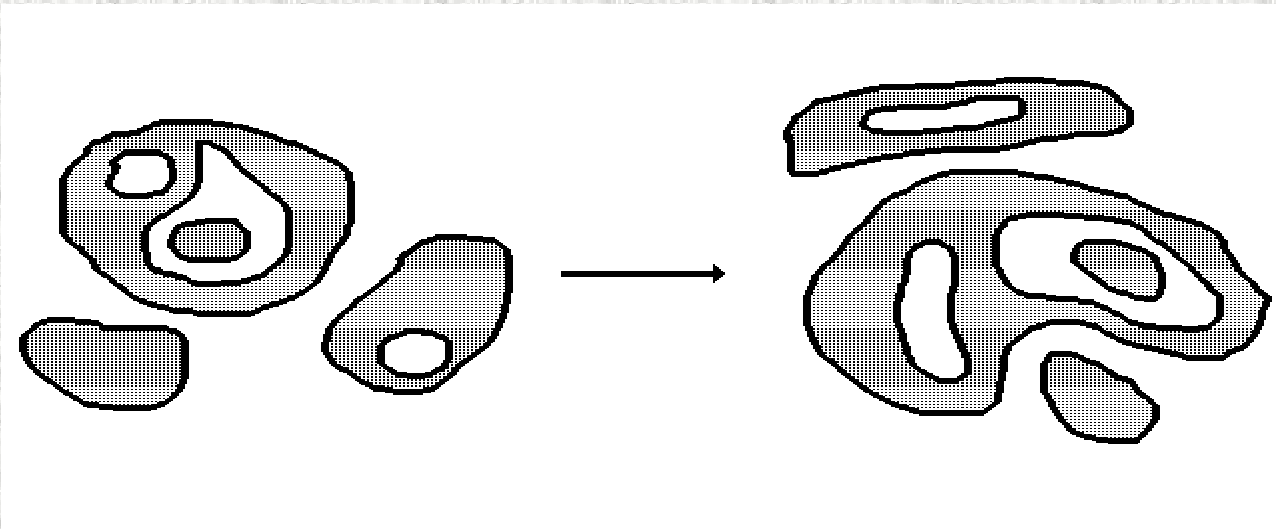
• Homotopy is an equivalence relation

• Two arc-connected spaces are homotopic if it is possible to apply one on the other by a continuous sequence of deformations)

Homotopic transformations

A transformation ψ is homotopic if the initial set X and the transformed set Y= ψ (X) are homotopic, that is if there exists a bicontinuous transformation from one to the other such that:

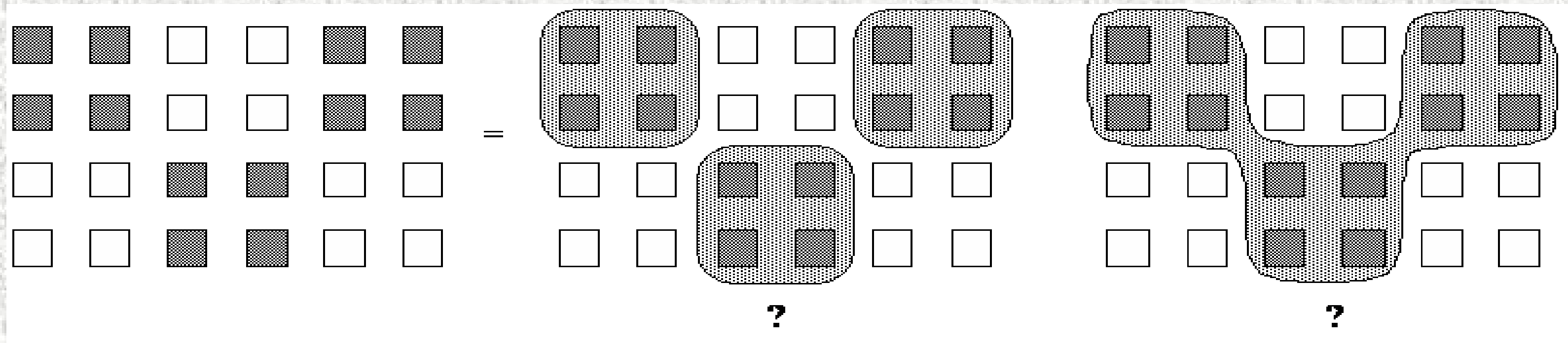
- each connected component of X contains the same number of holes as its transformed set in Y,
- each hole of X contains the same number of connected components as its transformed set in Y.



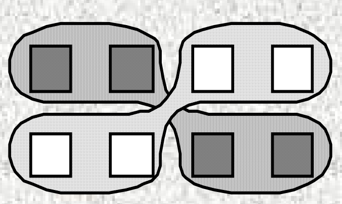
Homotopy describes the organisation (hierarchy) of the connected components and of the holes.

Connectivity of digital spaces

In digital case, the definition of homotopy depends on the definition of an arc-connectivity. However it is not simple to decide how many connected components appear in the following figure:



Connection rules applied to the diagonal configurations must be chosen so that the connectivity be defined on a planar graph.



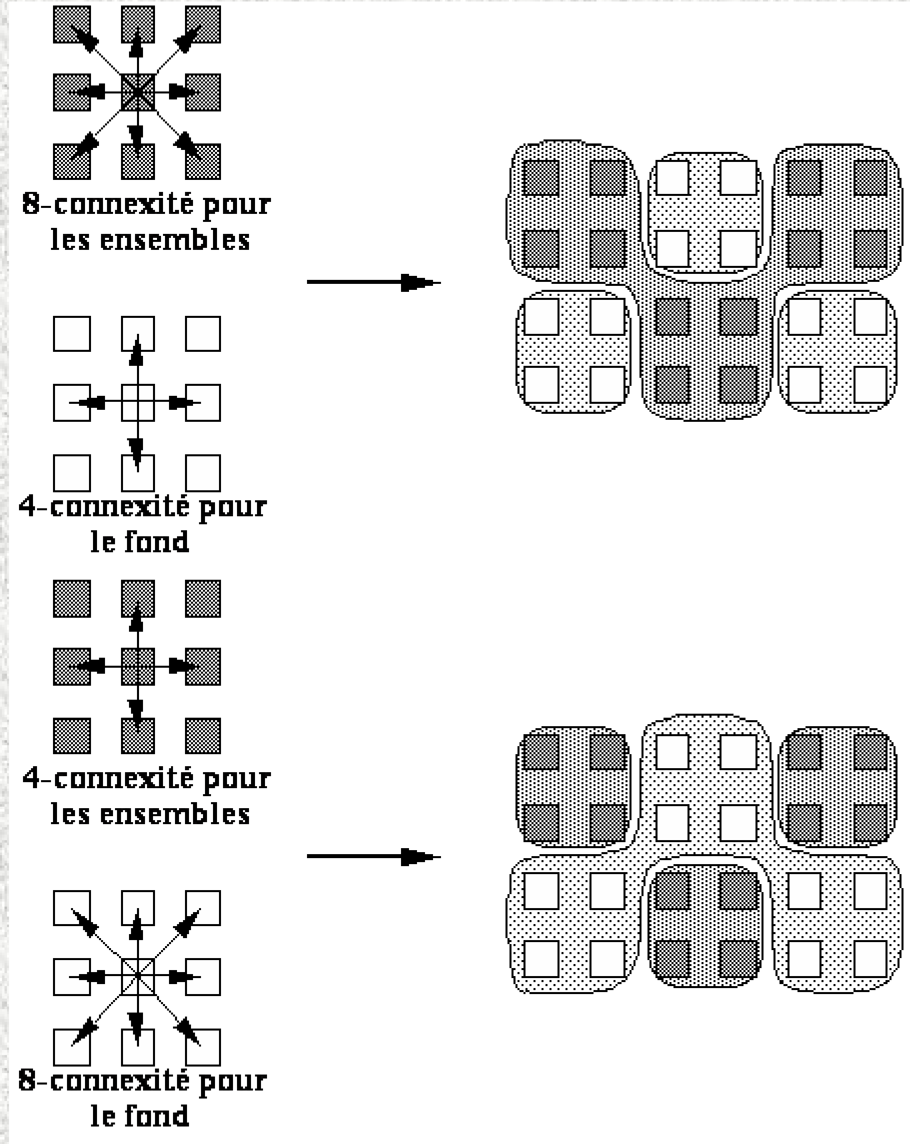
Connectivity and square grid

On the square grid, two kinds of connectivity are available:

• 8-connectivity for the sets and 4-connectivity for the background

• 4-connectivity for the sets and 8-connectivity for the background

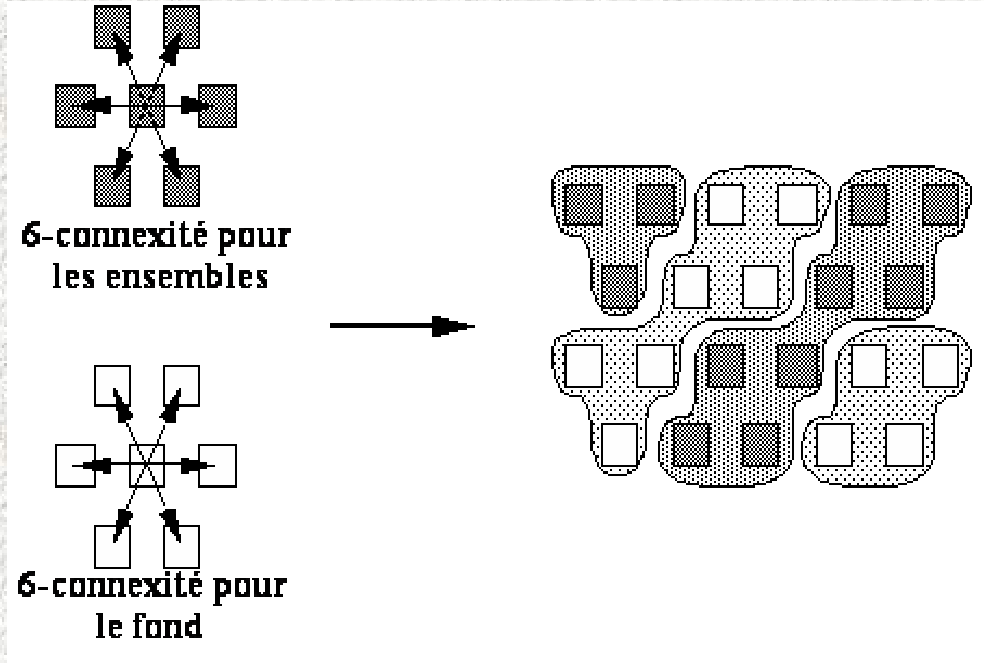
This structure is very diadvantageous and complicate the topological operators



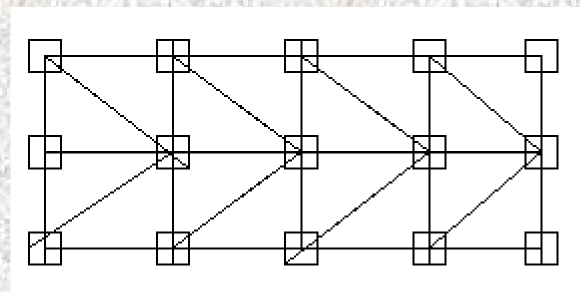
Connectivity on the hexagonal grid

On the hexagonal grid, a same 6-connectivity can be defined for the objects and for the background

Simplification of the algorithms (less neighbours and auto-dual configurations)



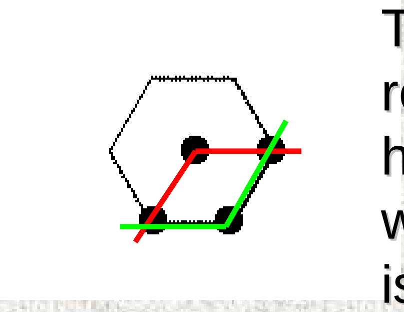
In practice, the hexagonal grid can be built from the square one.



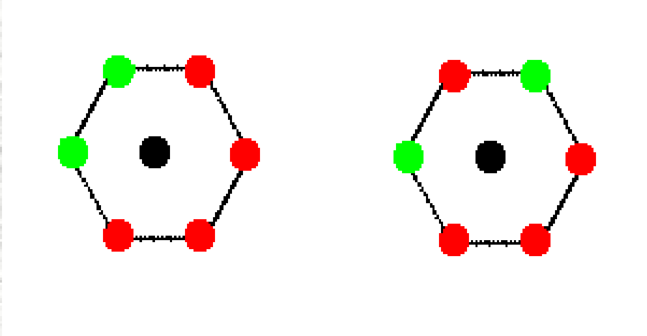
Homotopic thinnings

All the possible structuring elements configurations T defined on the elementary ball (up to rotations and symmetries) can be analysed and those which preserve the homotopy can be determined

Case of the hexagonal grid



The red path can be replaced by the homotopic green one when the central point is removed



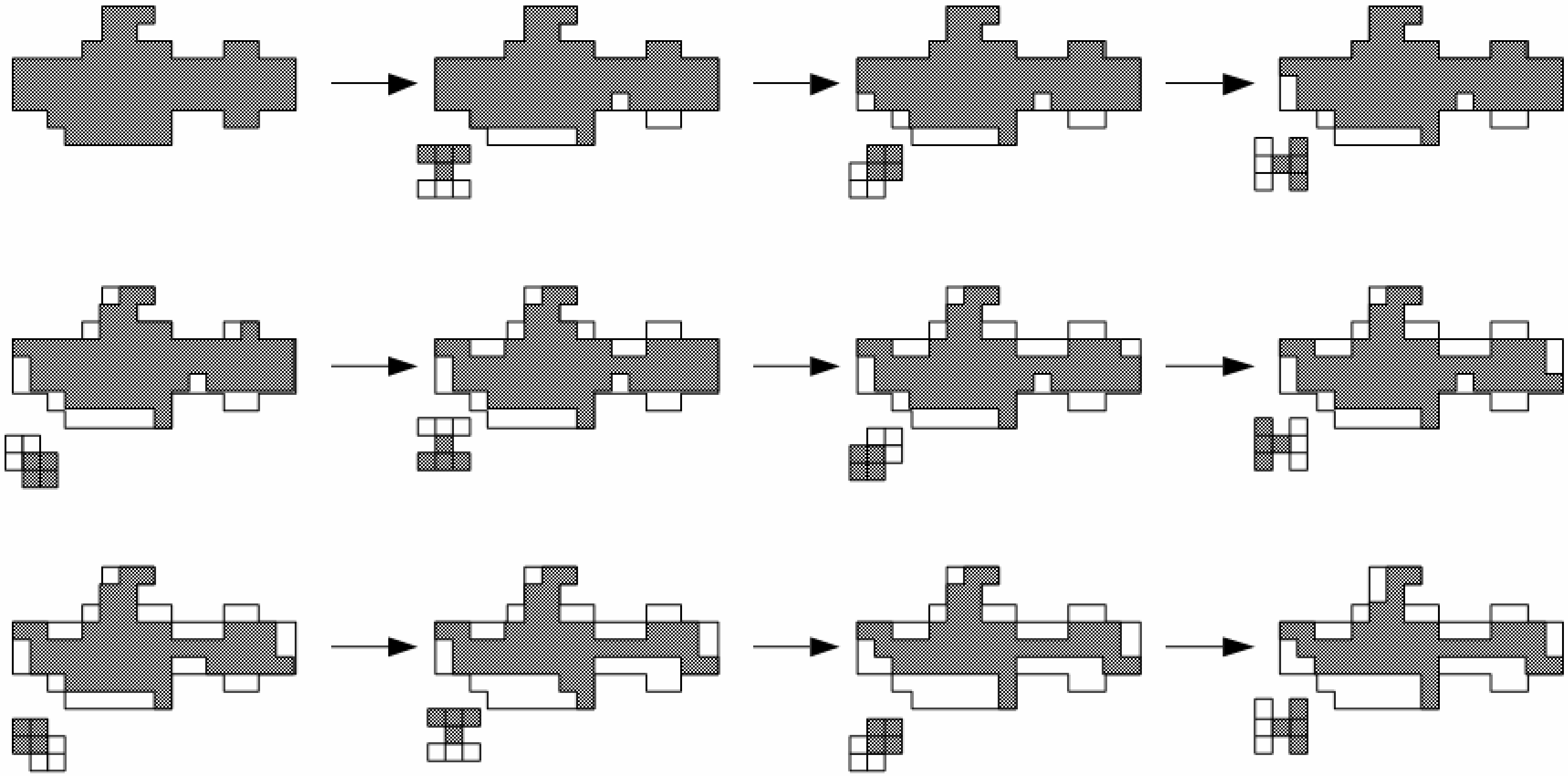
Only the configurations where T' and T'' are simply connected produce homotopic thinnings

Structuring elements L, M and D

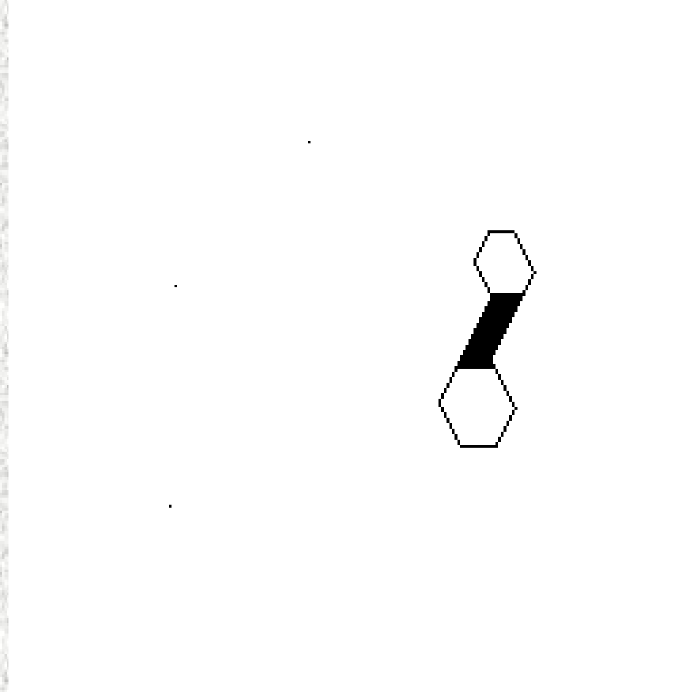
On the hexagonal grid, by grouping some configurations, we define three families of structuring elements producing homotopic thinnings provided that they are used sequentially

1 1	1 1	1 •
• • •	• • 1	• • 0
0 0	0 •	0 0
L	**M**	**D**

Example of sequential thinning

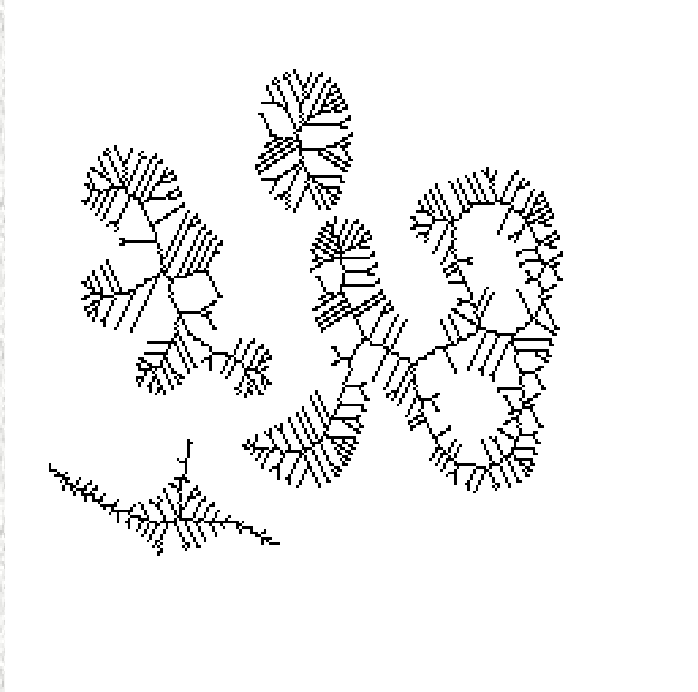


Thinning with D



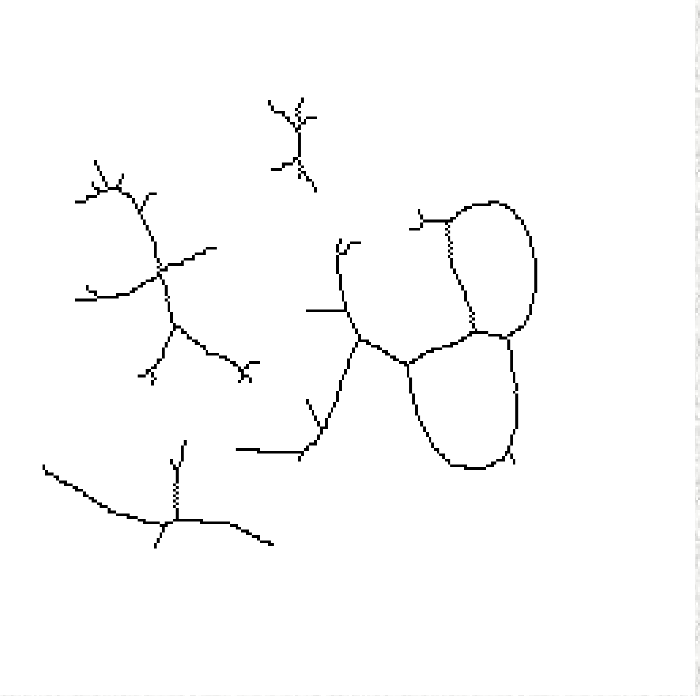
- D is used to emphasize the simply connected components (they are homotopic to a point)
- It is used also to build a centroid of the connected component (it is not the geodesic center)

Thinning with M

- This thinning is seldom used (the result is too chaotic!)
- This thinning is mainly used in a geodesic context (geodesic SKIZ)

L Skeleton



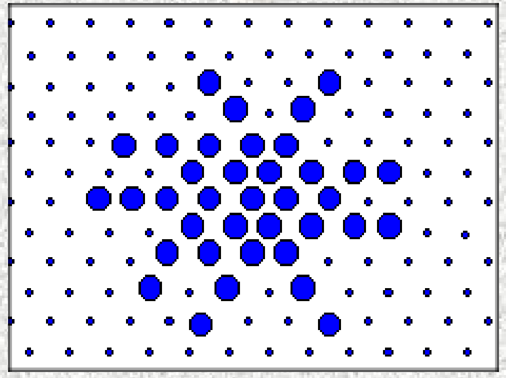
This thinning is often called skeleton because:

- It is connected
- It is built with a propagation algorithm
- The result « looks like » a skeleton

However, it has many defects

Biases of the L skeleton

- It does not contain the maximal balls skeleton
- The final result varies with the sequence of structuring elements used
- this skeleton can be very thick!
- The final result is often biased and the biased may be very important



Enhancements

Various enhancements have been proposed. They aim at including the maximal balls skeleton in the skeleton obtained by homotopic thinnings

- Use of intersections of homotopic thinnings (allows to define isotropic thinnings, not depending on the rotations order)

- Use of geodesic thinnings

Intersections of homotopic thinnings

It can be shown that the maximal balls skeleton can be obtained by an intersection of thinnings by all the rotations of the following structuring elements:

• 0	• •	0 •
1 **1** •	1 **1** 0	1 **1** •
1 1 1	1 1 1	1 1 1
1 1	1 1	1 1

These structuring elements are not built on the elementary hexagon!

Intersections of homotopic thinnings (2)

By sorting among these configurations those preserving the homotopy, it can be shown that only the following configurations:

$\bar{2}$ 0

1 **1** $\bar{2}$

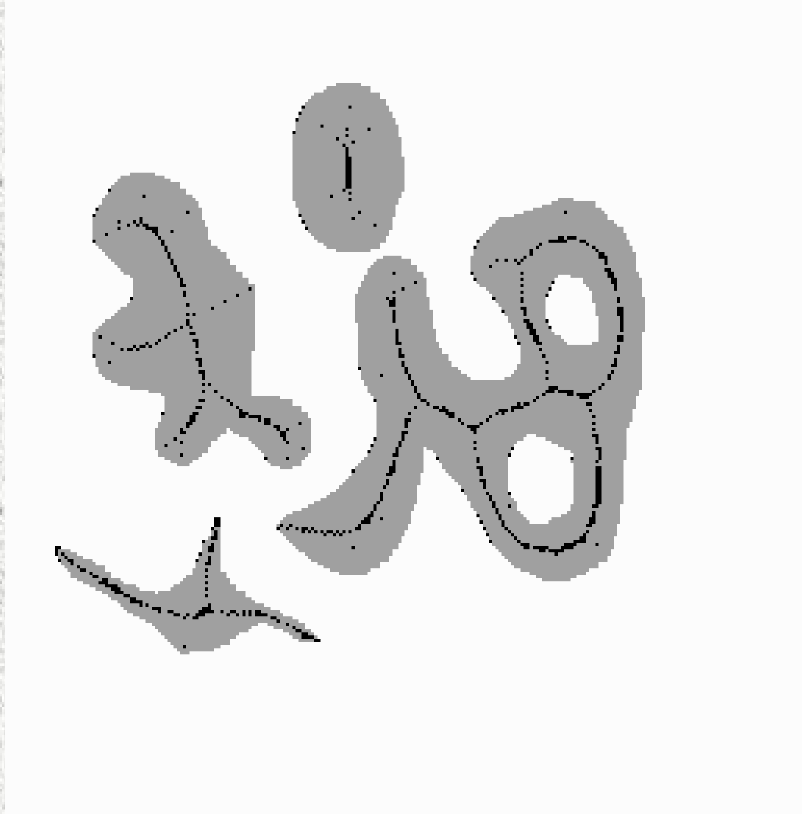
1 1 1

1 1

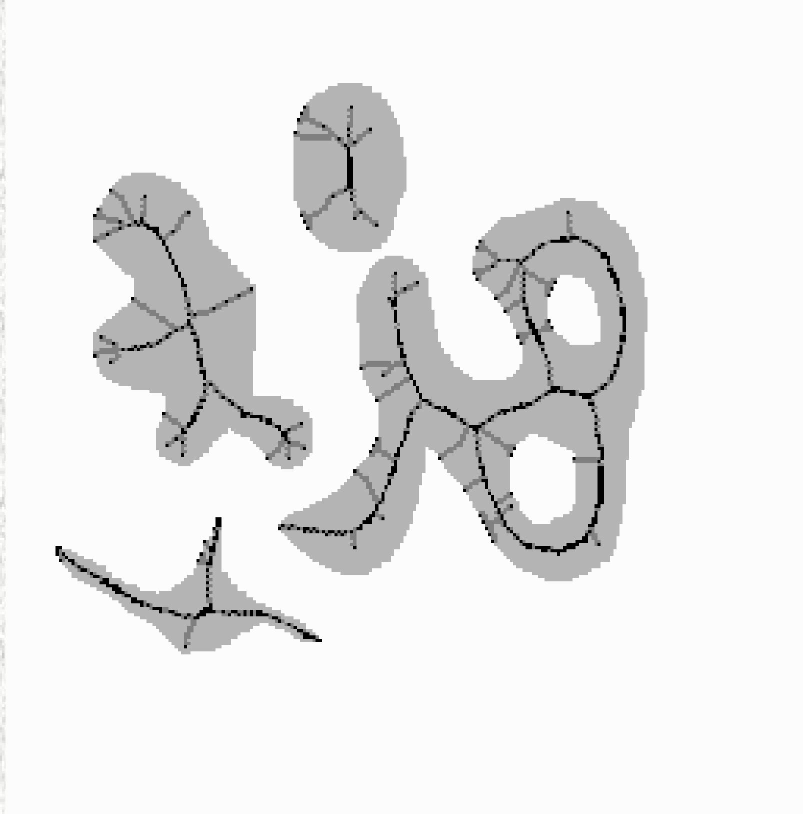
and all their rotations produce a connected skeleton containing the maximal ball skeleton

- The 1-points must be included in set X
- The 0-point must be included in set X^c
- The 2-points must not belong to the residue $X \backslash \gamma(X)$

Example of non biased connected skeleton



Skeleton by maximal balls



Connected skeleton containing the maximal balls skeleton

Use of the skeleton

The skeleton by itself is not very interesting:

• The maximal balls skeleton is not a good shape descriptor

• Representing a set by its skeleton and its quench function do not lead to the definition of more powerful algorithms for the elementary morphological transformationss

•The connected skeleton has many defects

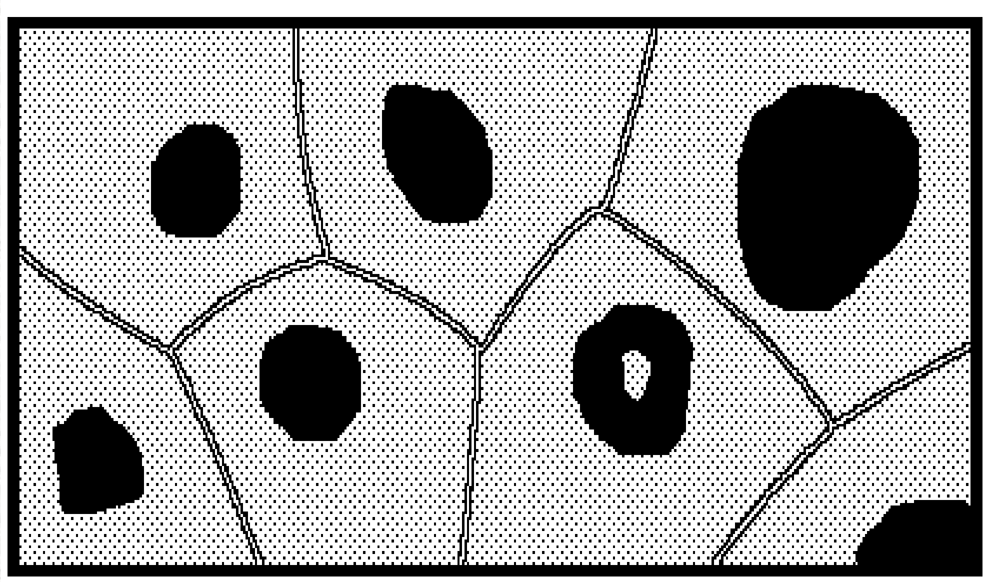
The skeleton by influence zones (SKIZ) is much more used.

Skeleton by influence zones (SKIZ)

X, set made of n connected components $\{X_i\}$

- Influence zone $Z(X_i)$ of X_i: set of points closer to X_i than to any other connected component of X:

$$z(X_i) = \{x : \forall\ j \neq i, d(x, X_i) < d(x, X_j)\}$$



Building the SKIZ

The SKIZ is built with homotopic thickenings combined with a clipping operator

On the hexagonal grid, the thickening is performed with M (compulsory if a connected component is reduced to a point) and the clipping is made with a structuring element denoted E:

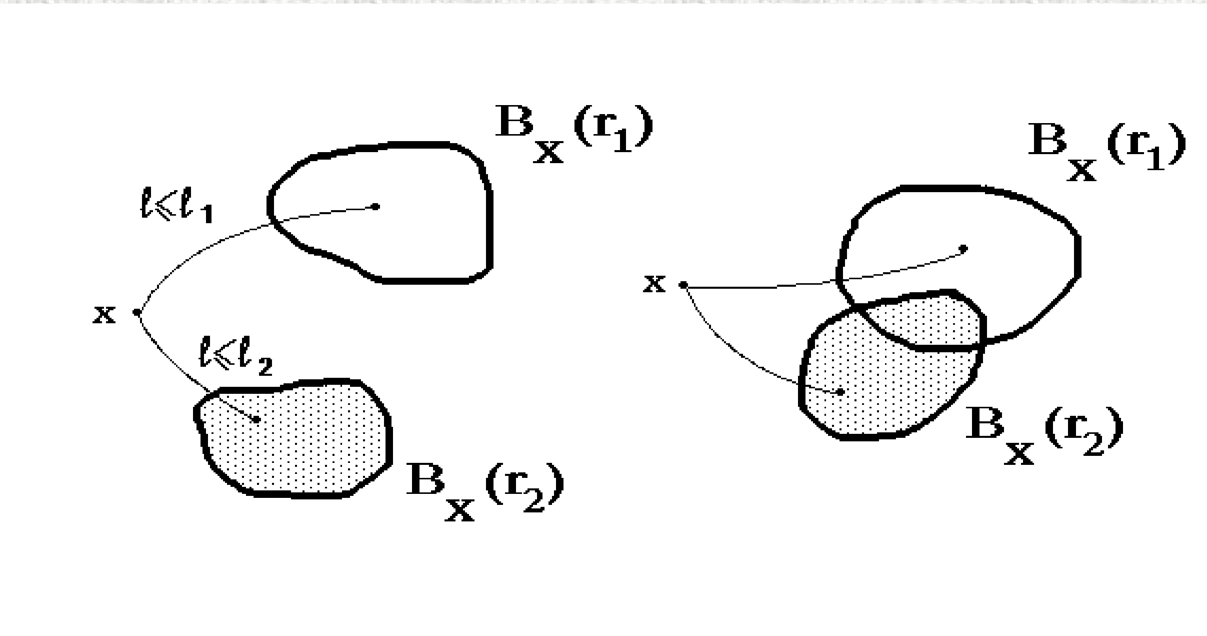
	0 0		. .
M	. . 0	E	1 0 1
	1 .		1 1

The SKIZ is not an homotopic transform (holes inside the connected components are suppressed).

The algorithm by thickenings produces some biases.

Geodesic thinnings and thickenings

Geodesic thinnings and thickenings can be defined. The structuring elements are defined with geodesic balls and geodesic distances. These structuring elements are not «stiff », they can be distorted.



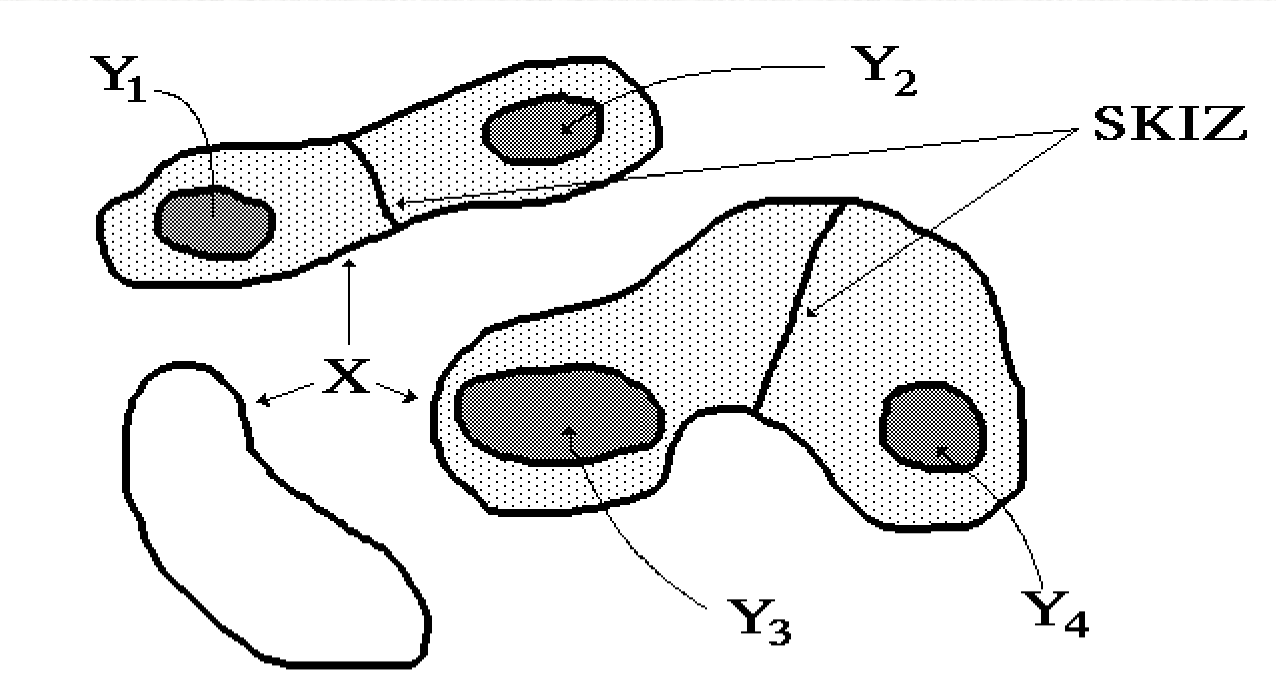
Geodesic SKIZ

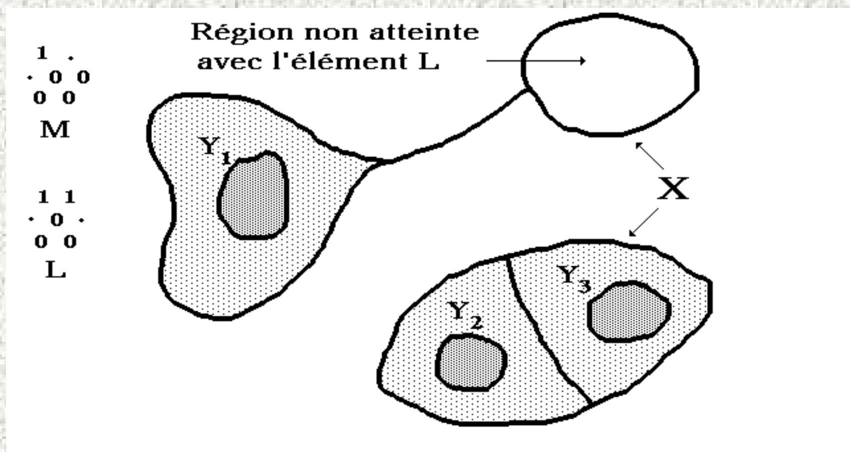
Set Y made of various connected components and included in a geodesic space X

Influence zone of a connected component: set of points of X at a finite geodesic distance from the connected component and closer to this component than to any other connected component:

$$Z_X(Y_i) = \left\{ \begin{array}{l} x \in X : d_X(x, Y_i) < +\infty \\ \text{and} \\ \forall j \neq i, d_X(x, Y_i) < d_X(x, Y_j) \end{array} \right\}$$

Geodesic SKIZ, construction





The use of structuring element M is compulsory to insure propagations in regions of low thickness of X

Summary, list of the main hexagonal structuring elements

Structuring element		Sequential Thinning	Sequential Thickening	Hit or miss
L	● □ ■ ■ □ ■ ●	**Skeleton of the shape**	**Skeleton of background**	
M	● □ ■ ■ ● ■ ■	**Skeleton of the shape with branches**	**Thickenning from isolated points**	
D	● □ ■ ■ □ ● □	**Homotopic marker**	**Quasi-convexe hull**	
E	● ◉ □ ■ □ □ □	**Pruning of skeleton**	**Pruning of background**	**Extreme points**
F	□ ■ ■ ■ □ □ ■			**Triple points**
I	□ □ □ ■ □ □ □			**Isolated points**

Homotopic

Nonhomotopic

Third part

Numerical residues

Elementary residues for functions can be defined by difference between two transformations ψ and ζ (with $\zeta \leq \psi$)

The most classical examples of elementary numerical residues are the morphological gradient and the top-hat transform:

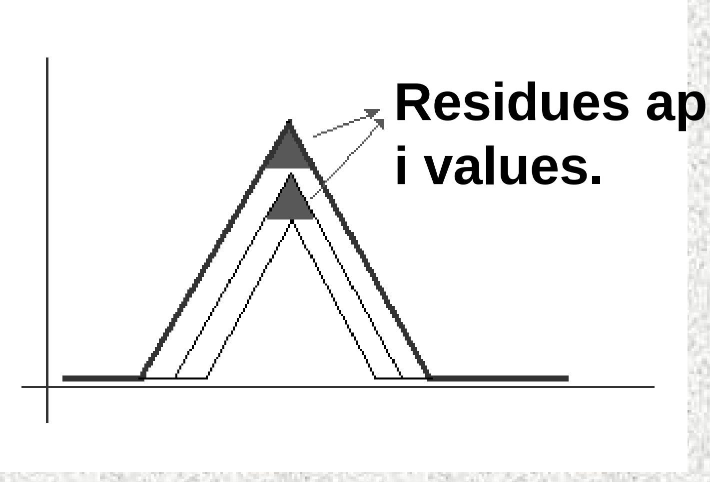
• Morphological gradient $\delta_i - \varepsilon_i$
(semi-gradients $I - \varepsilon_i$ et $\delta_i - I$)

• Top-Hat transform $I - \gamma_i$

Residues in numerical morphology

We can try to extend to functions the definitions of the residues for sets.
This extension presents some difficulties:

• The set difference and the subtraction of functions are not really equivalent.
• Different residues may appear at a point x $\longrightarrow$ problem of the definition of the associated function.



Numerical residual functions definition

Definition based on the observation of the vertical evolution of the image during its transformation.

<u>Definitions</u>

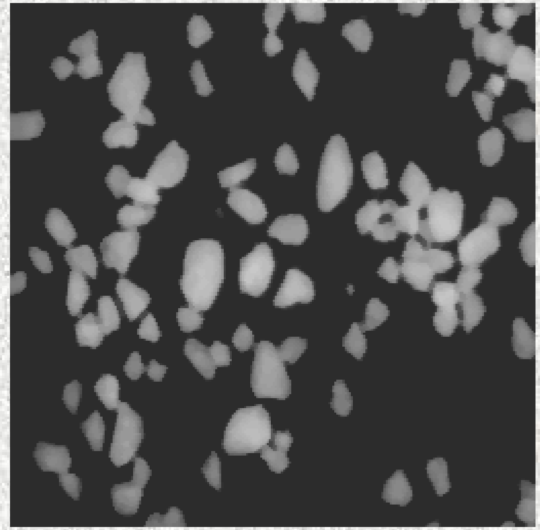
- Transformation $$\theta = \underset{i \in I}{Sup}\left(\psi_i - \zeta_i\right)$$

- Associated function $$q = \arg\max\left(r_i\right) + 1 = \arg\max\left(\psi_i - \zeta_i\right) + 1$$ $$q(x) = \max(i) + 1 \quad r_i(x) > 0 \text{ and maximum}$$

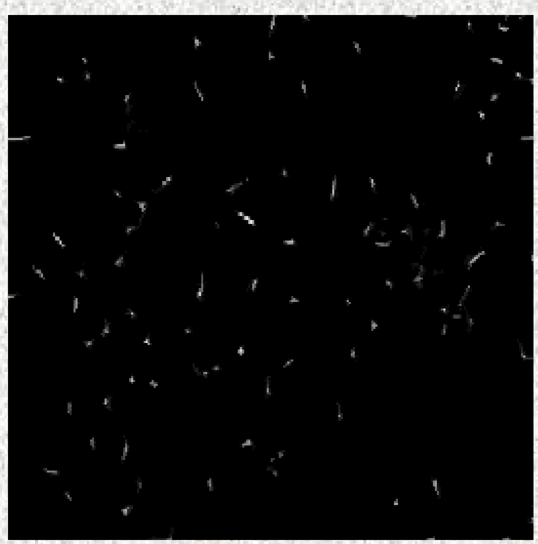
In the binary case, this definition and the classical one are identical.

Examples (1)

Ultime erosion



θ

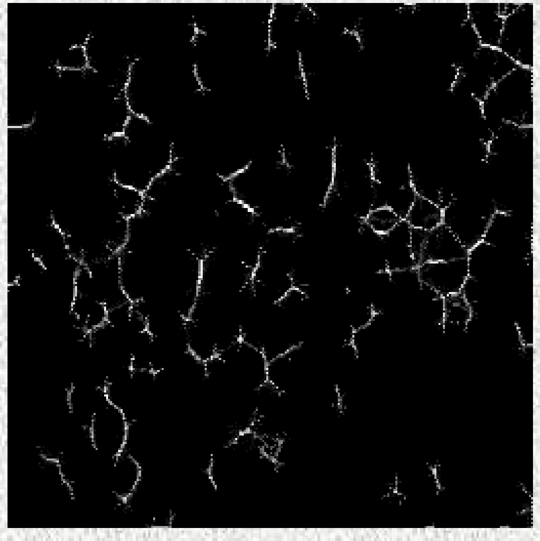


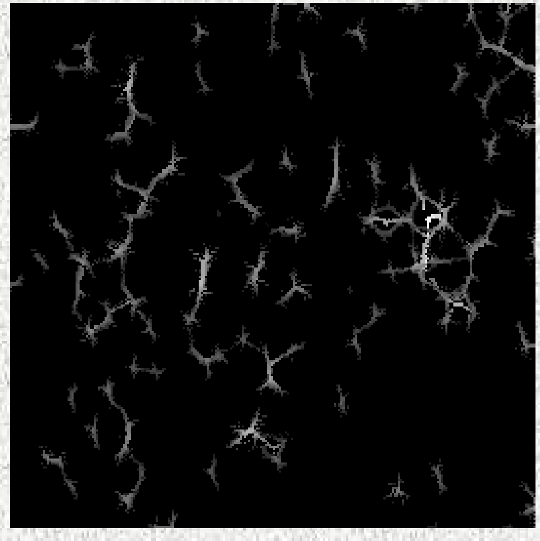
q

Examples (2)

Skeleton by « significant maximal cylinders »

θ

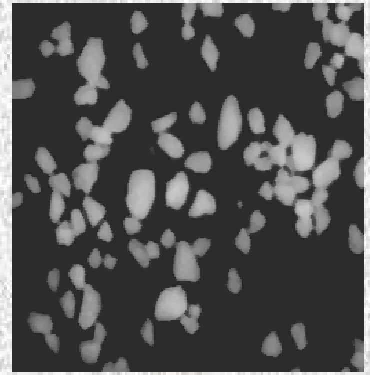


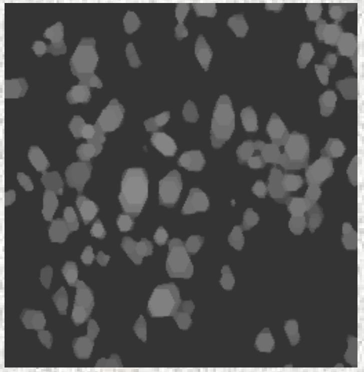


q

Problem of the reconstruction: it not possible in general to entirely rebuild the function from its skeleton

$$\rho(f) = \sup_{x \in E} \left(\theta(x) \oplus B_{q(x)}\right)$$





New residues

• The extension of the definition of set residues as the ultimate erosion or the skeleton by openings is interesting.

• This definition of residues especially the introduction, for functions but also for sets, of new powerful residual transforms and associated functions.

We shall introduce in particular:

• The **ultimate opening** (with some variants)

• The **quasi-distance**

• residues based on **pilings**

Ultimate opening

$$\psi_i = \gamma_i$$

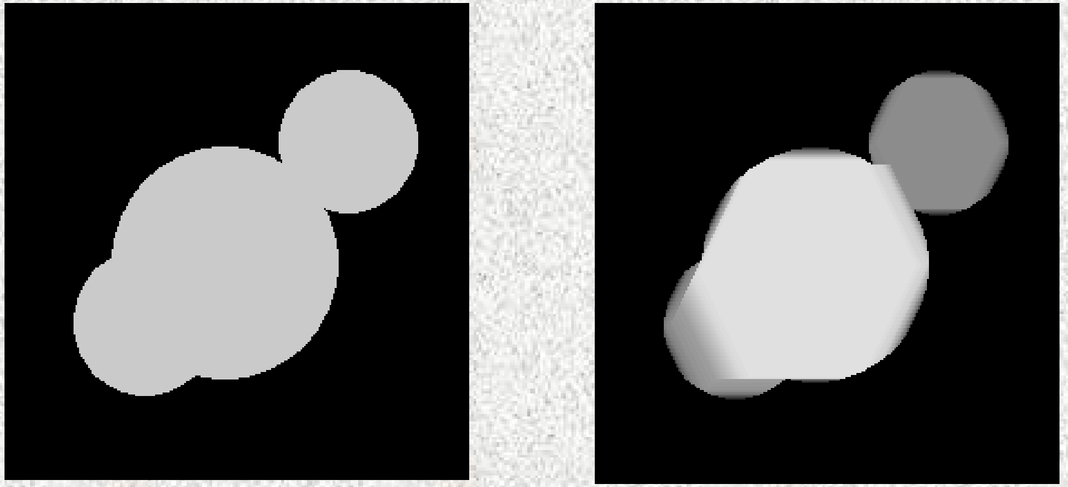
$$\zeta_i = \gamma_{i+1}$$

In binary, the transform θ has no interest (θ = I).

The associated function q is called granulometric function.

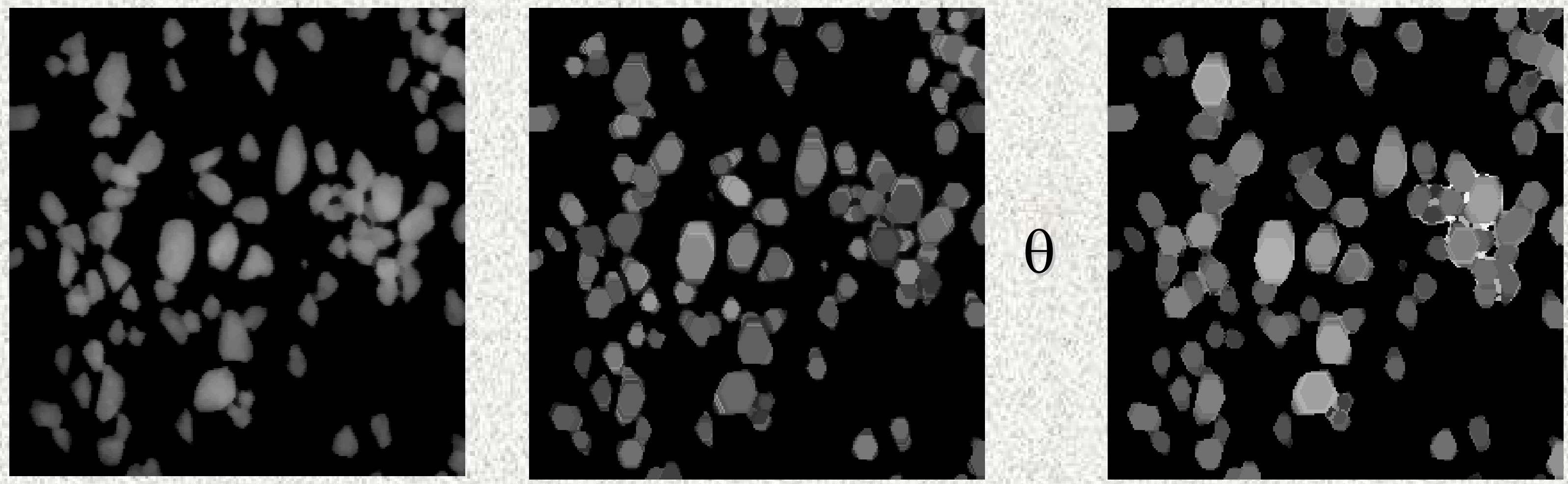
At every point x, q(x) is equal (up to the unit value) to the size of the greatest ball covering this point x in the binary case, to the radius of the biggest significant cylinder of the partial reconstruction covering x in the numerical case.

Ultimate opening (2)



q

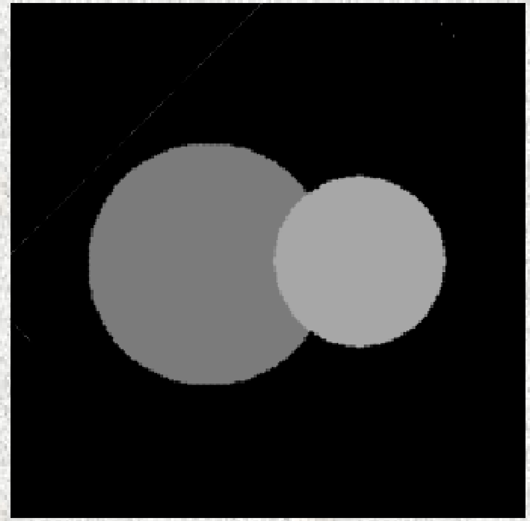
Associated function of a set ultimate opening



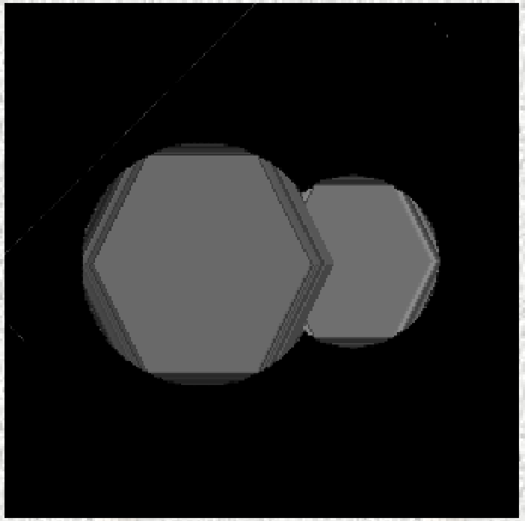
θ q

Numerical ultimate opening and associated function

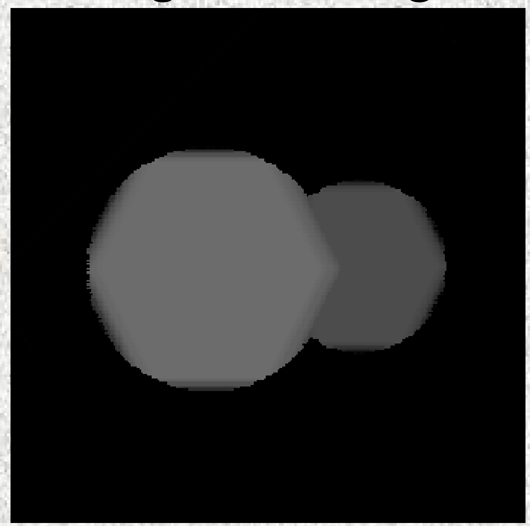
Ultimate opening, reconstruction



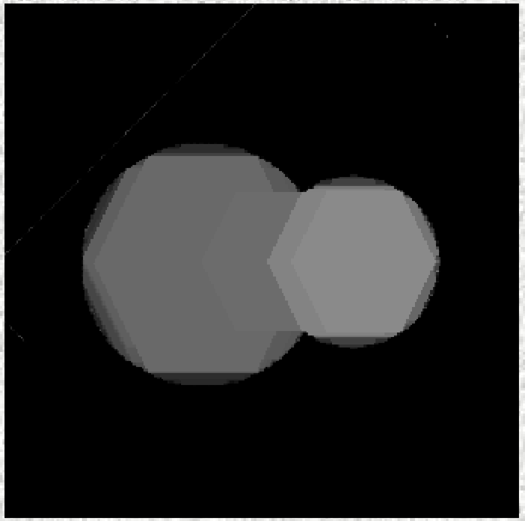
Original image



Ultimate opening



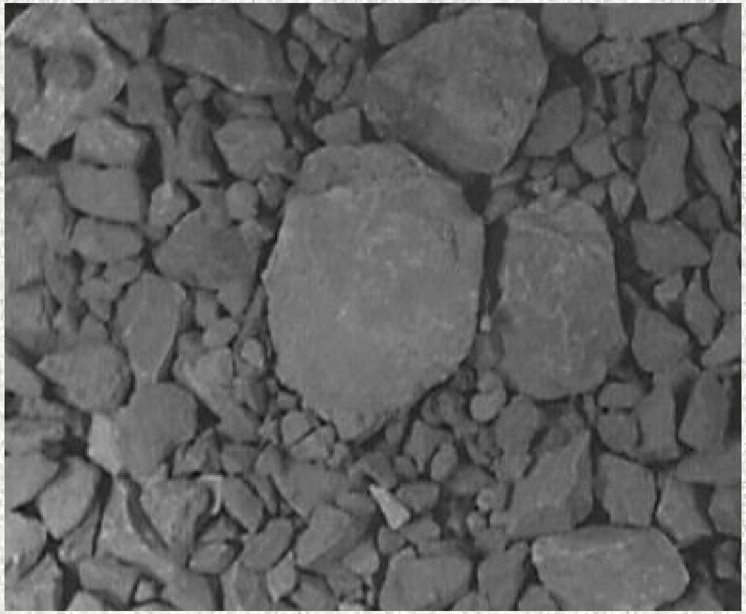
Granulometric function

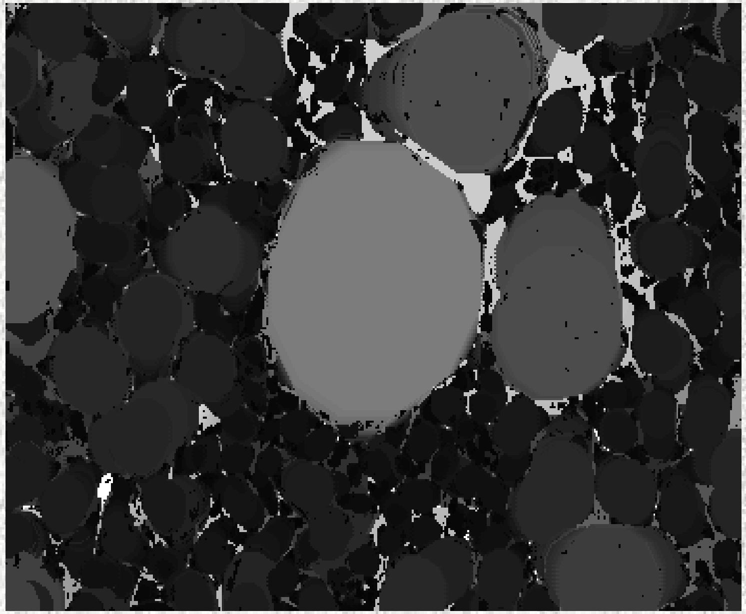


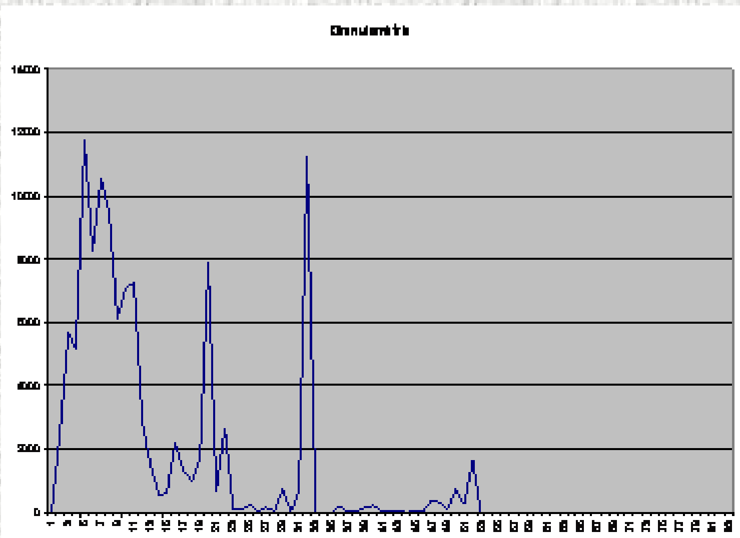
Reconstruction by the skeleton

Size distributions and segmentation

<u>Heap of rocks</u>: Determing the blocks size distribution







Granulometric function

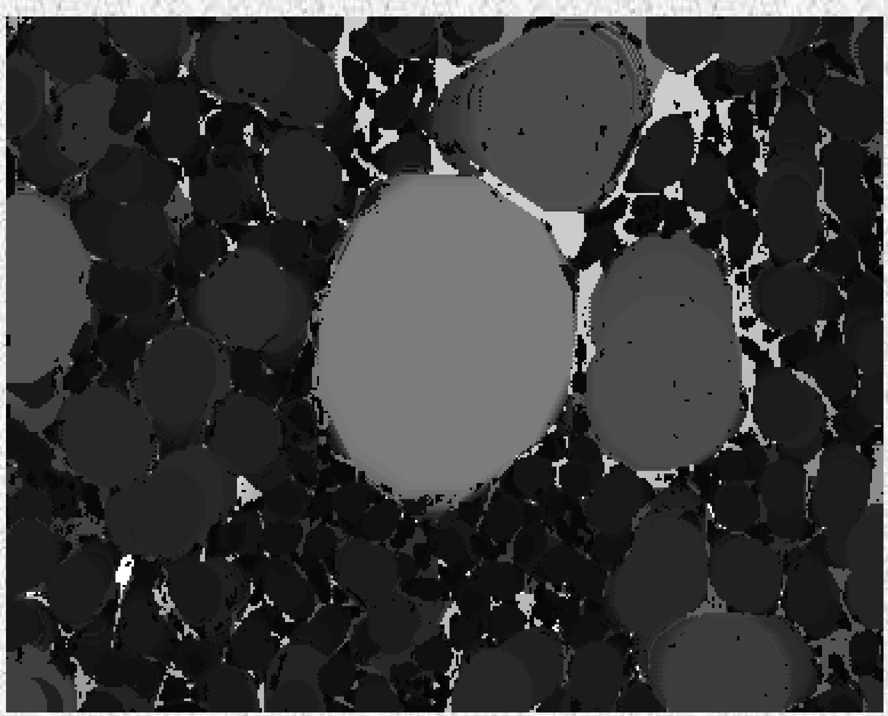
These functions allow to define the size distribution of the more or less homogeneous regions of the image BEFORE segmenting them

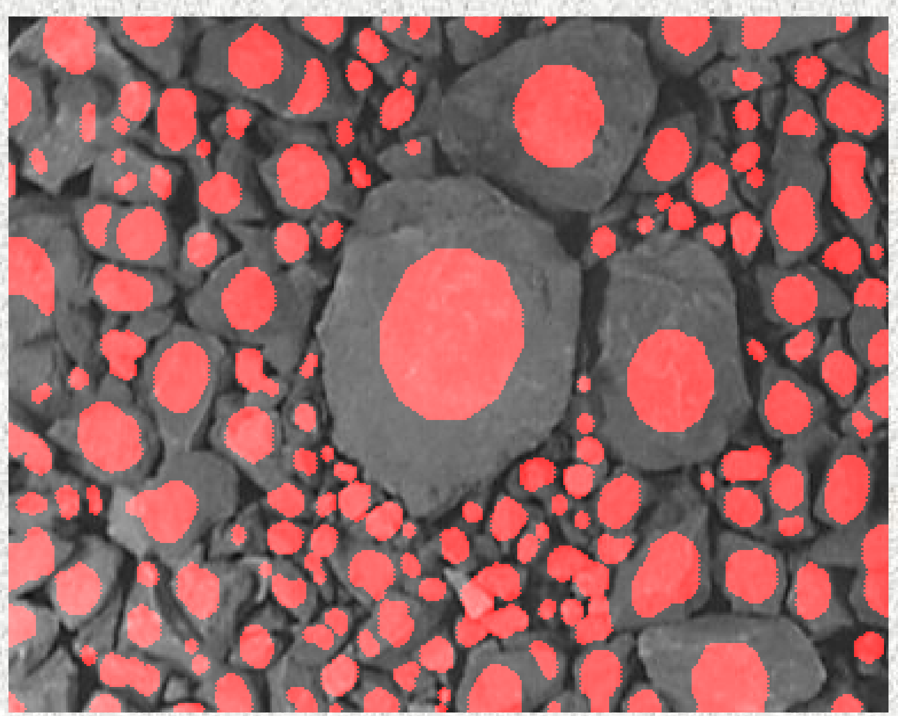
Size distributions and segmentation (2)

Definition of markers for counting and segmentation.

For each threshold i of the ganulometric function:

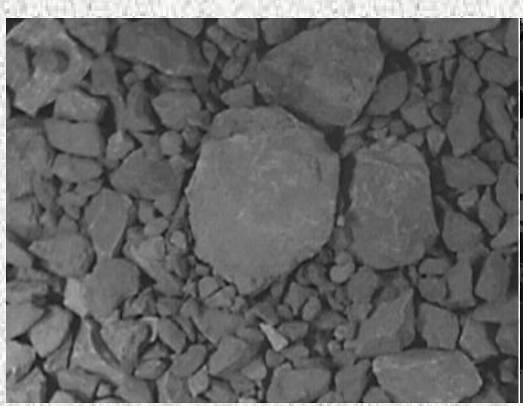
- holes filling
- erosion of size j = max (k .i, c), k <1

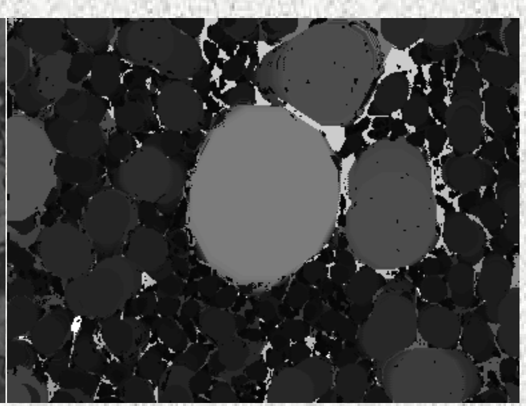


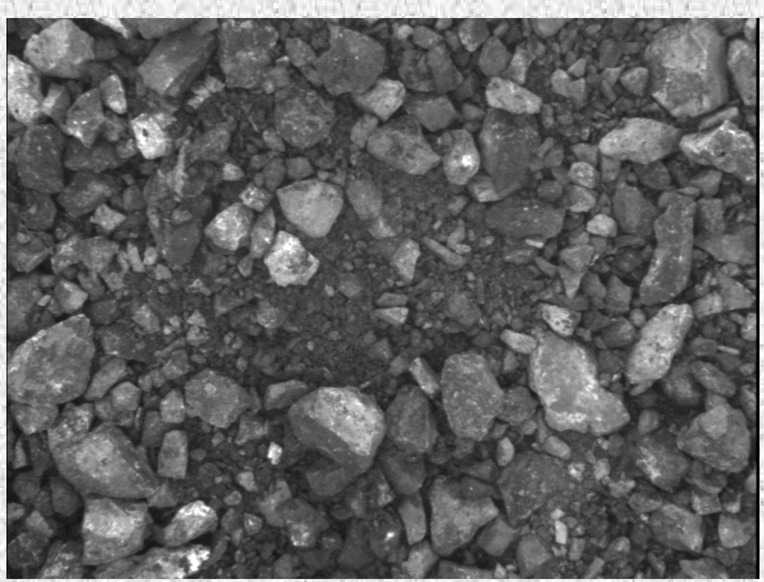


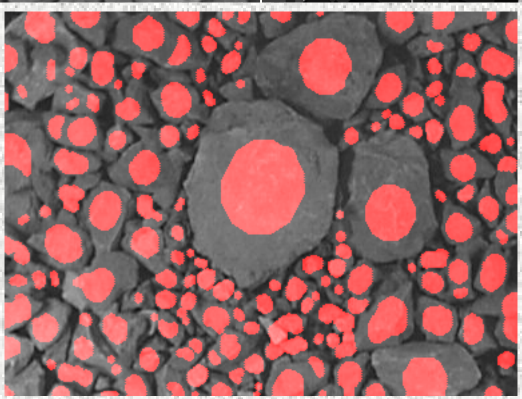
Markers of the blocks generated from the granulometric function

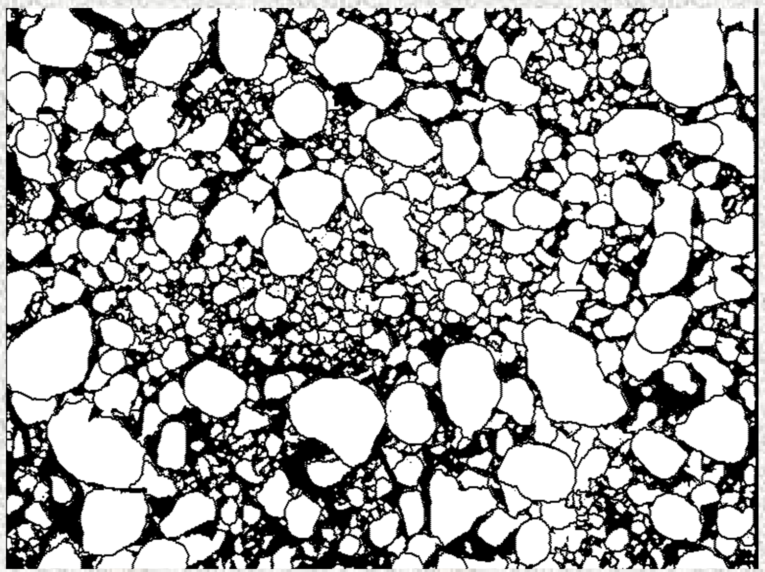
Size distribution and segmentation (3)





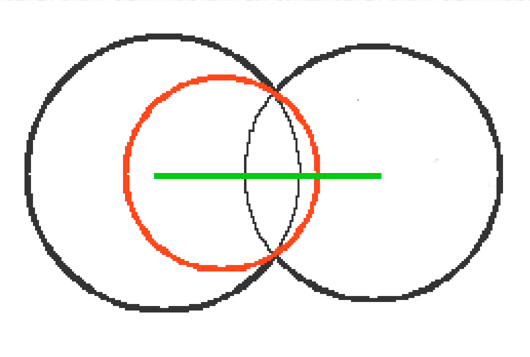






Blocks markers built from
the granulometric function

Maximal balls and critical balls



The knowledge of the maximal balls (position and radius) of a set X allows to rebuild X.

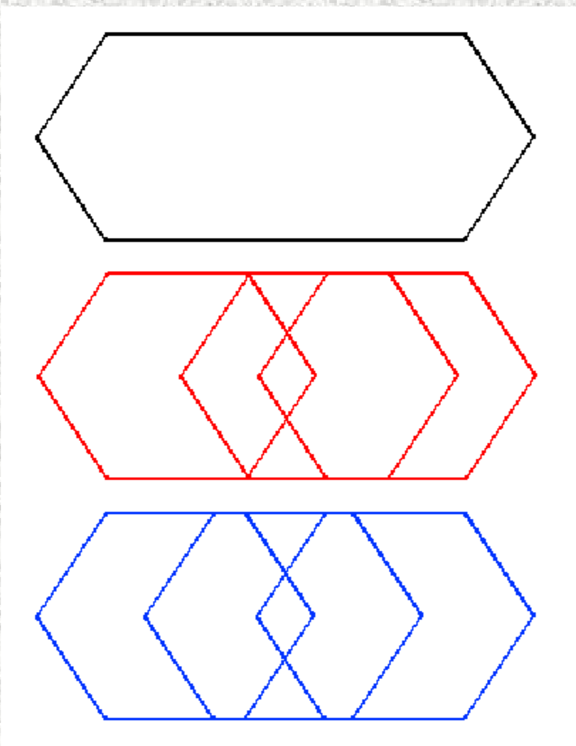
The maximal balls set (skeleton) is redundant for the reconstruction. Knowing the critical balls is sufficient.

Definition of a critical ball

A maximal ball B is critical when there exists no combination of the other maximal balls which covers B

The shape description of a set is easier by means of its critical balls

Digital critical balls



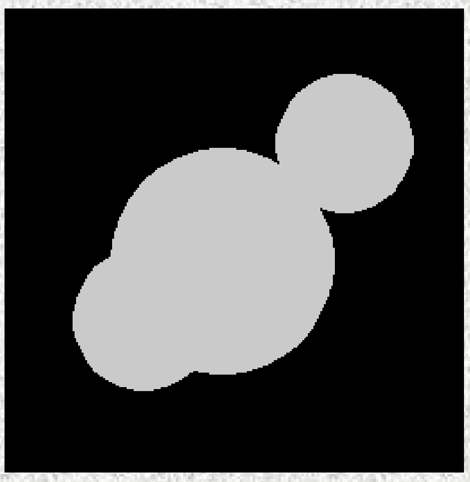
A digital maximal ball B_i of size i is critical if there exists no combination of maximal balls B_j of size different of i which covers B_i.

The granulometric function allows to sort the critical balls

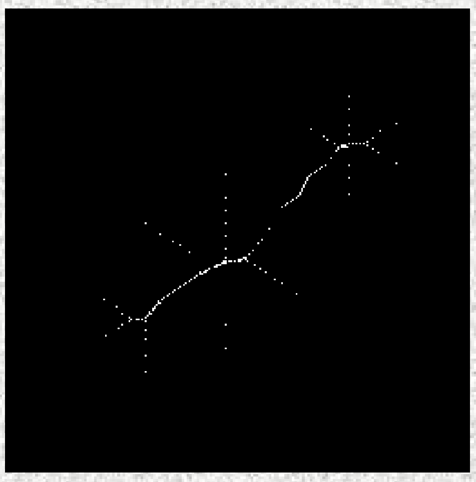
Two steps:

- Removal of the centers of the centers of the balls covered by larger balls
- Removal of the centers of the remaining balls covered by smaller balls

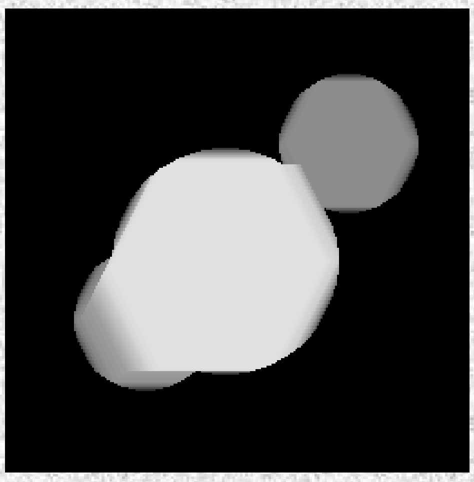
Digital critical balls (2)



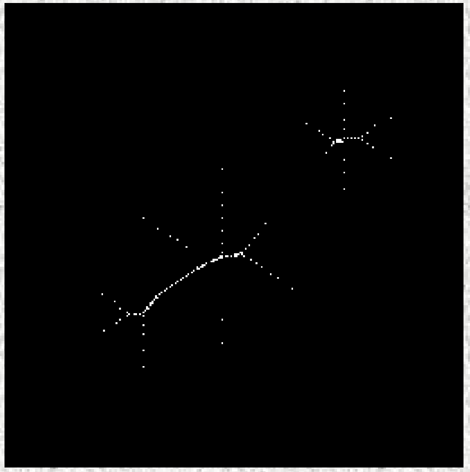
Initial set



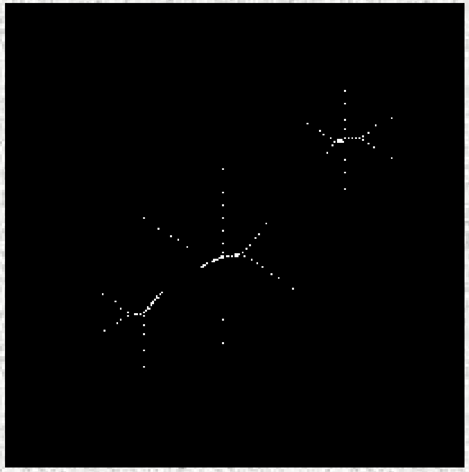
Quench function



Granulometric function



Filtering, 1st step

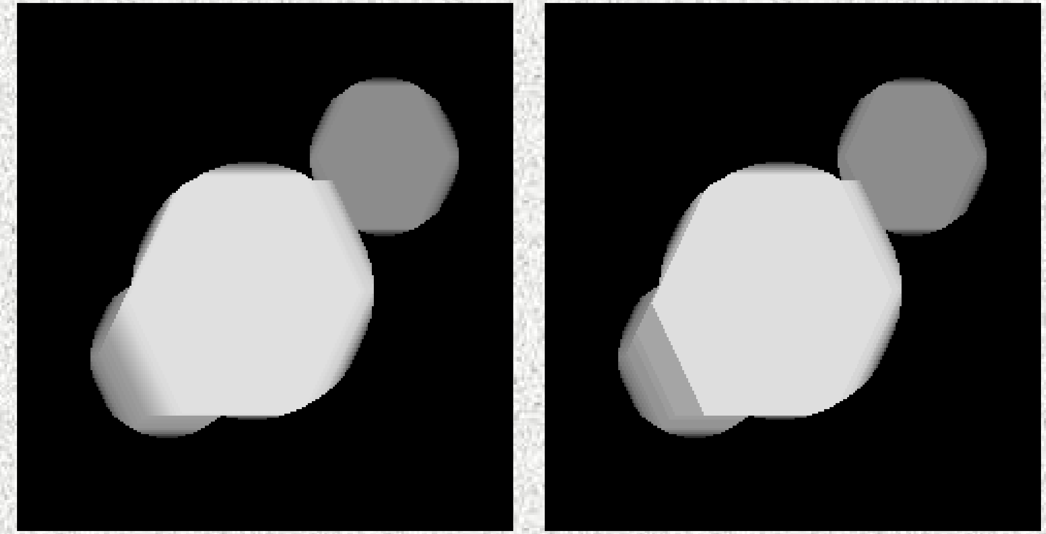


Centers of critical balls

Non critical centers

Digital critical balls (3)

The granulometric function can be rebuilt with the critical balls only, then the most important ones can be extracted.



Other ultimate openings

New ultimate openings can be defined by using openings based on criteria

- Area openings

- Opening based on Féret diameters or on sizes of bounding boxes

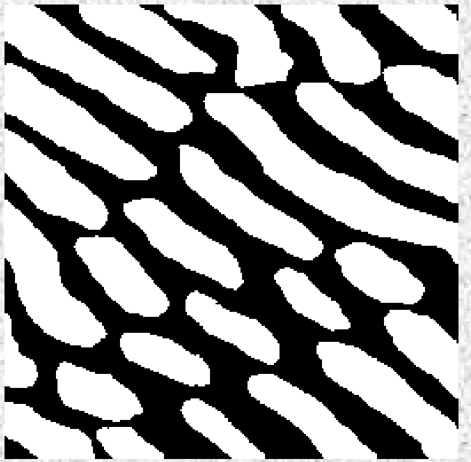
Area openings

Area openings for sets

$$\gamma_\lambda^a(X) = \left\{ x \in X \ / \ Area(C_x(X) \geq \lambda \right\}$$

If X_i is a connect component of X, $\gamma_\lambda^a(X)$ is equal to the union of X_i whose area is greater or equal to λ

$$\gamma_\lambda^a(X) = \bigcup \left\{ X_i \ / \ Area(X_i) \geq \lambda \right\}$$



original

1250 pixels

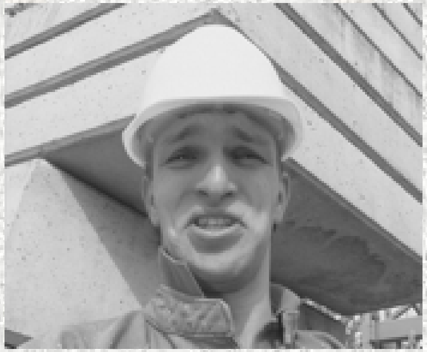
2000 pixels

Area openings

Area openings for functions

$$(\gamma_\lambda^a(f))(x) = \sup\left\{h \leq f(x) \,/\, x \in \gamma_\lambda^a(T_h(f))\right\}$$

$T_h(f)$ is the threshold of f at level h: $T_h(f) = \{x : f(x) \geq h\}$



original

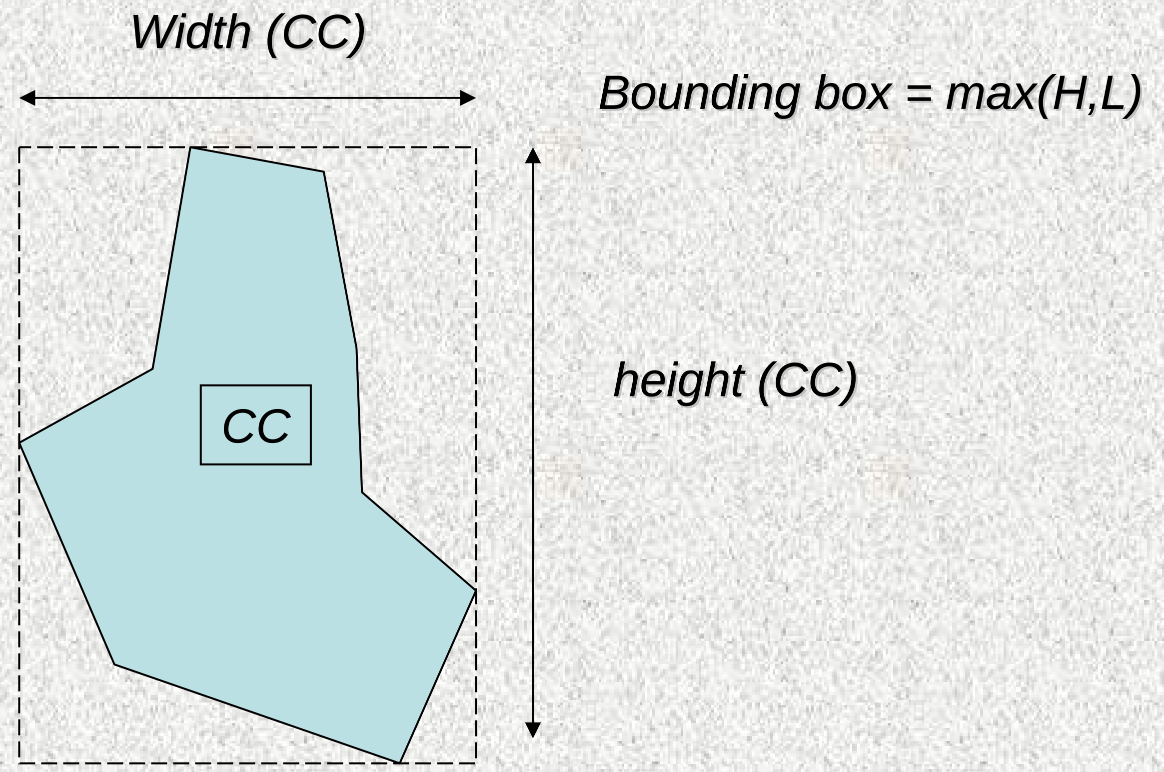
Size 100 pixels

Size 500 pixels

The area opening is obtained by stacking the area openings of all the thresholds of the function

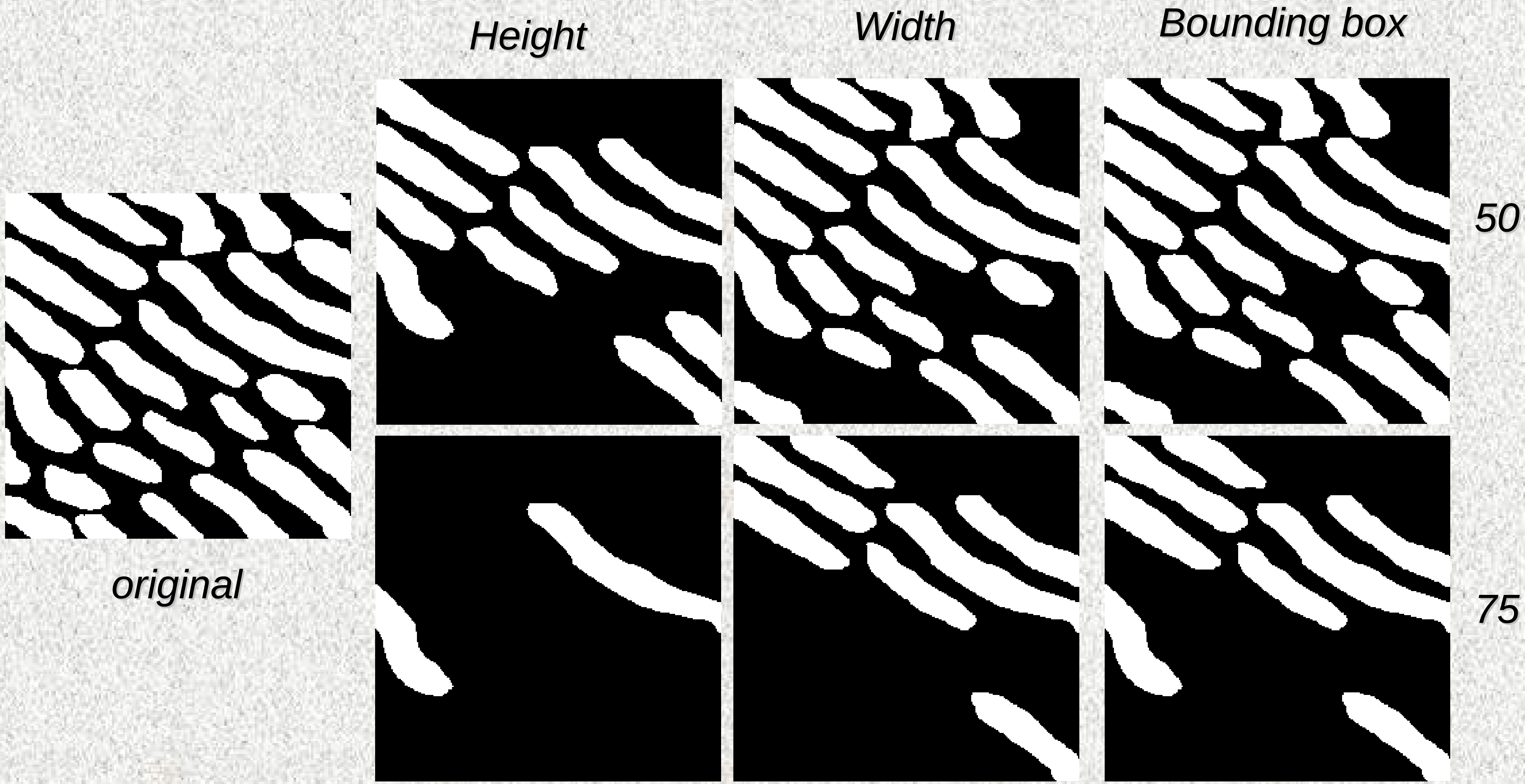
Bounding boxes openings

Rather than the area, horizontal and vertical Féret diameters of the connected components of X can be used. We can also use the size of the bounding box



Example of bounding boxes opening

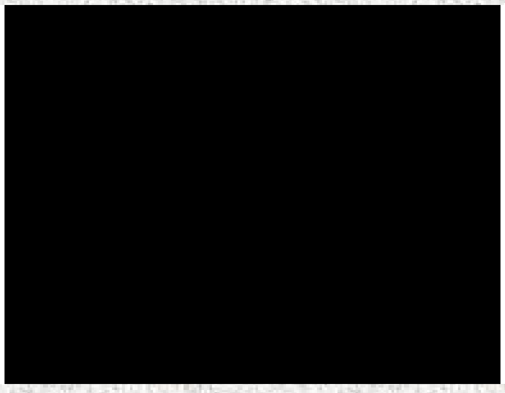
The bounding boxes opening is a union of openings



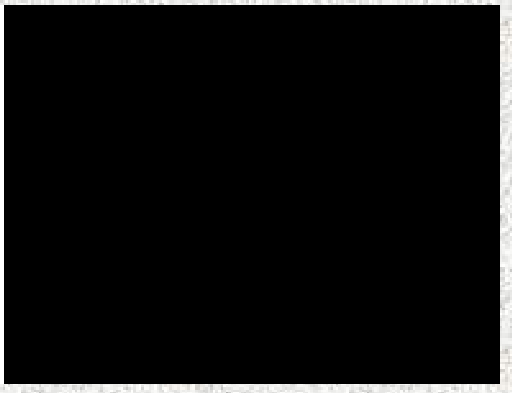
Criteria ultimate openings

Detection of text embedded in pictures

Criteria opening
Féret diameter



Maximal residue



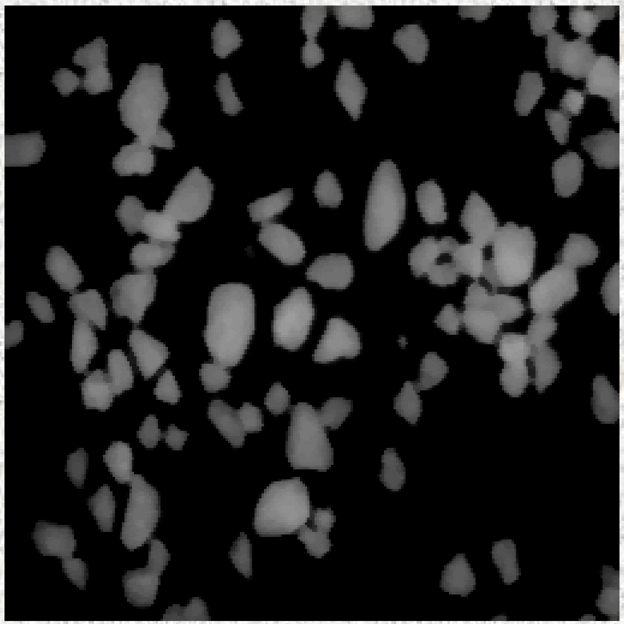
Size of the opening corresponding to the maximal residue

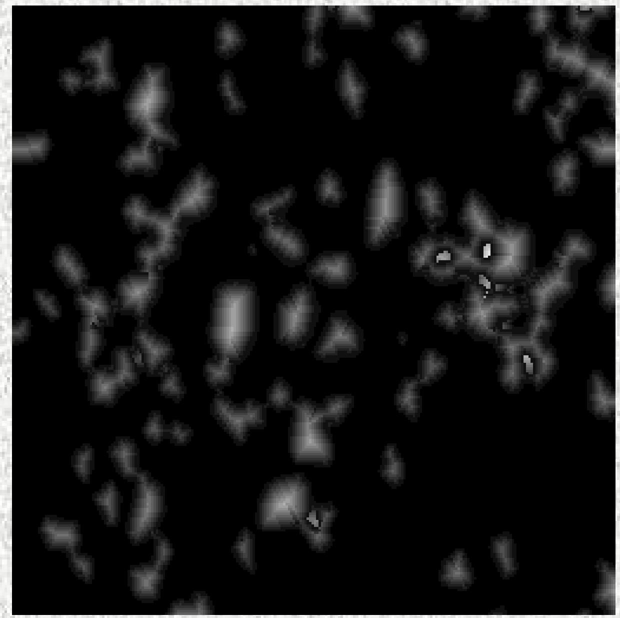
Quasi-distance

$$\psi_i = \varepsilon_i$$

$$\zeta_i = \varepsilon_{i+1}$$

- In binary, θ and q are not interesting (θ=I) and q is the distance function)
- With functions, q is called quasi-distance.



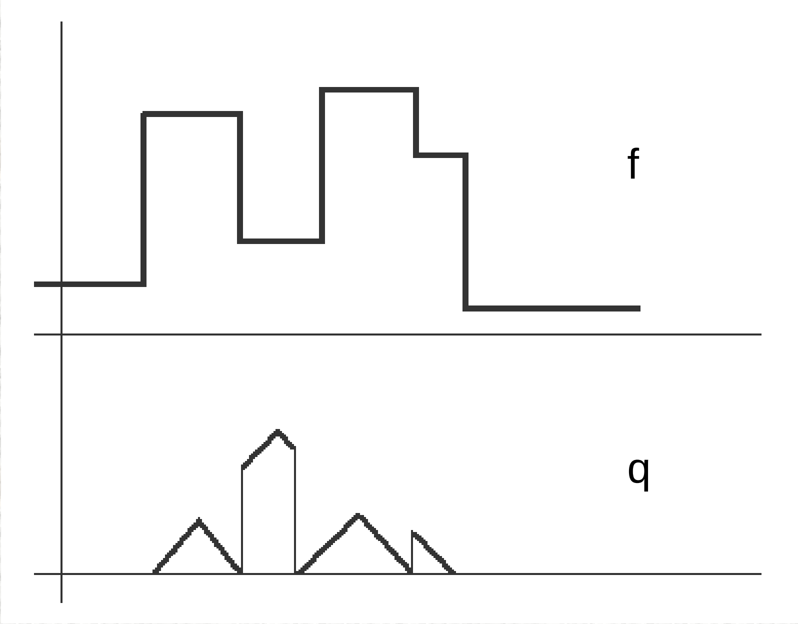


« hung up » distances appear.

Quasi-distance (2)

The quasi-distance is not 1-Lipschitzian.

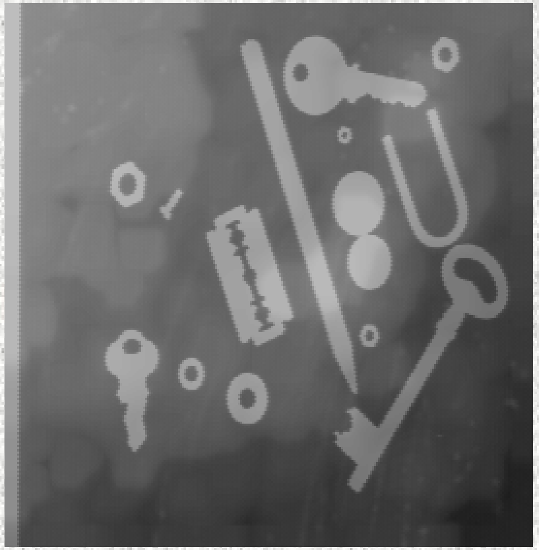
One can make the quasi-distance 1-Lipschitzian by using a « hung up distances descent » iterative operator.



- For every point x where $[q-\varepsilon(q)](x)>1$, do $q(x)=\varepsilon(q)(x)+1$
- Iterate until idempotence.

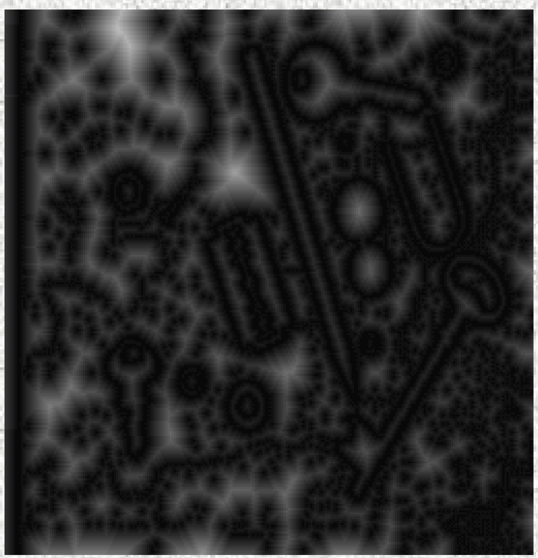
Quasi-distance (3)

Initial and corrected quasi-distances



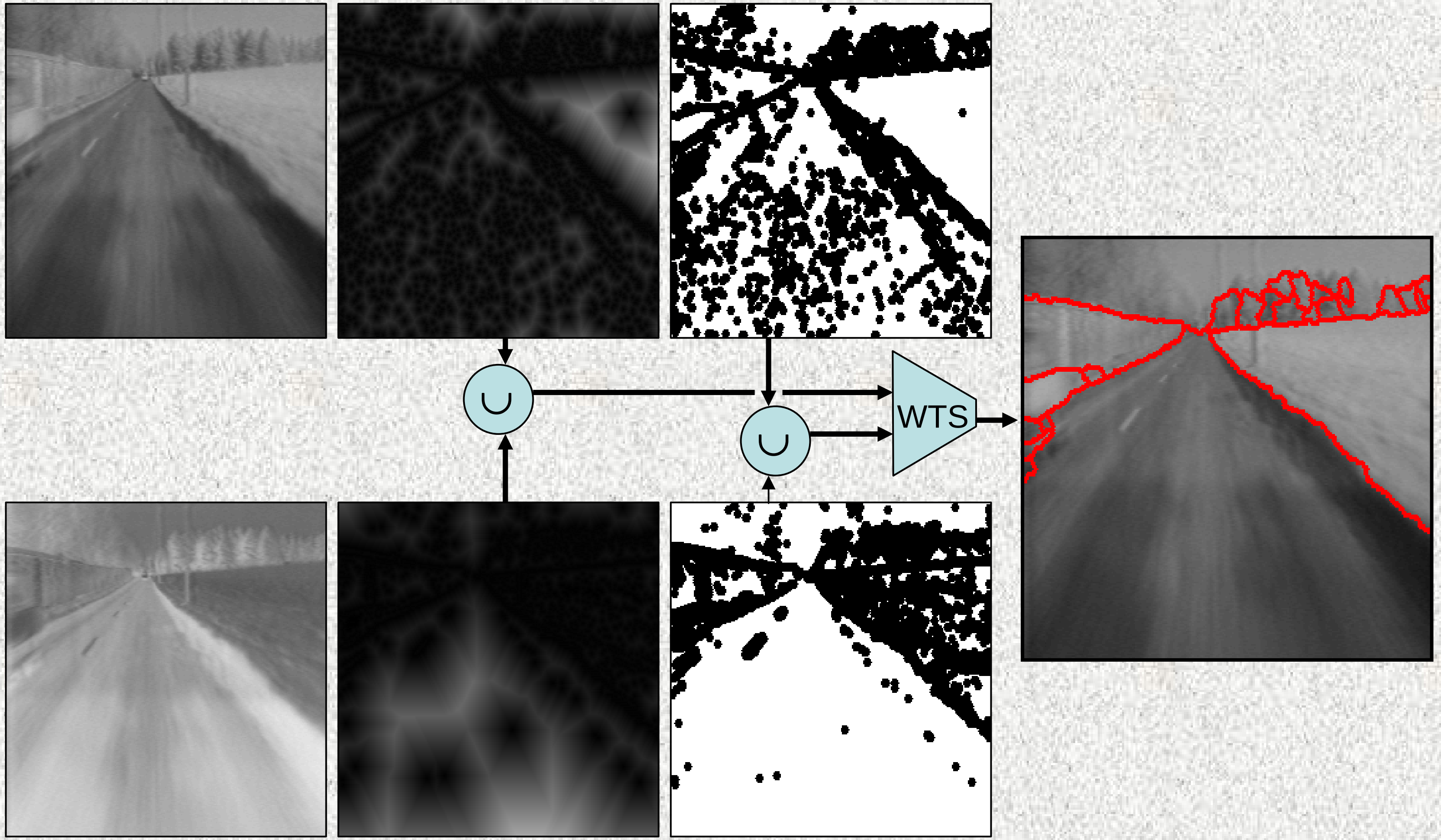
Initial image

Initial quasi-distance



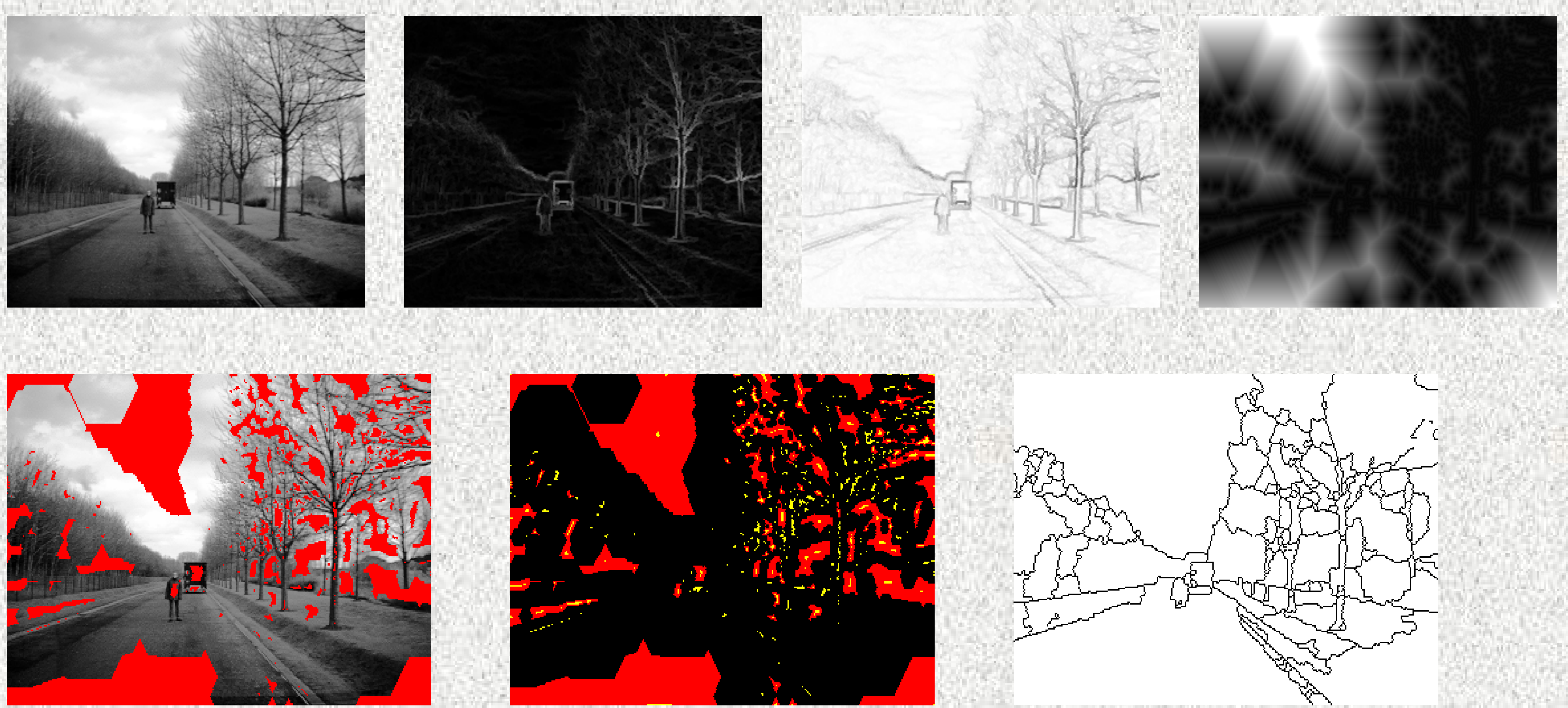
Corrected quasi-distance

« Granulometric » segmentation



When applied to a greytone image, the quasi-distance allows to define the distances, and therefore the sizes of the flat regions → Markers for a segmentation based on the size and shape of regions.

Gradient and quasi-distance



Quasi-distance computed on an inverted gradient

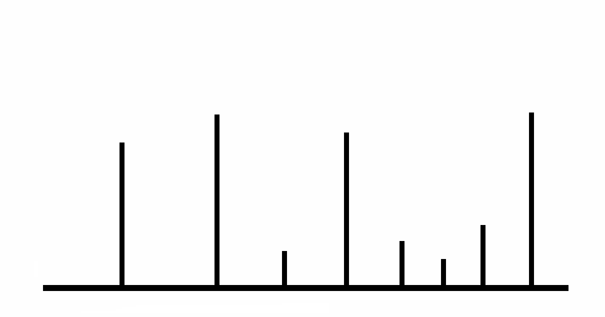
- Only one quasi-distance is calculated
- Hierarchy of regions based on their relative contrasts (similar to the waterfall algorithm)
- The sizes and shapes of regions are taken into account (closing of regions which are not perfectly closed)

Introduction to pilings

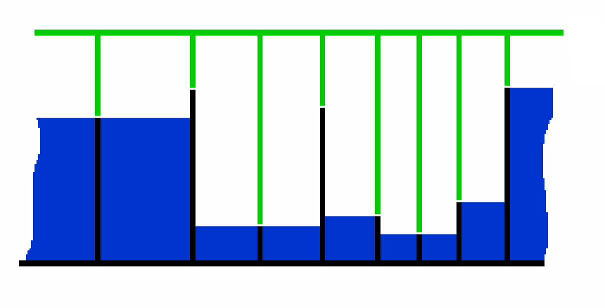
- Approach different from the waterfall algorithm but with the same premises
- Definition of a residual transform

Approach defined on valued WTS (supports of CB and of LCB are the same)

$w_0 = \psi_0$



ψ_1



We define a function:

$\xi_0 = \psi_0$ on $Min^c(\psi_0)$

ξ_0 = max on $Min(\psi_0)$

Then, we define:

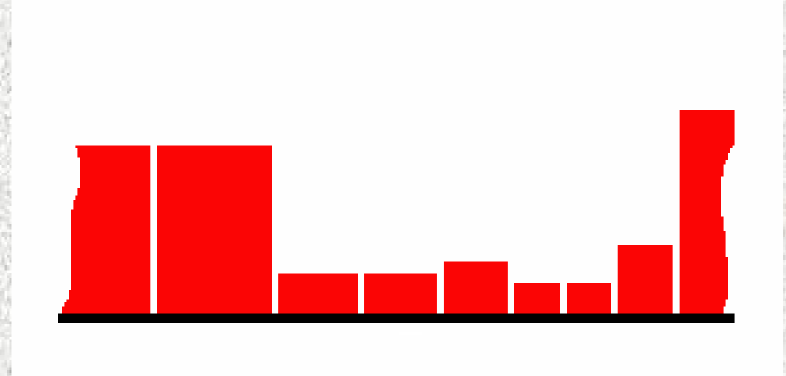
$\psi_1 = R^*_{\xi_0}(\psi_0)$

At the first step, ψ_1 is the hierarchical image because W_0, the WTS is the complementary set of the minima of ψ_0

Residues of pilings

Then, we can define a first residue r_1 as the difference between ψ_1 and ψ_0:

$$r_1 = \psi_1 - \psi_0$$

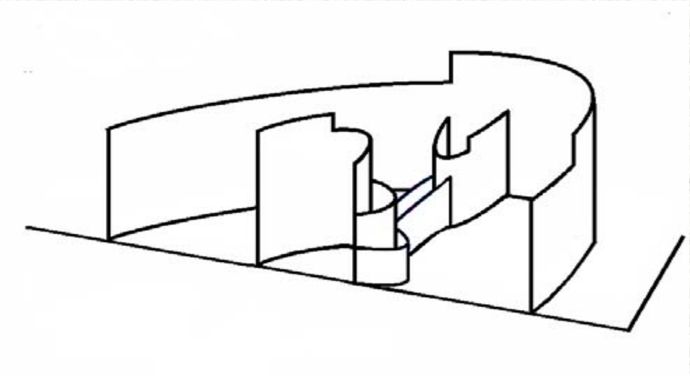


This residue corresponds to the pilings which are necessary to fill the minima of ψ_0

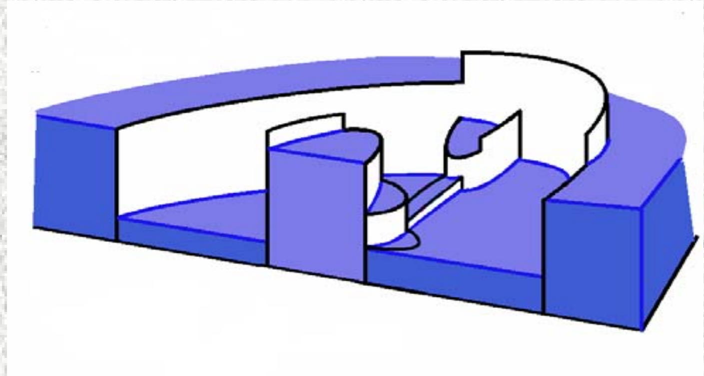
At the next step, the same transformation is defined by using the function ξ_1, itself defined from the minima of ψ_1

Residues of pilings (2)

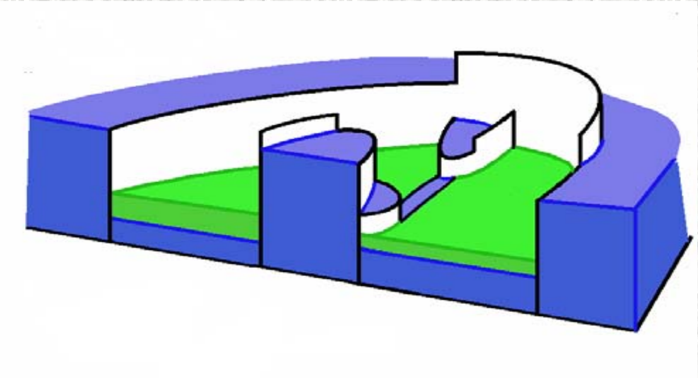
ψ_0



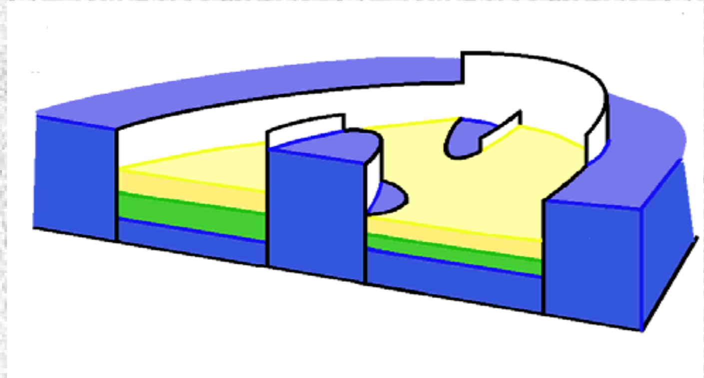
ψ_1



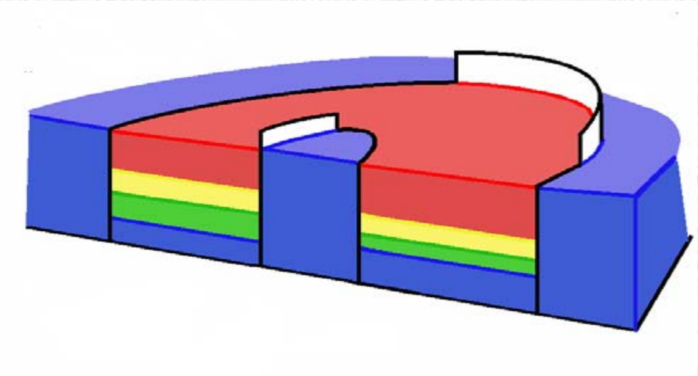
ψ_2



ψ_3



ψ_4



Transformations ψ_i and successive residues r_i (different colours)

Residual transformation

Definition of a residual transform by iterations

ψ_i is defined from ψ_{i-1}:

$$\psi_i = R^*_{\xi_{i-1}} (\psi_{i-1})$$

with:

$\xi_{i-1} = \psi_{i-1}$ on $Min^c(\psi_{i-1})$

ξ_{i-1} = max on $Min(\psi_{i-1})$

The residue r_i is equal to:

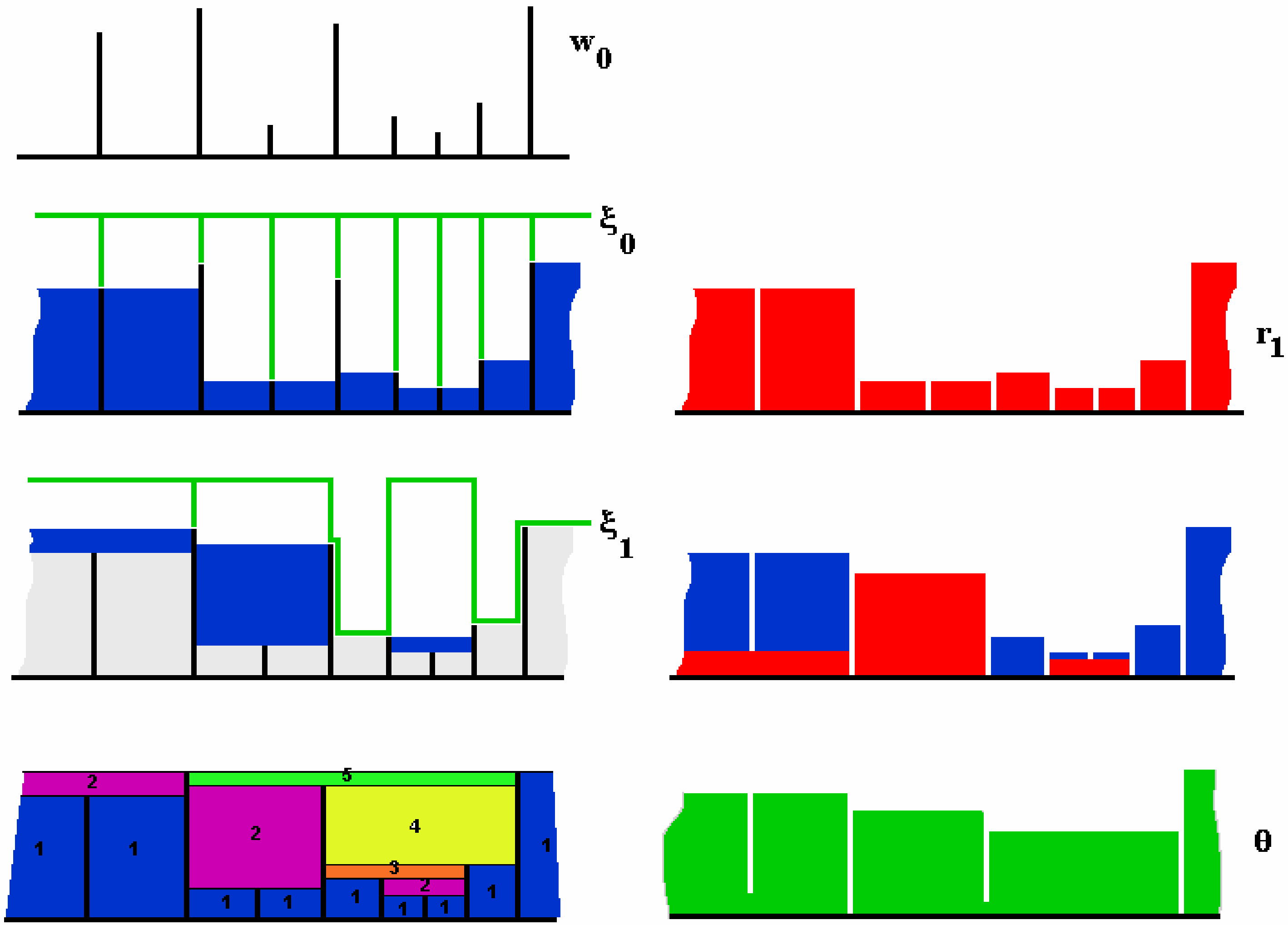
$$r_i = \psi_i - \psi_{i-1}$$

Finally, two functions θ and q are defined:

$$\theta = \sup_{i \in I} (r_i) = \sup_{i \in I}(\psi_i - \psi_{i-1})$$

$$q = \arg\max(r_i) = \arg\max(\psi_i - \psi_{i-1})$$

Residual transformation (2)



Steps of the construction of the function θ

Pilings and hierarchy

- The pilings cover some contours whilst other are preserved (function θ)
- The preserved contours remain in $\sup(w_0,\theta)$. They can be extracted with a Top-hat transform

What is the criterion for keeping some contours?

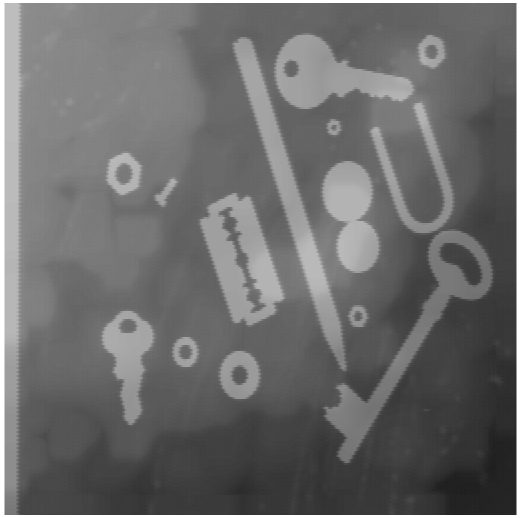
- Waterfall algorithm (primitive):

  Contours separating regions where the contours heights are lower than the maximum height of the preserved contours

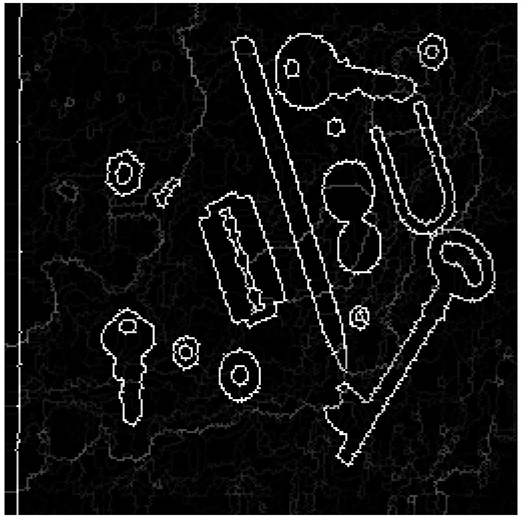
- Residues of pilings:

  Contours separating regions where the contours heights (if the contours exist) are at least twice lower than the minimal height of the preserved contours

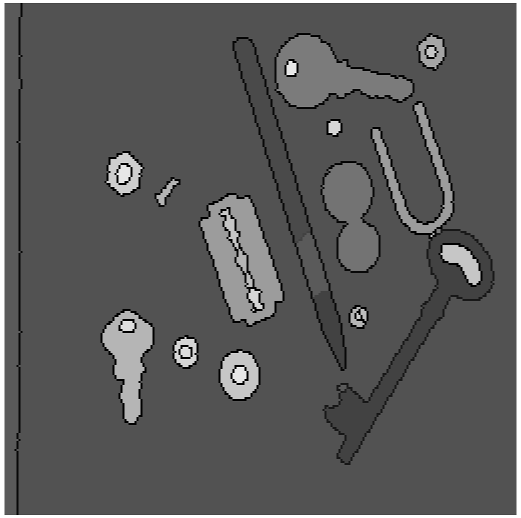
Example



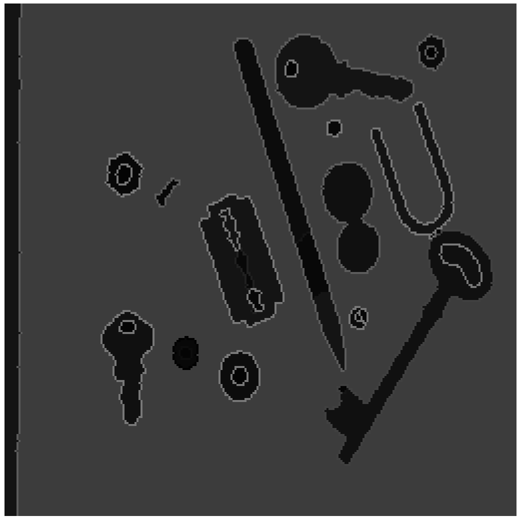
Original image



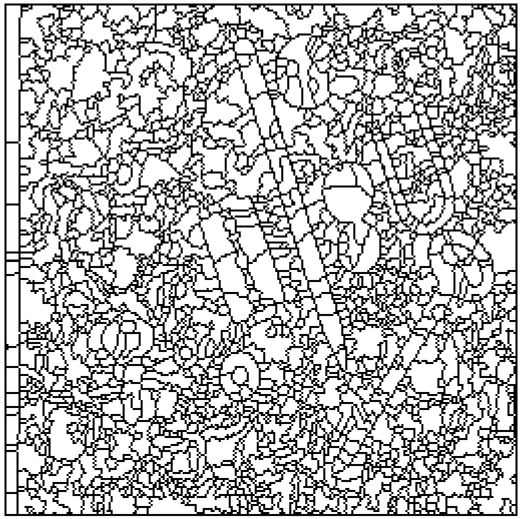
ψ_0



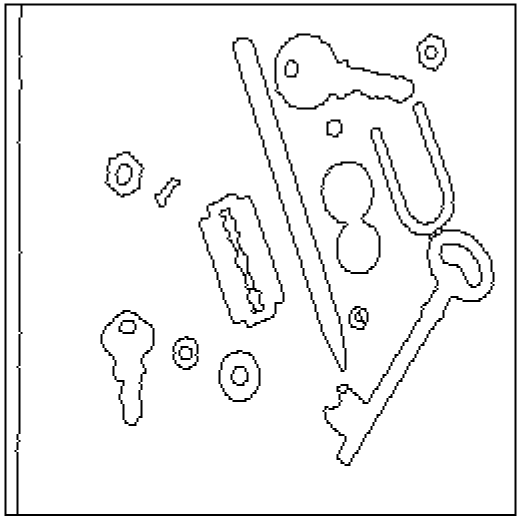
θ



q



initial contours



Contours of θ

Perspectives

The residual transformations offer interesting propects:

• Granulometric functions on greytone images
• More refined segmentations of sets (better handling of « potatoe-shaped » sets)
• unsupervised image segmentation based at the same time on a contrast criterion AND a shape/size criterion (alternate solution to hierarchisation) ⟶ Non parametric approaches
• Descriptions using stackings of cylinders (connection with some random sets models)
• Use of various openings (by reconstruction, by criteria, etc.)