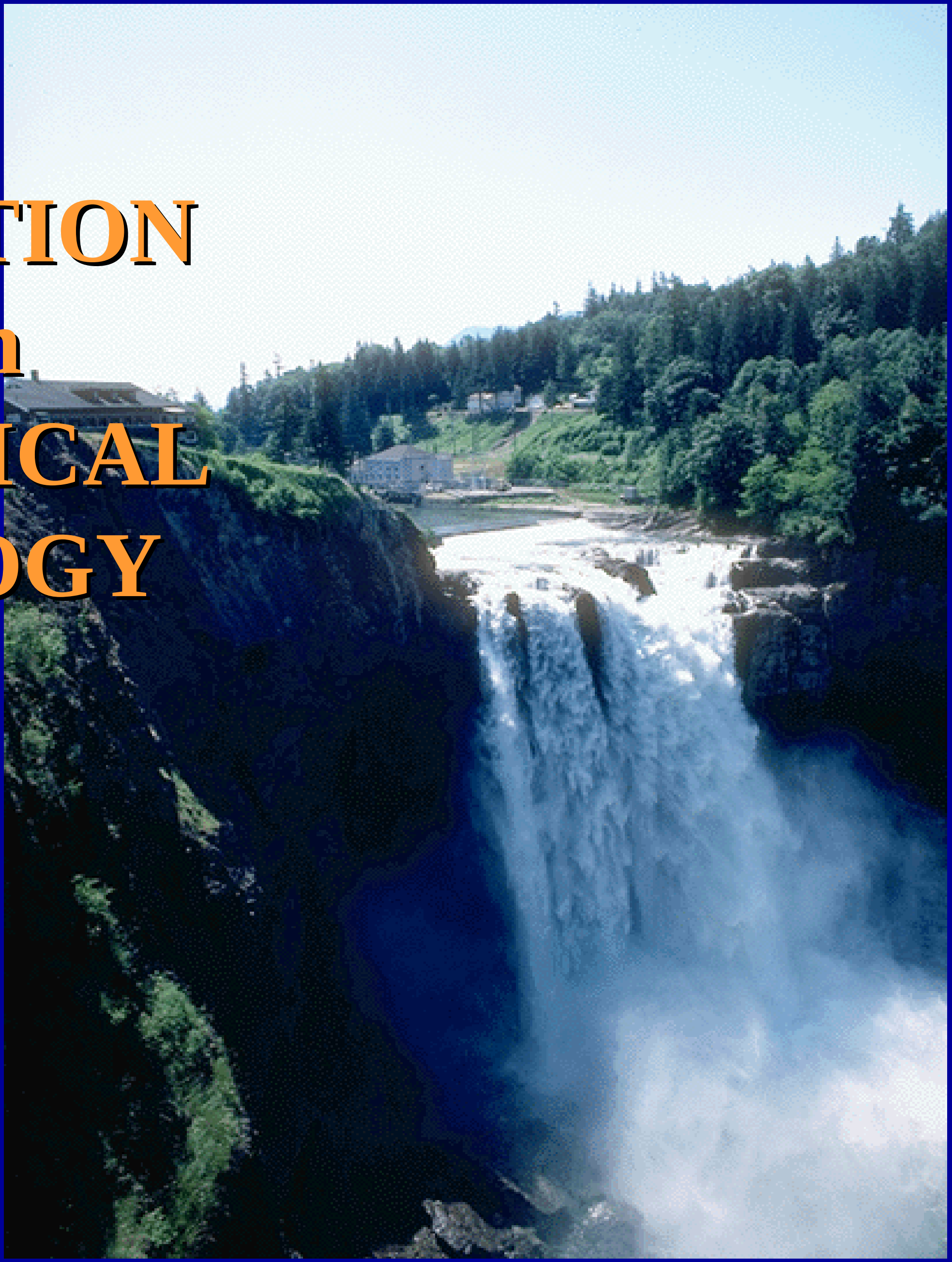
SEGMENTATION TOOLS in MATHEMATICAL MORPHOLOGY



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PRELIMINARY REMARKS

- **There is no general definition of image segmentation**

- **The morphological segmentation approach is a pragmatic one**

- **Nevertheless, this approach is proposing a segmentation methodology, a « user guide » for the segmentation tools**

- **It is important to keep in mind the various properties of these tools to avoid some traps. Their implementation must be as accurate as possible to insure a good quality of the results**

WHAT IS IT ALL ABOUT?

- **The segmentation tool in MM: the Watershed transformation**
  - Definition, description
  - How it can be built
  - Bias, problems, falsities
- **How to segment with fhe watershed transform**
  - The initial idea
  - Why it does not work well
  - Marker-controlled watershed
  - The segmentation toolbox and its user handbook
  - Old and new tools
- **Hierarchical segmentation**
  - Focus on the waterfall transformation
  - Pilings, another approach

APPLICATION EXAMPLES

- **Future developments**

A MACHINE-TOOL, THE WATERSHED TRANSFORM (WTS)

- In MM, image segmentation is organised around a transformation, the watershed transform (1979)

- The introduction of markers enhanced dramatically the efficiency of the watershed (1982)

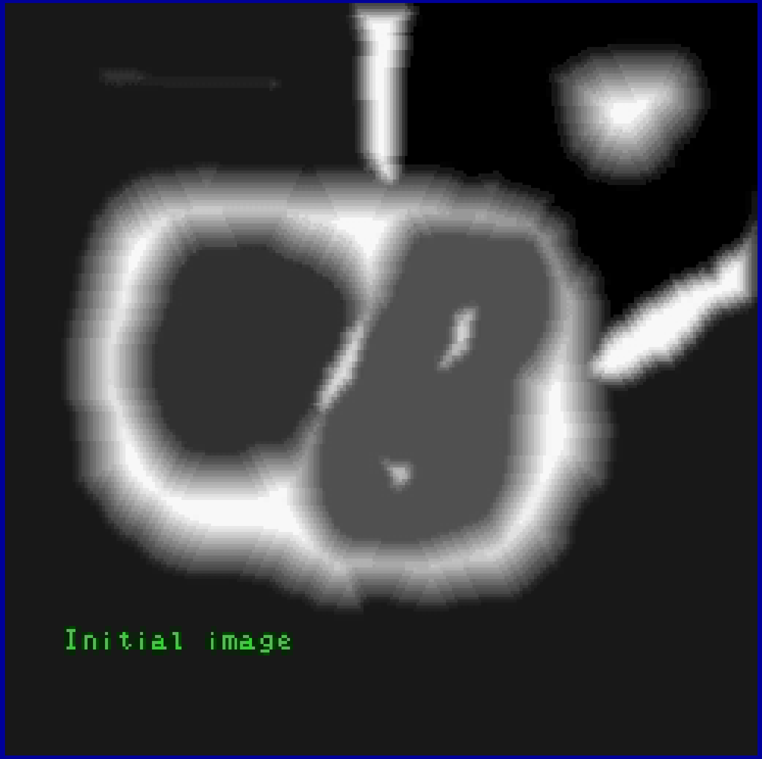
- This transformation belongs to the morphological operators « family » and especially to a class of operators named geodesic transformations

- The WTS can be realized on various image structures or representations: 3D images, videos, graphs, etc. This capability has led to hierarchical segmentation solutions (1990)

- Recent developments: new hierarchical tools, new criteria (2005-2007)

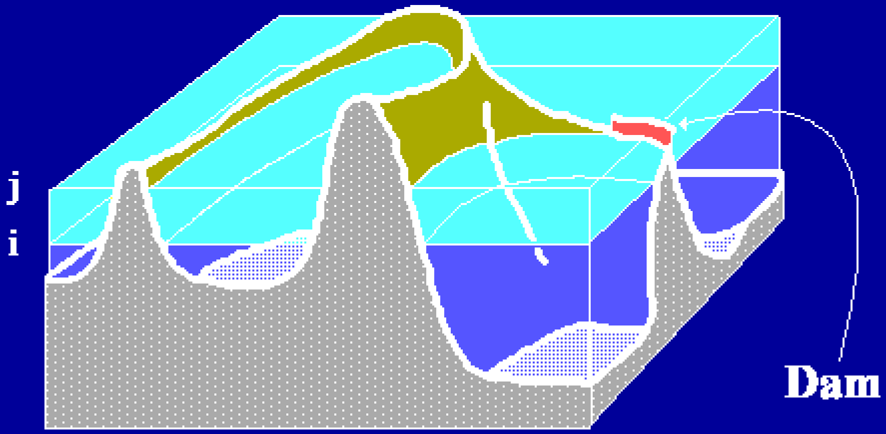
THE CLASSICAL WATERSHED ALGORITHM

- **It's a flooding process**
- **Flooding sources are the minima of the function**



Two hierarchies are used:

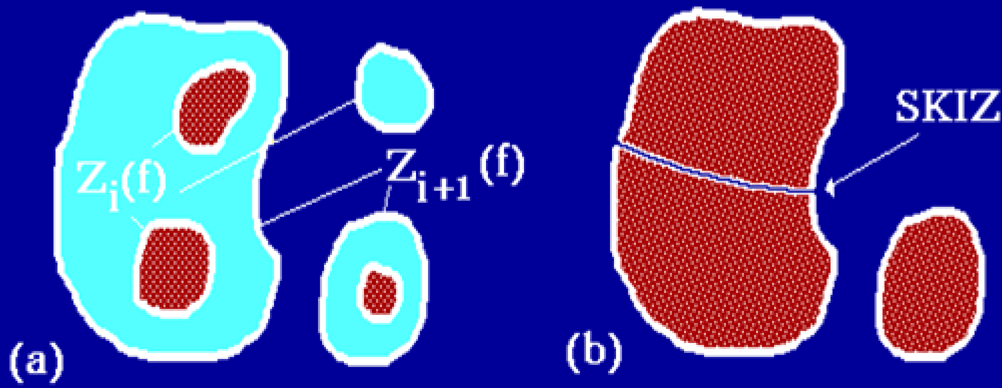
- **flood progression with the altitude (sequential process)**
- **flood on the plateaus/flat zones (parallel process)**



The result is a partition of the image into catchment basins and watershed lines (dams)

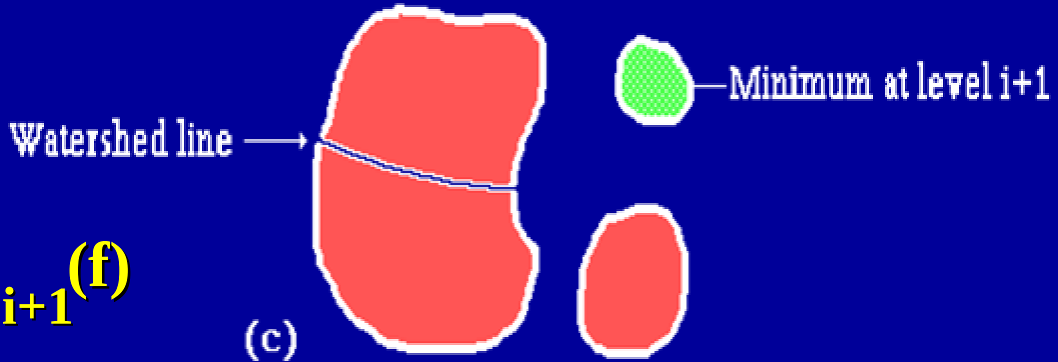
THE CLASSICAL WATERSHED ALGORITHM (2)

The transformation can be expressed on the successive levels Z_i of the function f:



$$W_0 = m_0(f)$$

The catchment basins at level 0 are the minima at this level



$$W_{i+1} = [\,SKIZ_{Z_{i+1}(f)}(W_i)] \cup m_{i+1}(f)$$

with:

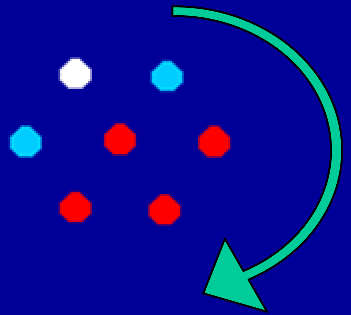
$$m_{i+1}(f) = Z_{i+1}(f) \,/\, R_{Z_{i+1}(f)}(Z_i(f))$$

R is the geodesic reconstruction

Use of the geodesic SKIZ transform to simulate the propagation without merging

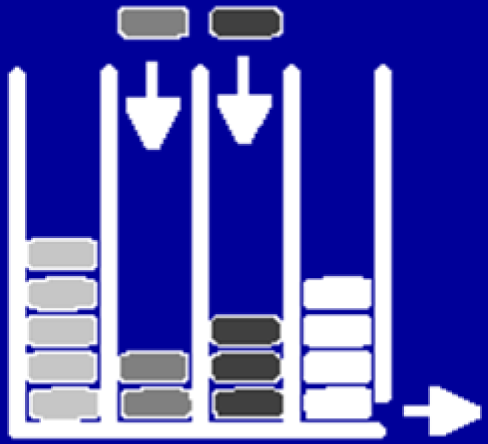
WATERSHED ALGORITHMS

- **Classical one (SKIZ using rotation thickenings)**



The use of rotating structuring elements in the SKIZ generates a non isotropic flooding on the plateaus

- **Hierarchical queues (a priori order defined in the queue)**



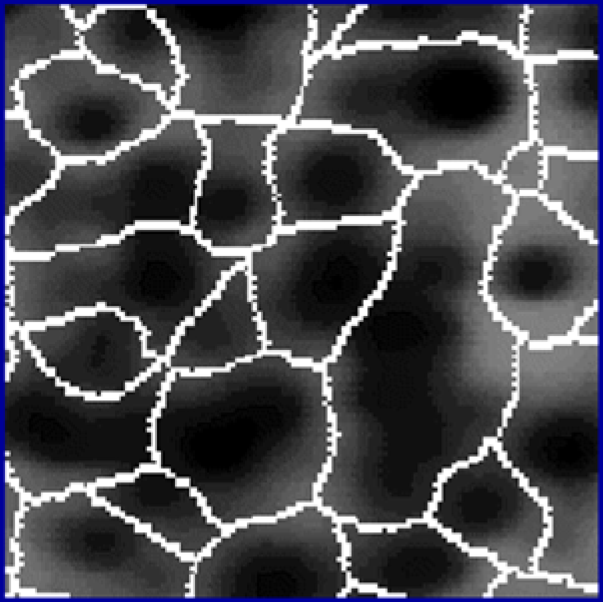
The tokens in the same stack should be processed at the same time

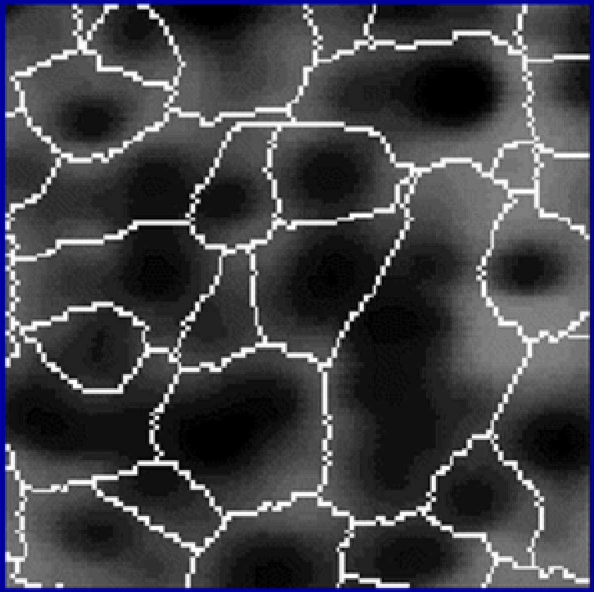
- **Watersheds based on graphs**

<u>Most of the watershed algorithms are biased</u>

BIASES AND FALSITIES WITH THE WATERSHED

For various reasons (complexity, computation speed, laziness…), the unbiased watershed transforms are seldom used.





Comparison between a true watershed (left) and the result of a "classical" algorithm.

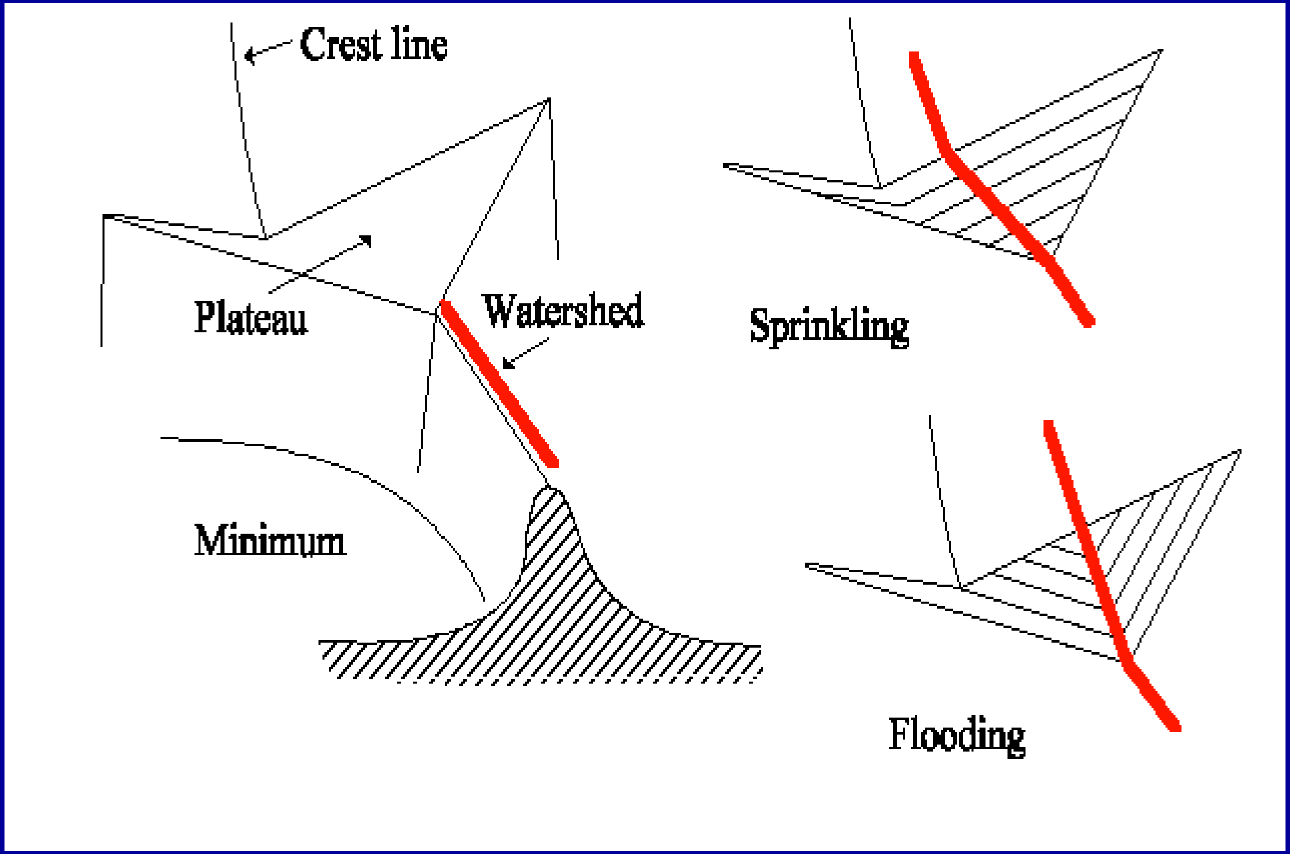
Due to this bias, the watershed transform is not UNIQUE (should be unique…). It depends on the algorithm used to build it.

These biases may have dramatic consequences in hierarchical approaches based on the comparison of adjacent catchment basins.

BIASES AND FALSITIES WITH THE WATERSHED (2)

The watershed line cannot be built by computing the trajectory of rain drops streaming on the topographic surface (run off). FORGET IT!

The flooding on the plateaus is based on a MODEL (constant speed). It has mainly two advantages: it is simple and it has a physical meaning.



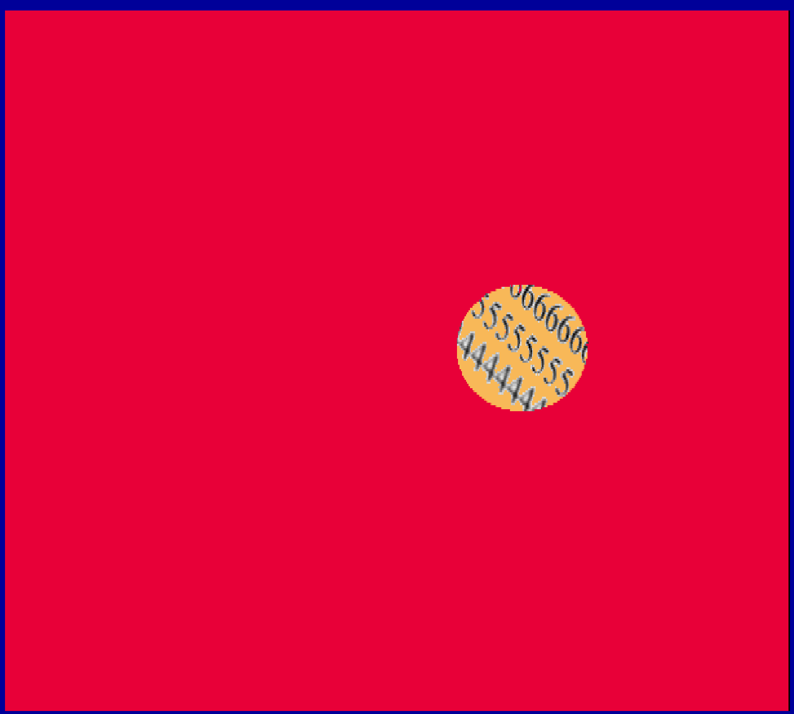
In any case, the results could not be identical (due to the propagation on the flat zones).

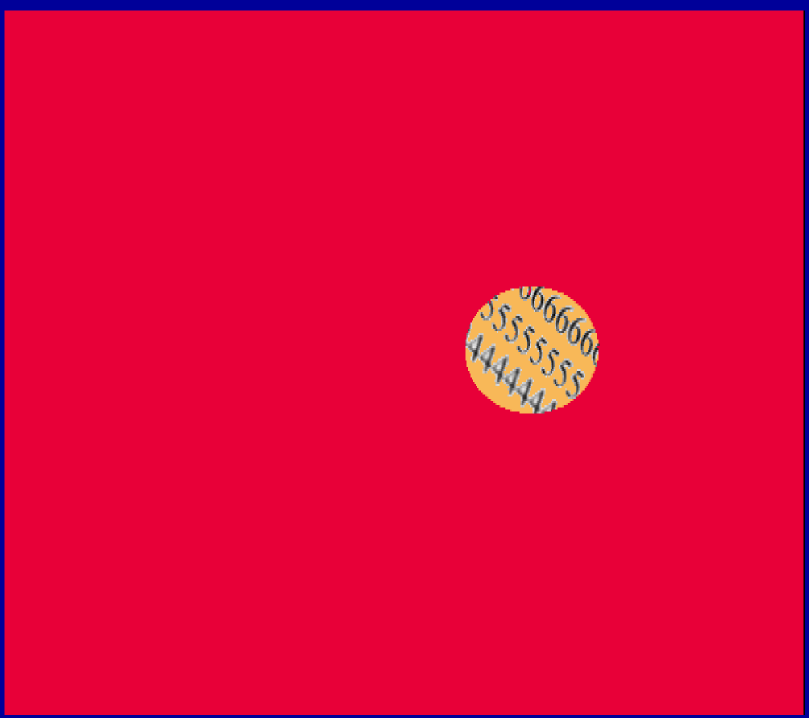
BIASES AND FALSITIES WITH THE WTS (3)

A watershed line is not a local feature. In particular, it is not linked to local structures (crest lines, ridges…). The watershed is not a LOCAL concept.

One can't, with the only local knowledge of the neighborhood of a point, answer the question:

Does this point belong to the watershed line?



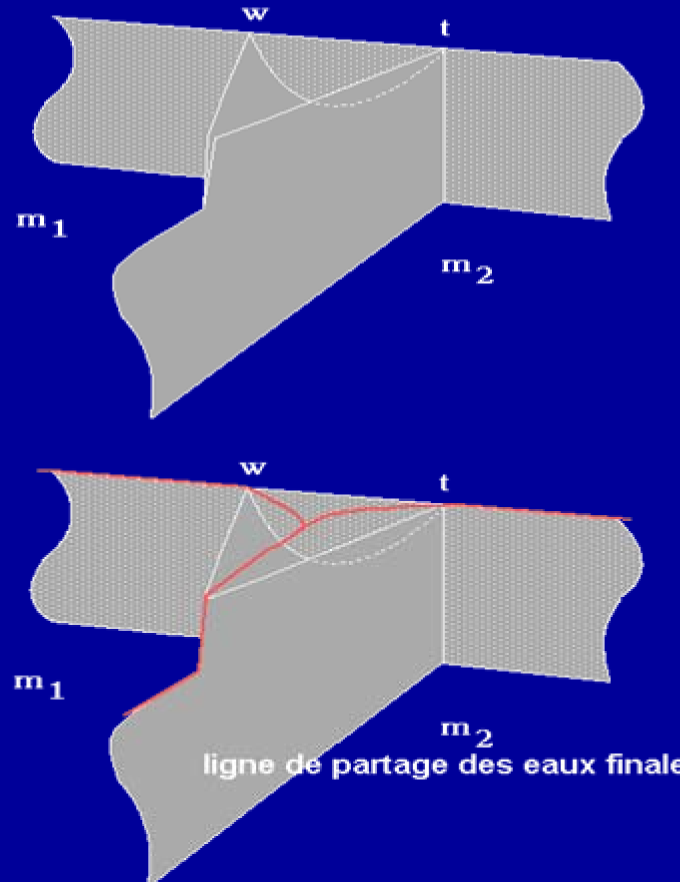


BIASES AND FALSITIES WITH THE WATERSHED (4)

Is flooding always an ascending process?
In other words, when flooding has reached altitude h, is it true that ALL the points at a lower altitude have been flooded?

The answer is NO! Counter-example: the button-hole

(true button-hole structure in french Gresivaudan)

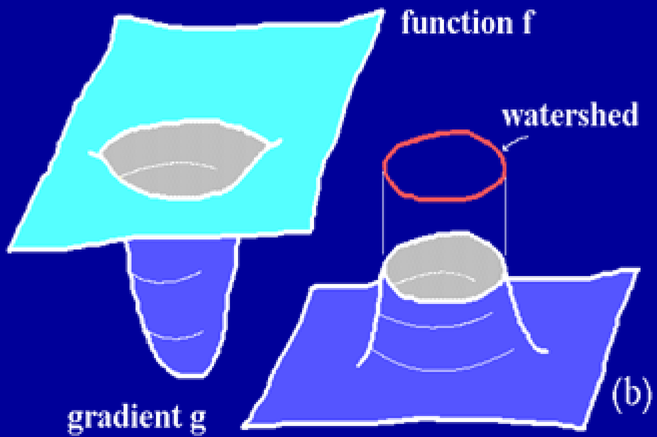


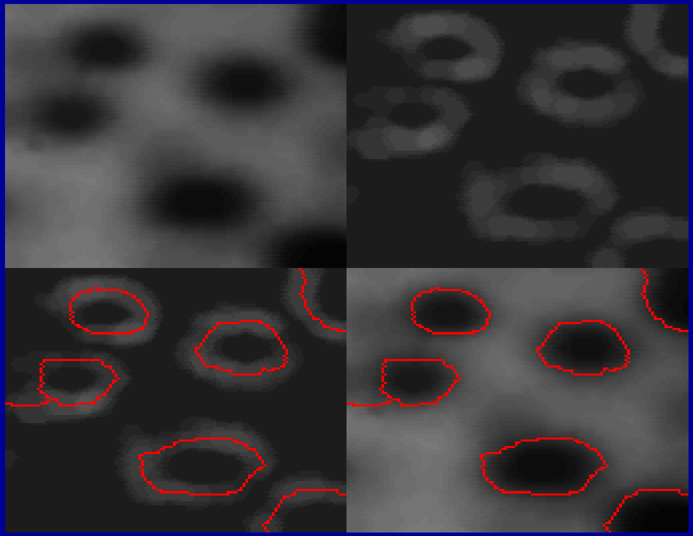
USE OF WATERSHED

The watershed transform is used for image segmentation

- **Greytone segmentation**

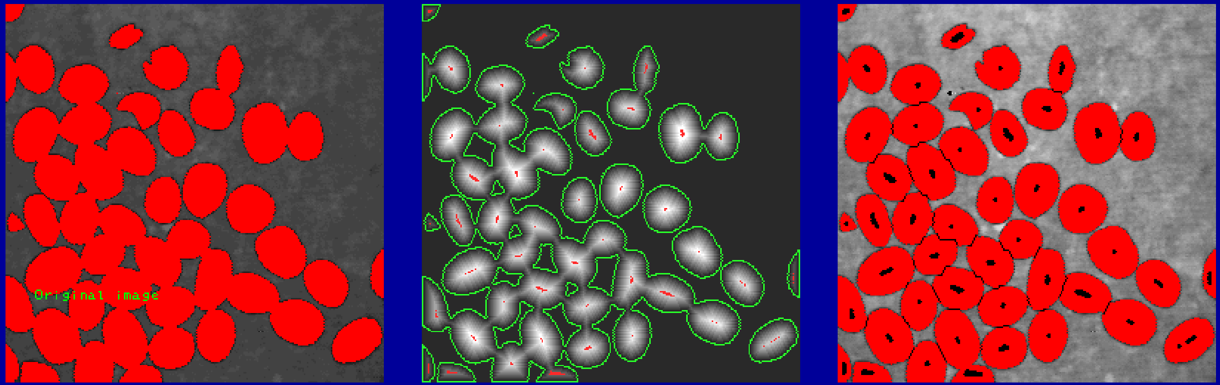
The watershed of the gradient corresponds to the contour lines





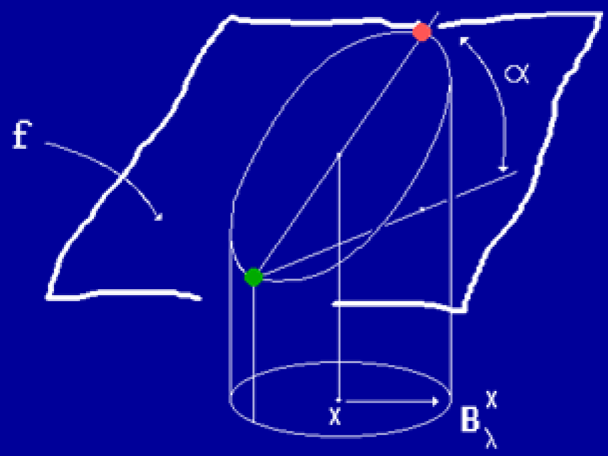
- **Shape segmentation**

Cutting objects into a union of “convex” sets by means of the watershed of the distance function



THE GRADIENT: A REMINDER

Morphological gradient



$g(f) = (f \oplus B) - (f \ominus B)$

Other morphological gradients (half-gradients) can also be defined:

$g\text{-}(f) = f - (f \ominus B)$

$g\text{+}(f) = (f \oplus B) – f$

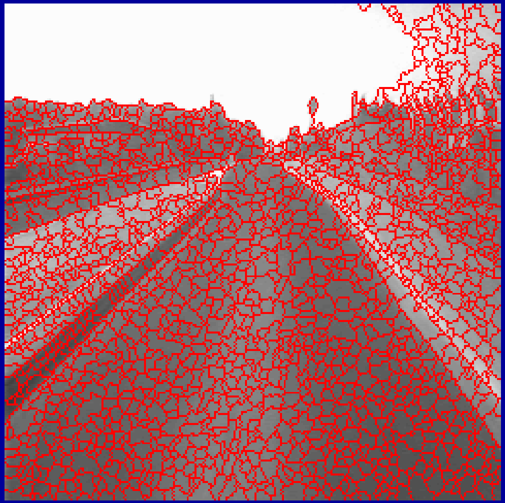
Thick gradients:

$g_i(f) = (f \oplus B_i) - (f \ominus B_i)$

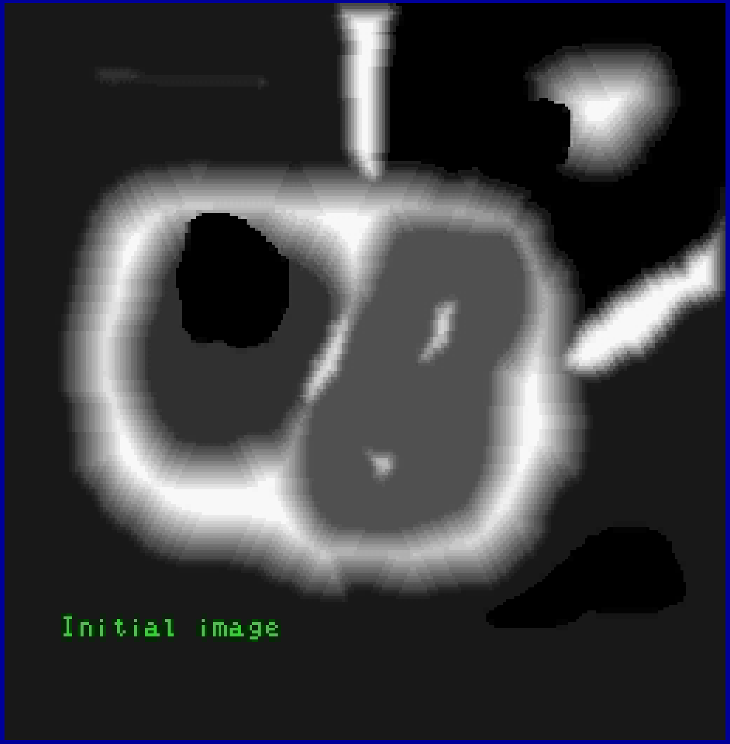
Regularized gradients

THE MARKER-CONTROLLED WATERSHED

The gradient watershed is over-segmented.



Gradient images are often noisy and contain a large number of minima. Each minimum generates a catchment basin in the WTS.

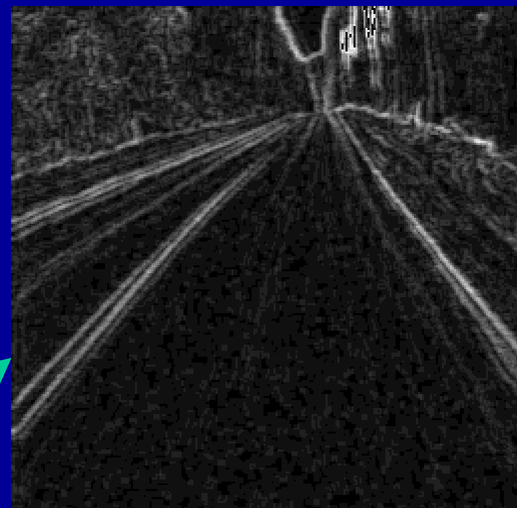


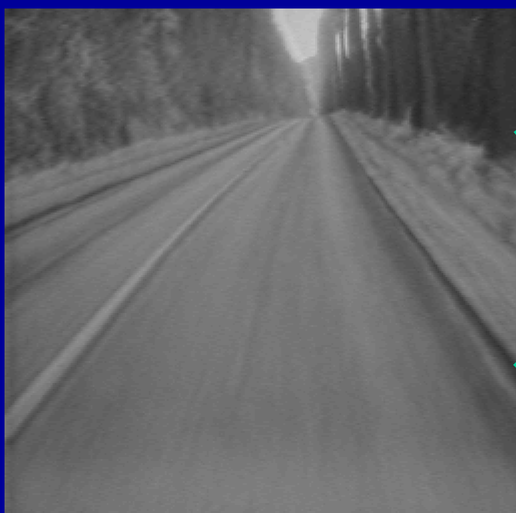
To avoid this over-segmentation due to numerous sources of flooding, one can select some of them (the markers) and perform the watershed transform controlled by these markers.

EXAMPLE OF MARKER-CONTROLLED WATERSHED

Road segmentation

gradient





Original image

Marker-controlled watershed of the gradient

markers

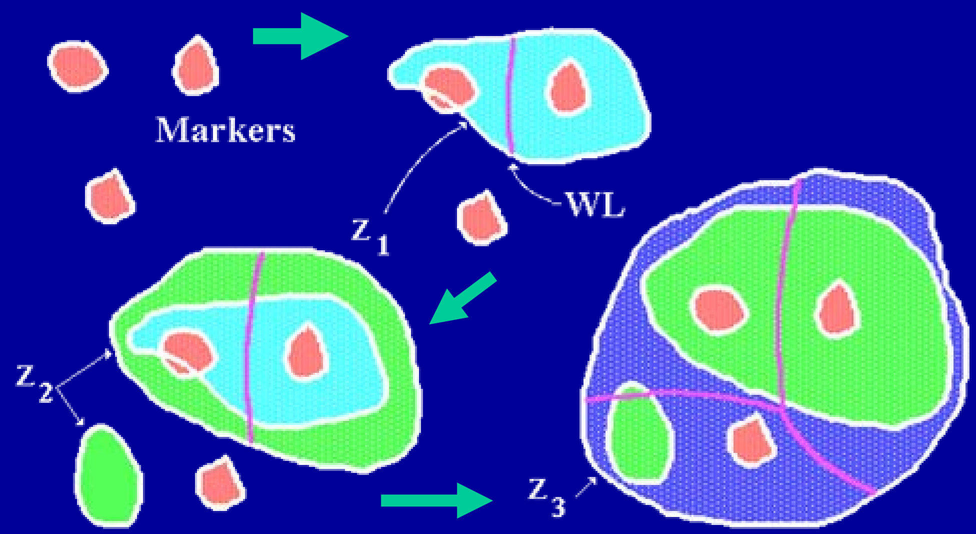
MARKER-CONTROLLED WATERSHED ALGORITHMS

- **Level by level flooding**

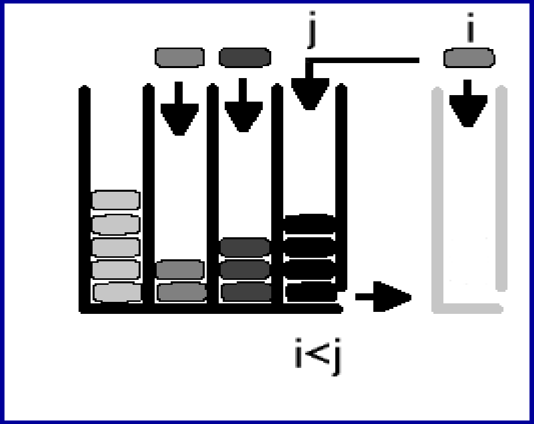
$$W_0 = M, \text{ marker set}$$

$$W_i = SKIZ_{Z_i(f) \cup M}(W_{i-1})$$

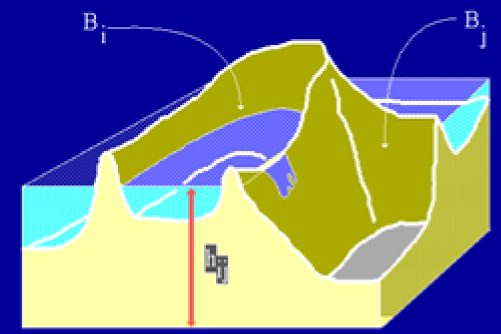
This algorithm is simpler than the classical one: there is no minima detection



- **Hierarchical queues**



A token at level i<j (the current level) may appear. In this case, it is treated as a token at level j (the i-queue is no longer alive)

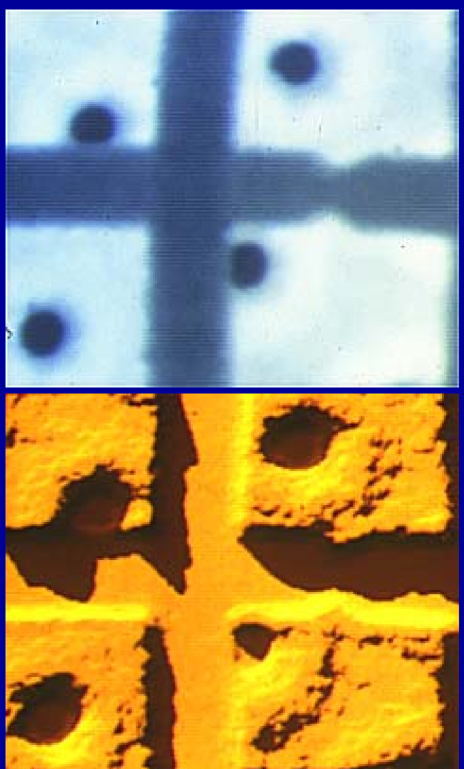


With marker-controlled watershed, overflow is the rule and not the exception

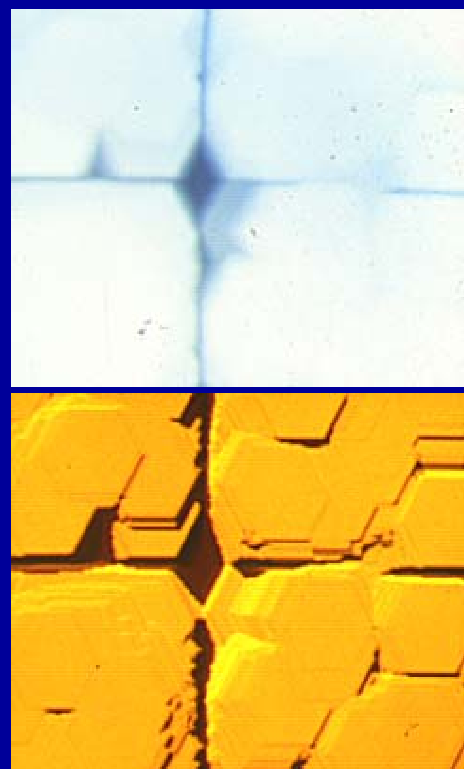
GEODESIC RECONSTRUCTION

The geodesic reconstruction is widely used in mathematical morphology:

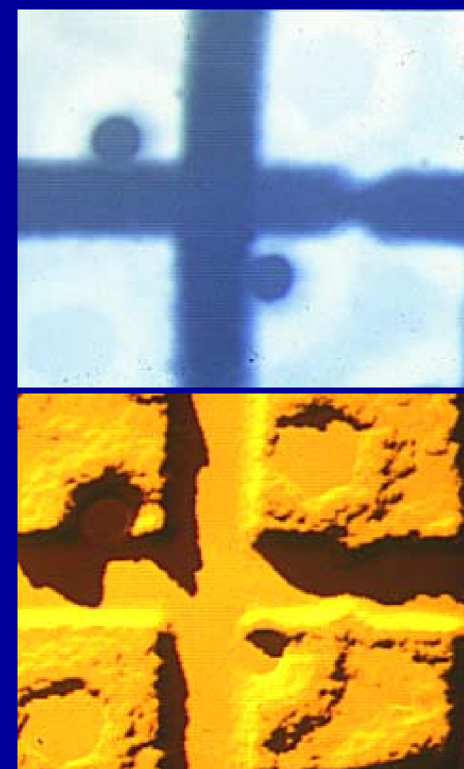
- **detection of extrema (minima, maxima)**
- **filtering (openings and closings by reconstruction)**
- **watersheds (swamping, homotopy modification)**
- **waterfalls**



g



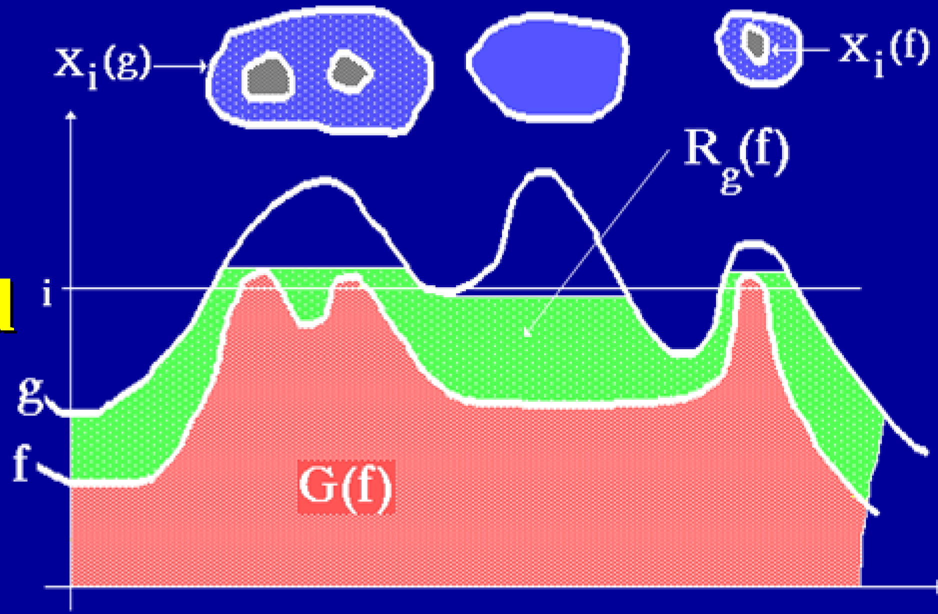
f

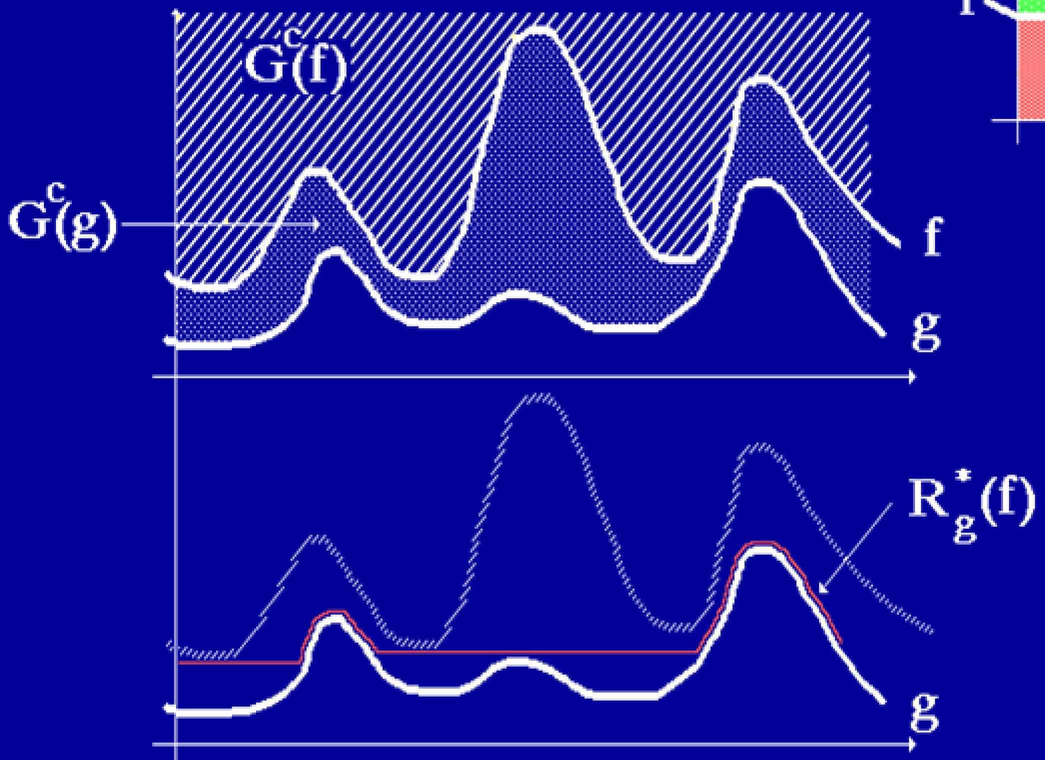


$R^*_g(f)$

GEODESIC RECONSTRUCTION (2)

The geodesic reconstruction is of outmost importance for performing and understanding the watershed transformation.

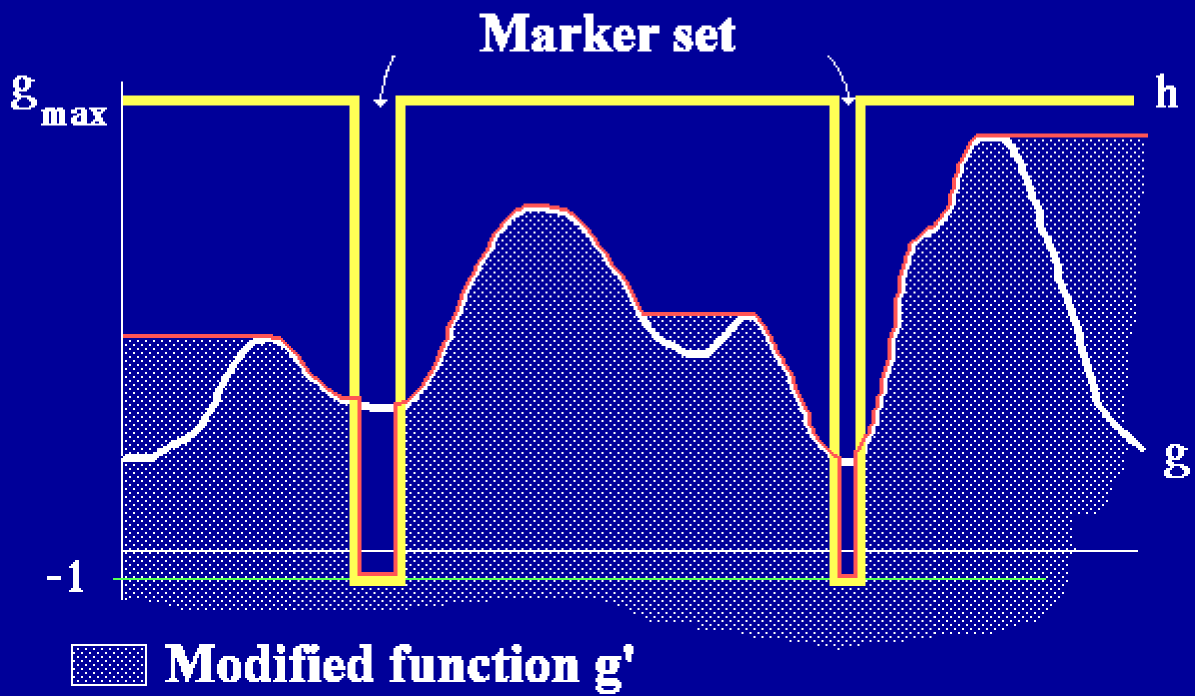




A dual reconstruction can also be defined (it uses geodesic erosions).

SWAMPING (HOMOTOPY MODIFICATION)

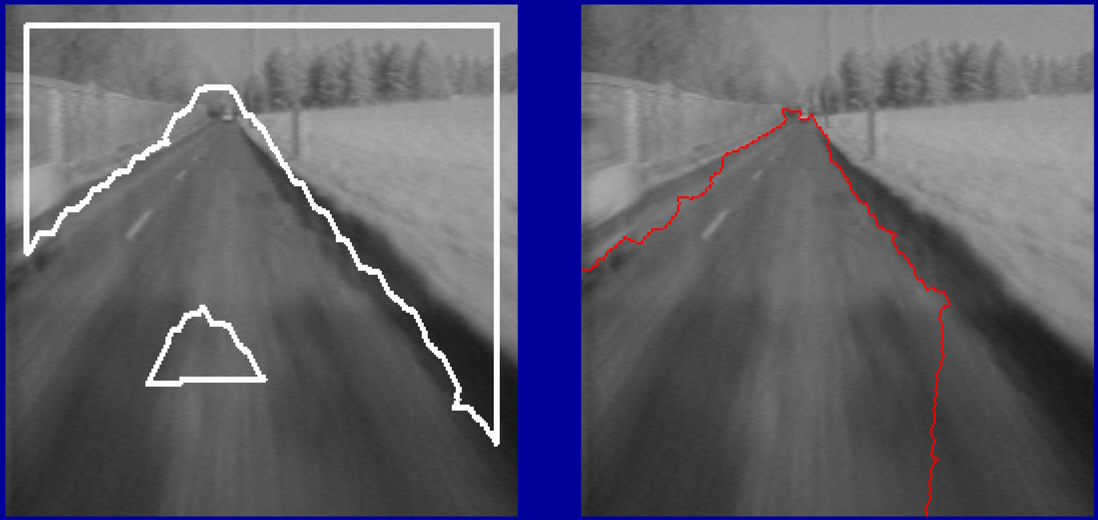
Based on reconstruction, swamping allows to build a new function whose minima correspond to the markers.



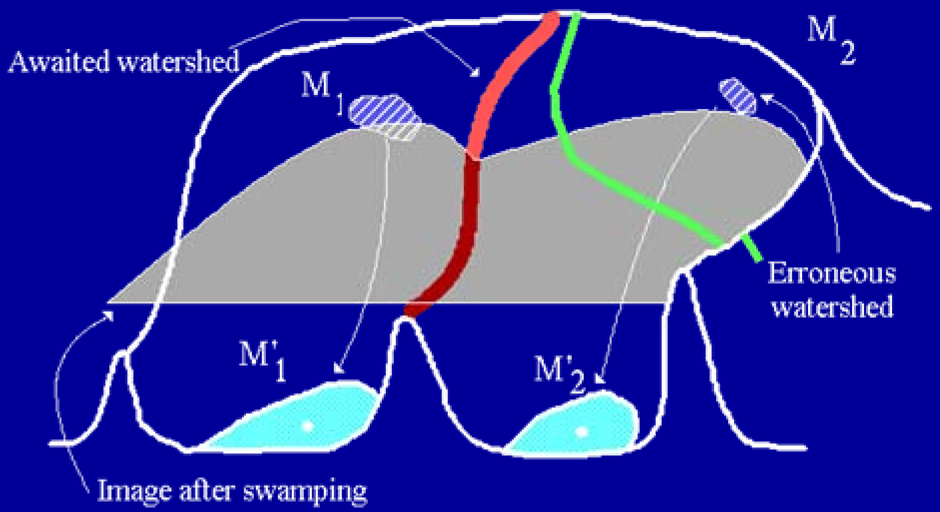
1) a marker function is defined:
$h(x) = -1$ iff $x \in M$
$h(x) = g_{max}$, if not

2) The reconstruction of h over $g' = \inf(g,h)$ is made:
$R^*_{g'}(h)$ → swamped function

POSITION OF THE MARKERS

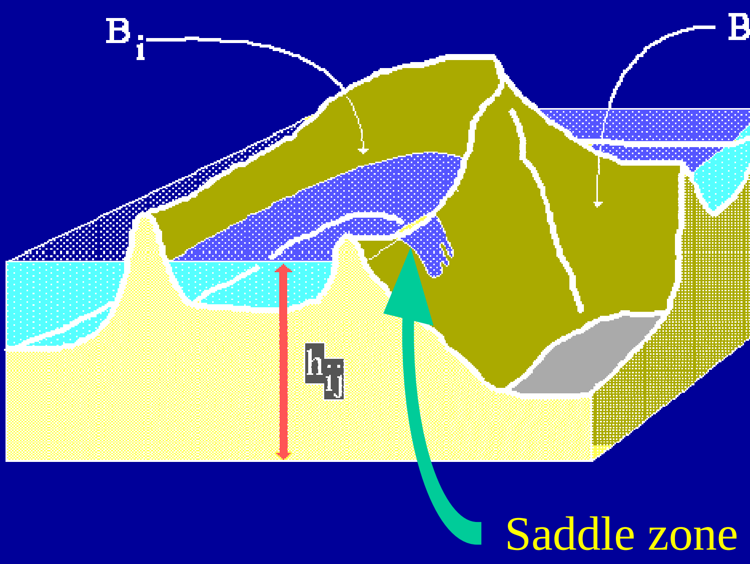


Segmentation obtained (right image) with a marker-controlled watershed of the gradient (markers on the left)

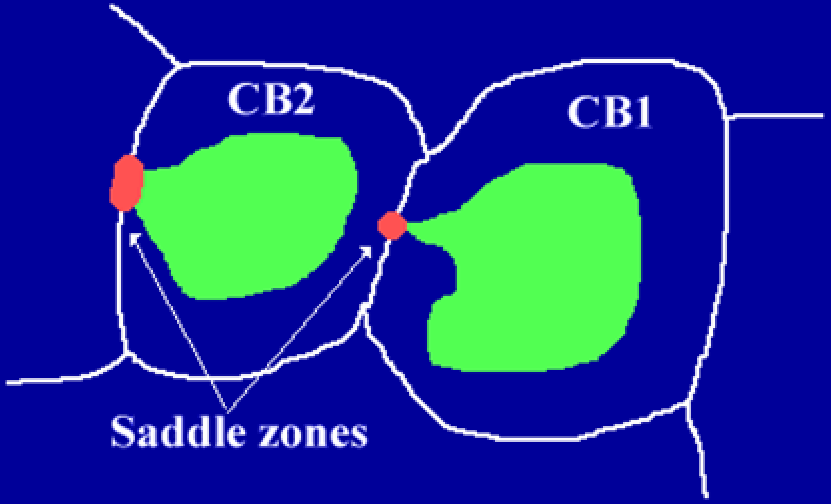


When minima are replaced by markers, it is of outmost importance to control the position of these markers

POSITION OF THE MARKERS (2)



Question:
if we replace the original minima by markers, where to put the markers to insure that the final watershed will be the same?



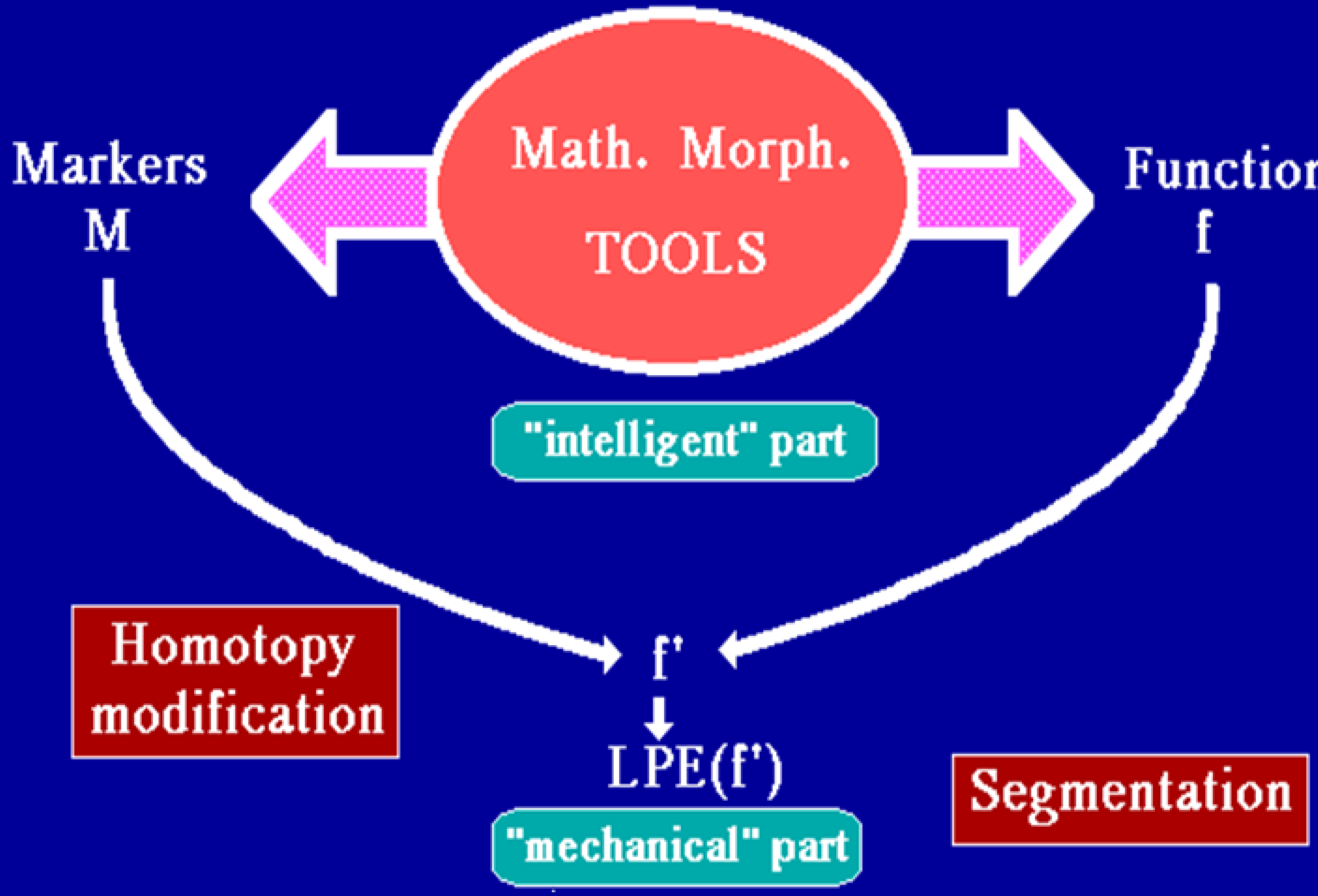
Notion of lower catchment basin

It’s the part of the catchment basin flooded before the first overflow (by the lower overflow zone)

Solution: the markers must be included in the lower catchment basins.

A one-to-one correspondence is not required provided that all the markers included in one lower catchment basin are given the same label.

THE SEGMENTATION PARADIGM

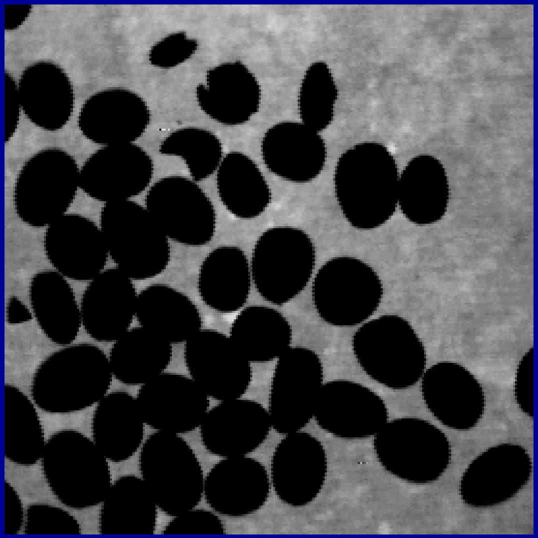


WHICH CRITERIA?

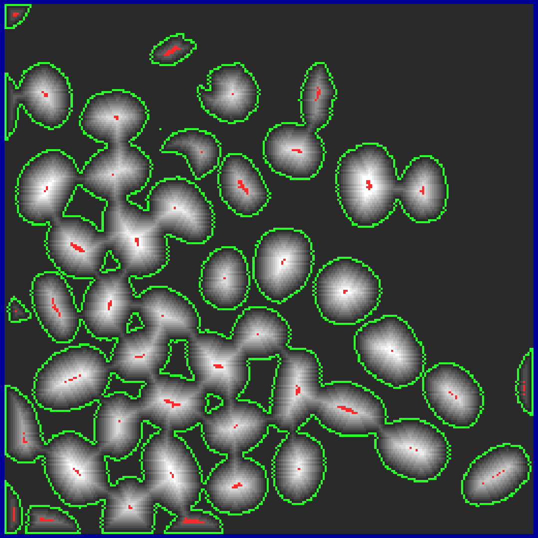
- **Contrast criteria**
  - **Gradient**
  - **Top-hat transform**
- **Size and shape criteria**
  - **Distance function**
  - **Granulometric function**
  - **Quasi-distance**
- **Combination of both criteria (?)**

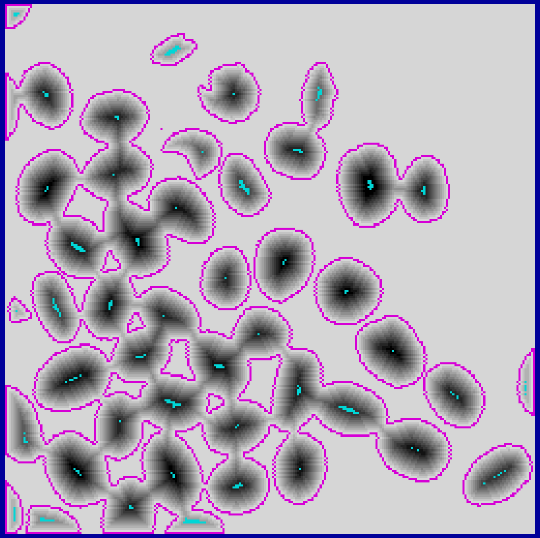
APPLICATIONS

Coffee grains



The **distance function** of the set is computed. This distance function is inverted and its watershed is performed. The marker set is made of the maxima of the distance function.



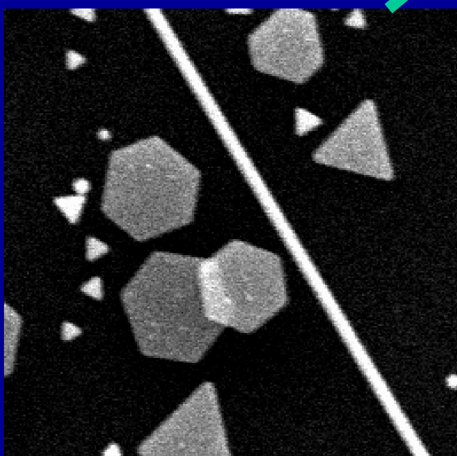


The watershed is performed on the support of the distance function. The maxima are filtered to avoid over-segmentation due to parity problems.

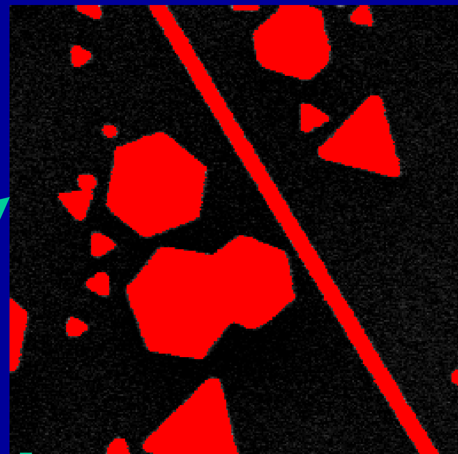
APPLICATIONS (2)

Silver nitrate grains on a film

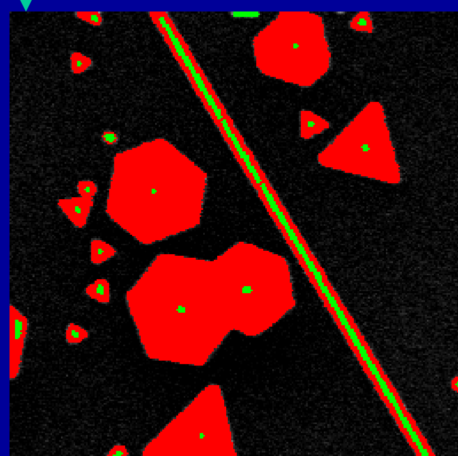
Problem: segmentation of the grains, even when overlapping



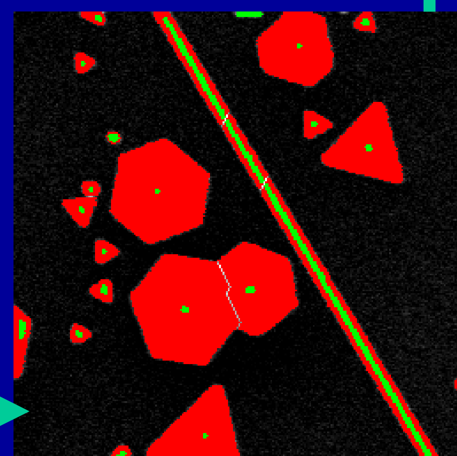
Original image



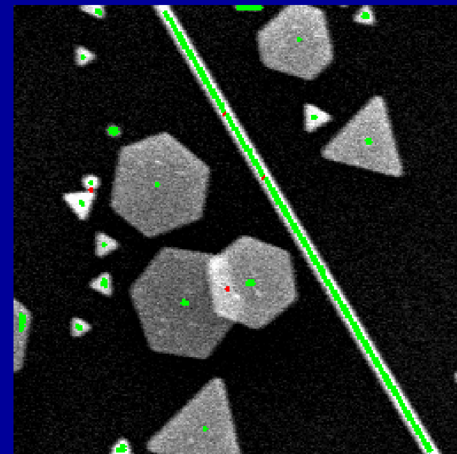
Mask of the grains



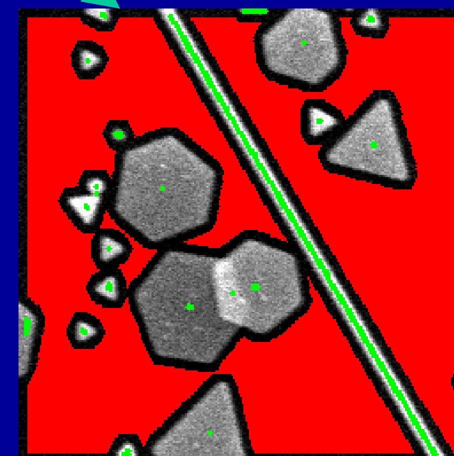
1st markers, maxima of distance function



Watershed of the distance function

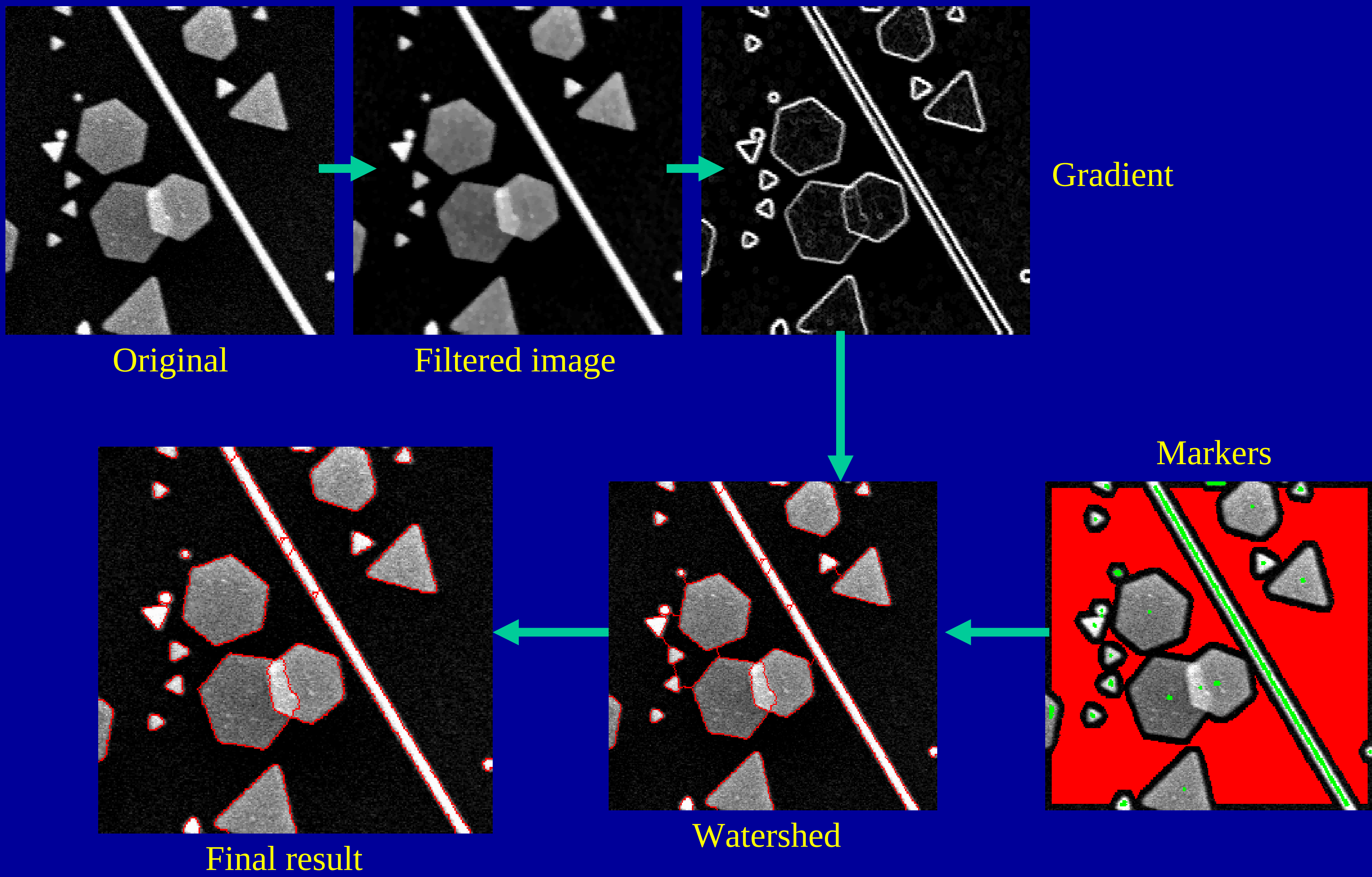


2nd marker set



The background marker is added. Final marker set

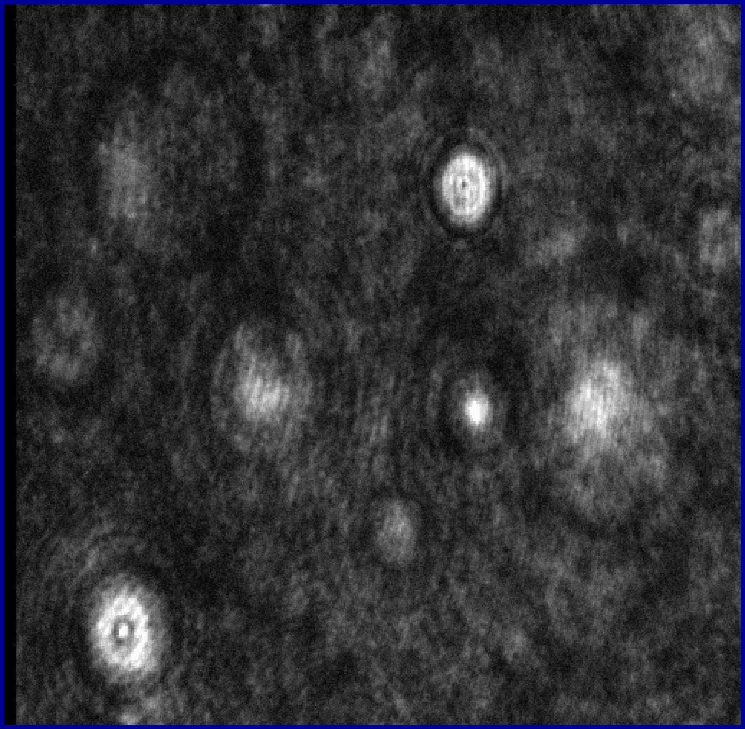
APPLICATIONS (3)



APPLICATIONS (4)

3D restitution of water drops from an hologram

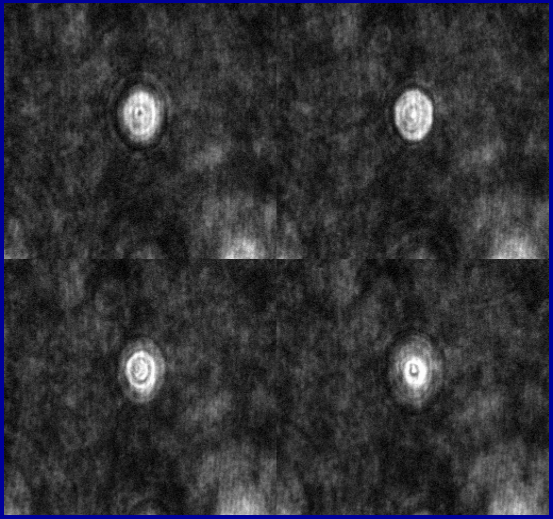
A 3D image of an aerosol (artificial fog) is generated from an hologram. Various sections of the 3D image are taken with a low focus depth camera.

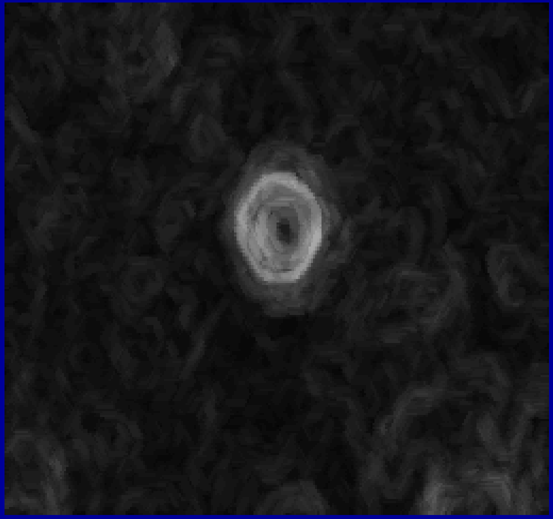


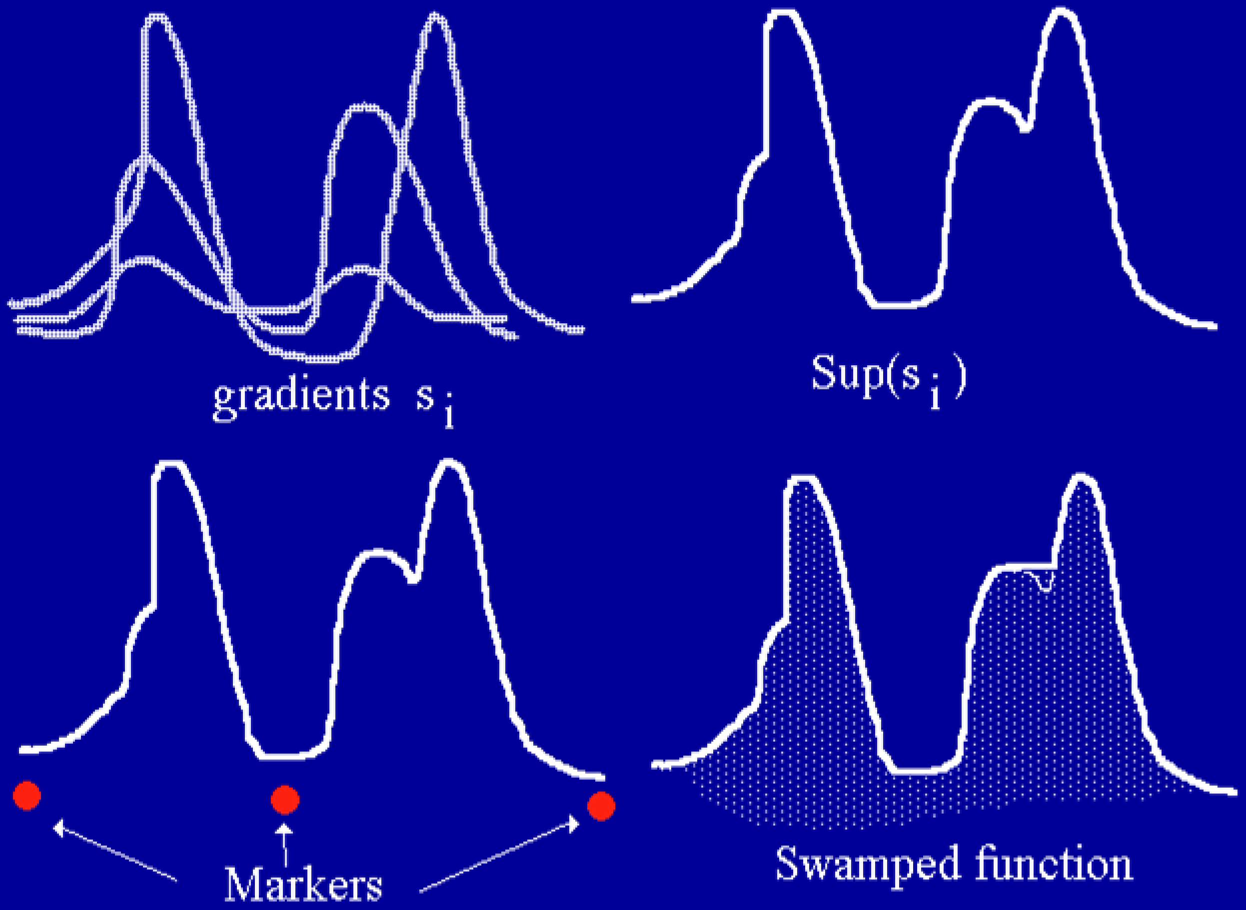
- **n sections s_i**
- **find the best contour**
- **position x, y, z of each drop**
- **volume**

APPLICATIONS (5)

Criterion:
Sup of the gradients



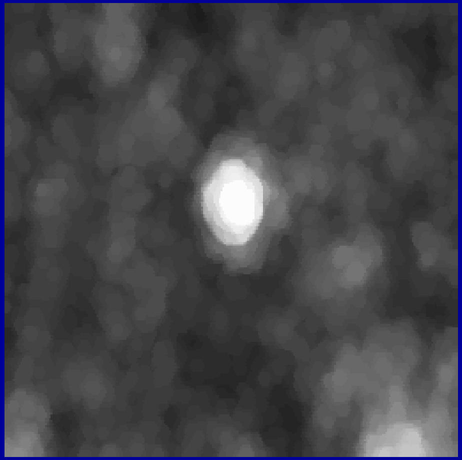


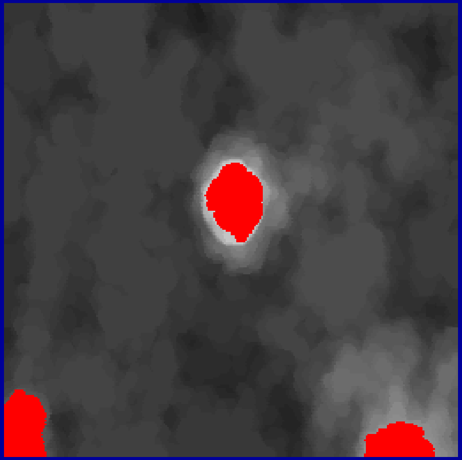


APPLICATIONS (6)

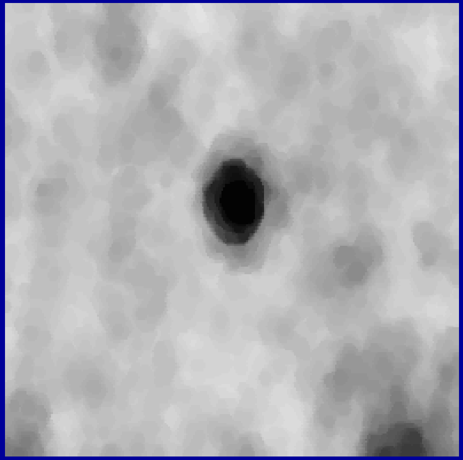
Markers:

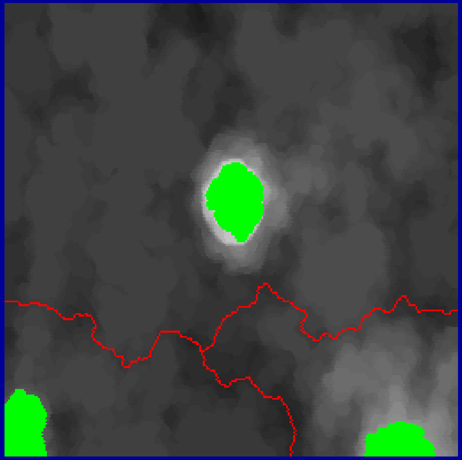
- **Drops → significant maxima of the filtered sup of all the sections**





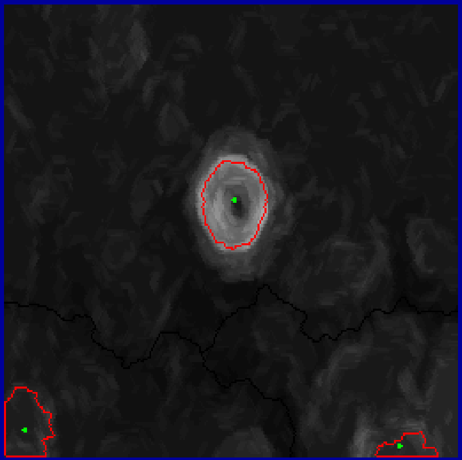
- **Background → watershed of the sup image (inverted)**



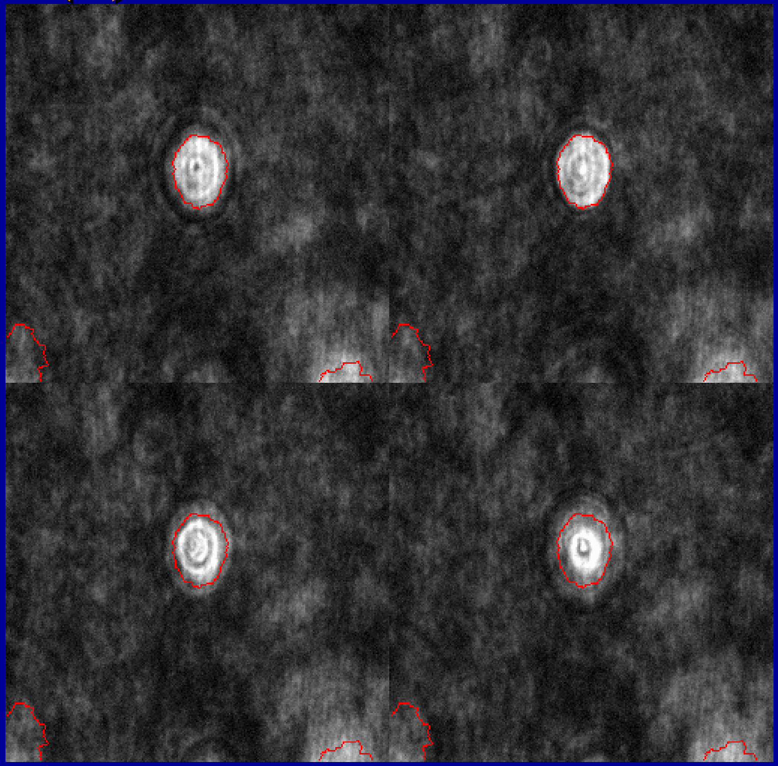


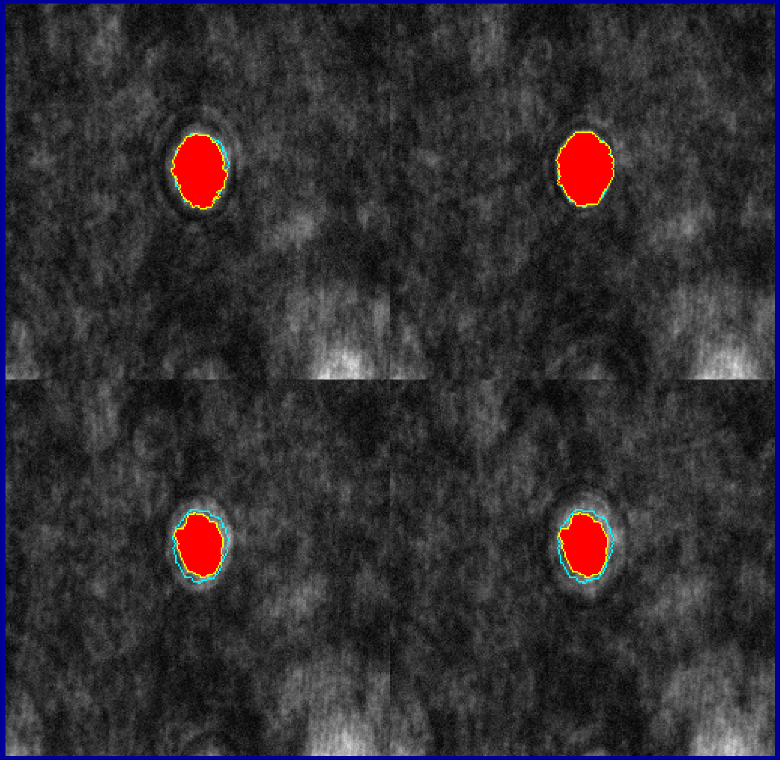
This watershed is a marker-controlled watershed (markers of the watershed are the drop markers)

APPLICATIONS (7)



Final watershed (left). The same watershed image superimposed on the different sections (right).





To find the best section, a marker-controlled gradient watershed is performed on each section with the same set of markers (result in blue) and the best fit with the previous contour is determined. The corresponding section gives the z-position of the drop.

APPLICATIONS (8)

Traffic lanes segmentation

The markers of the lanes are determined by an automatic thresholding. The marker of the background is the complementary set of a dilation.

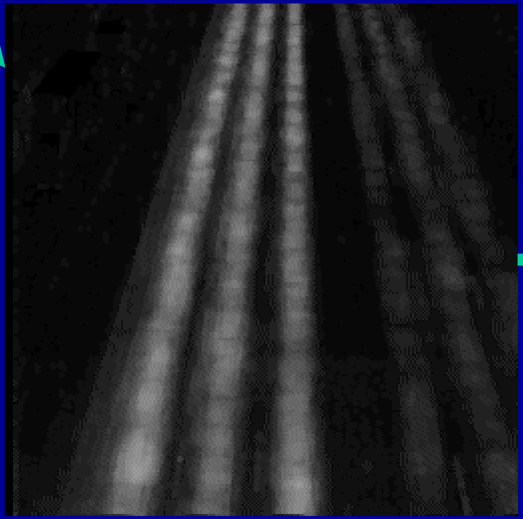
From a sequence of n images f_i , two images are computed:

- The mean,

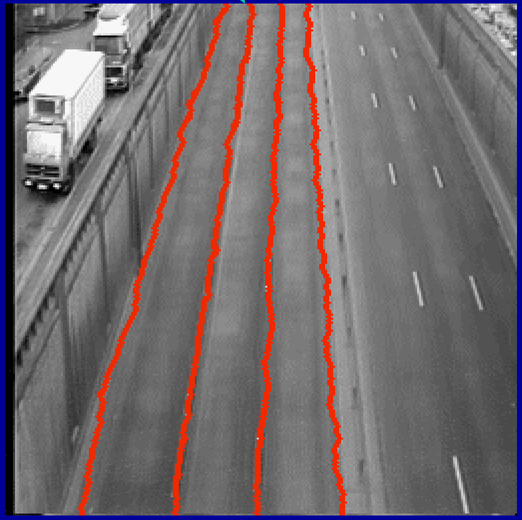
$$\Sigma f_i /n$$

- The mean of absolute differences,

$$\Sigma |f_i - f_j| /n$$



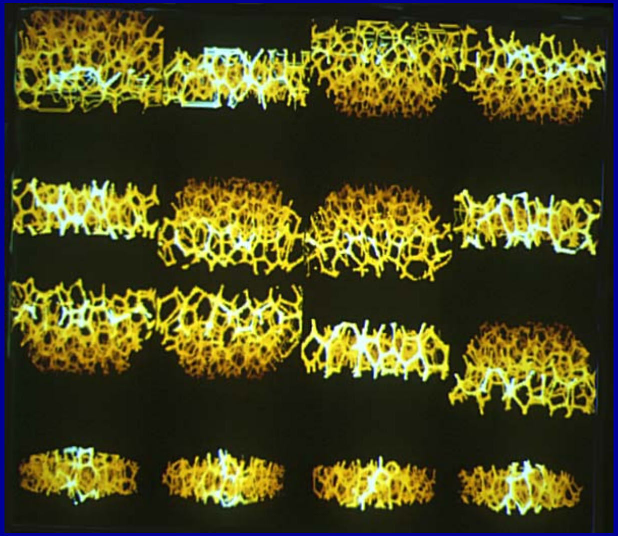
APPLICATIONS (9)



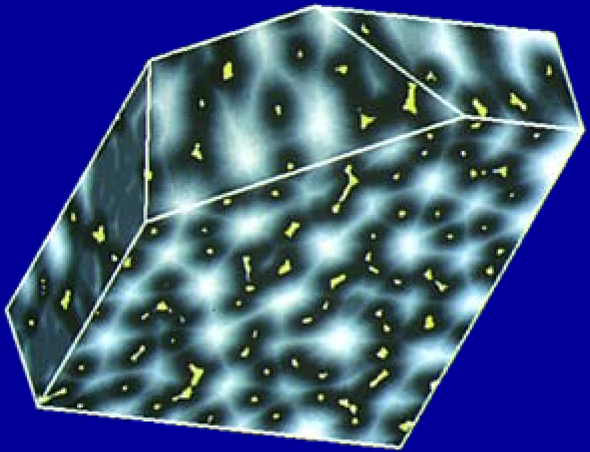
- **Extraction of road marking by a top-hat transform**
- **Calculation of the distance function of the road marking between the markers**
- **watershed of the distance function**

APPLICATIONS (10)

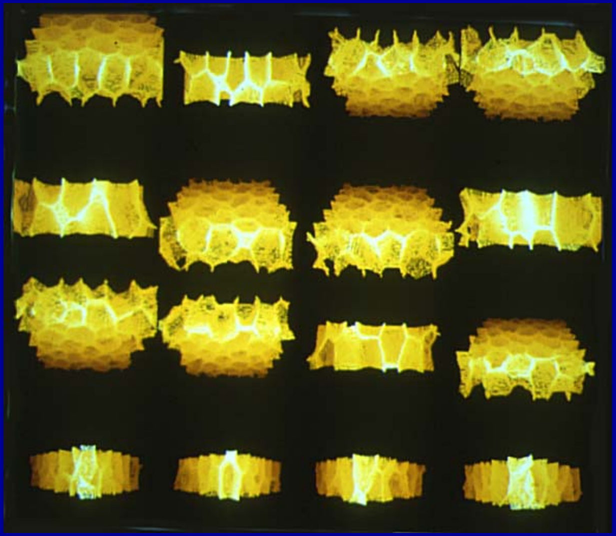
3D segmentations based on distance functions



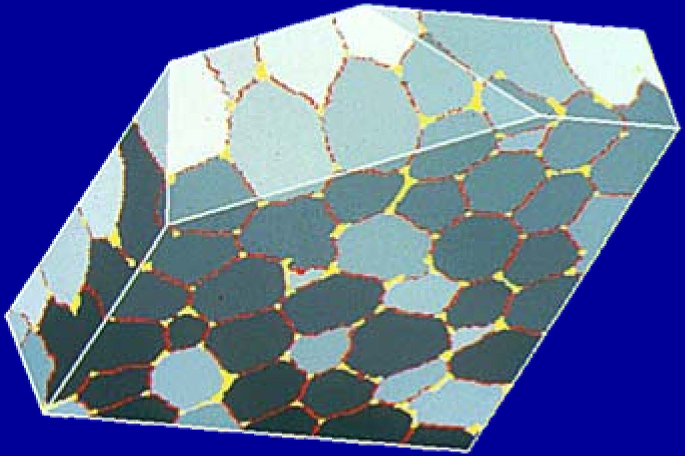
Polyester foam



Distance function



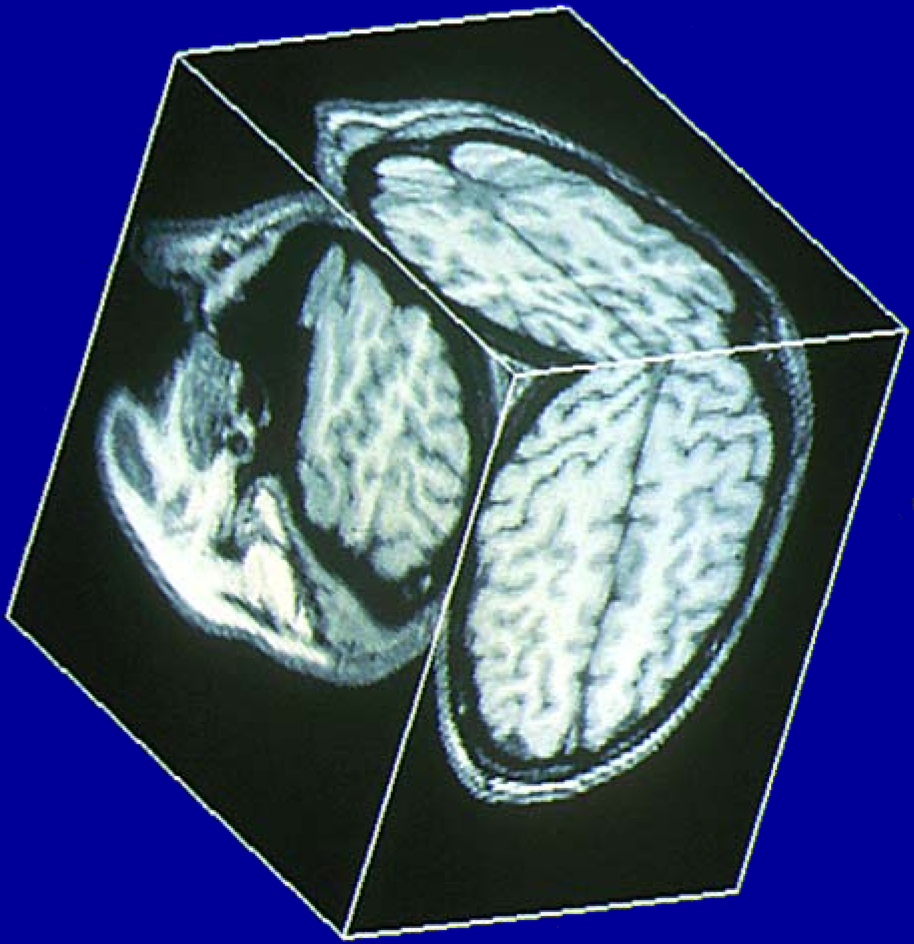
3D watershed

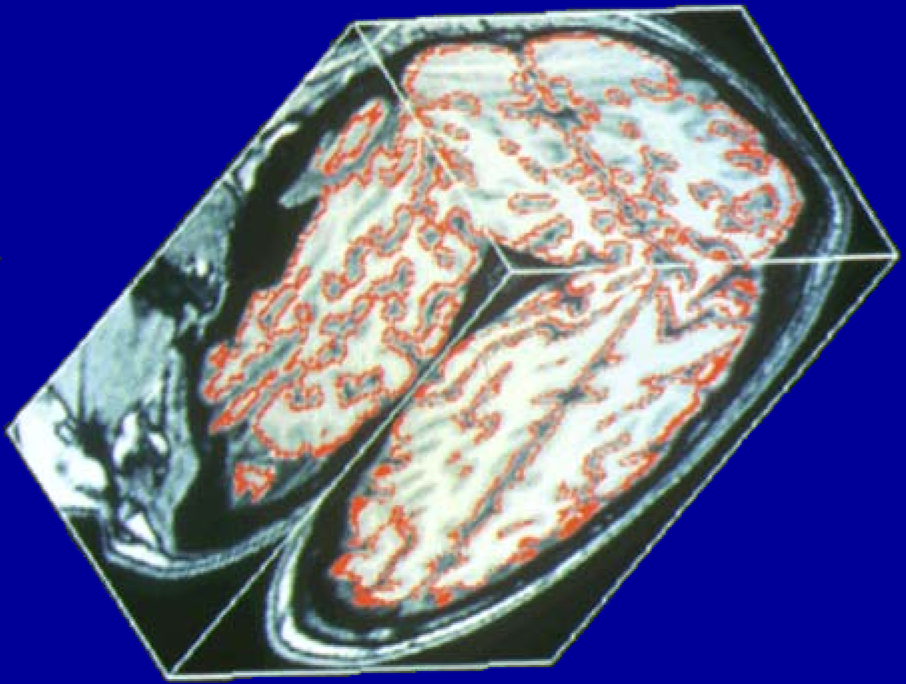


APPLICATIONS (11)

3D segmentations based on gradients

3D brain NMR image





CRITERIA BASED ON NUMERICAL RESIDUES

Starting from two sequences of transformations ψ_i and ζ_i with $\psi_i \geq \zeta_i$, we define two operators:

- **The residual Transformation** $\theta = \underset{i \in I}{Sup}\left(\psi_i - \zeta_i\right)$
- **Its associated function** $q = \arg\max\left(\psi_i - \zeta_i\right) + 1$

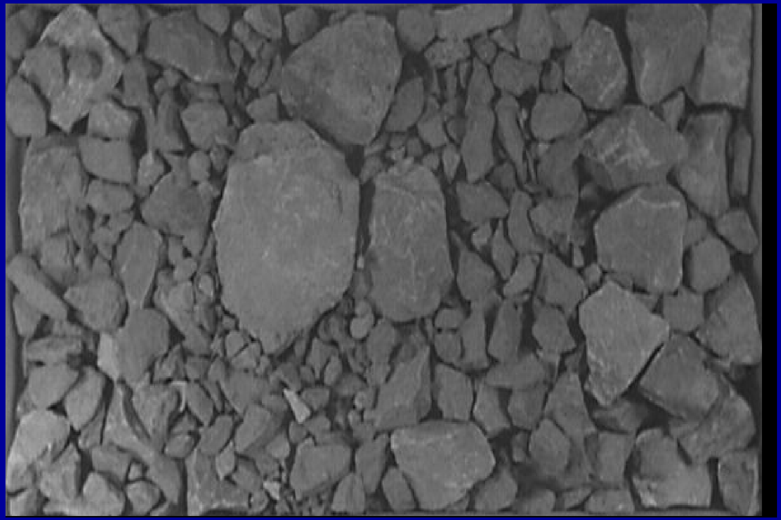
$$\left.\begin{array}{l}\psi_i = \gamma_i \\ \zeta_i = \gamma_{i+1}\end{array}\right\} \longrightarrow$$

θ is named ultimate opening
q is the granulometric function

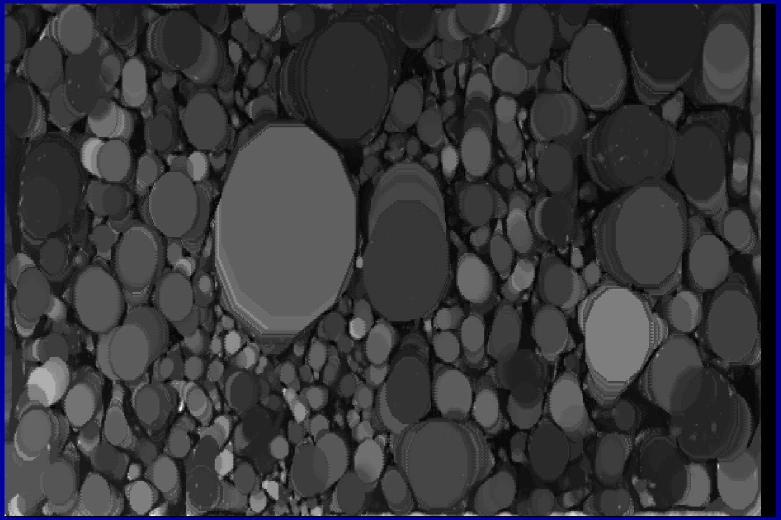
$$\left.\begin{array}{l}\psi_i = \varepsilon_i \\ \zeta_i = \varepsilon_{i+1}\end{array}\right\} \longrightarrow$$

q is called quasi- distance.

ULTIMATE OPENING GRANULOMETRIC FUNCTION

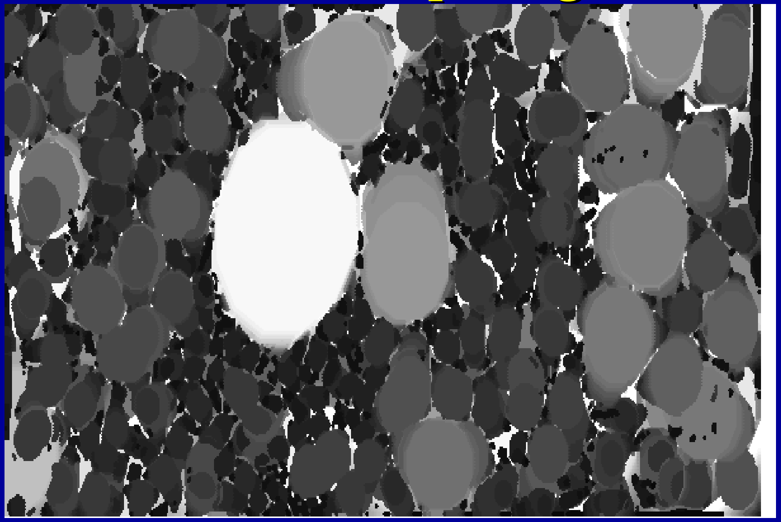


Heap of rocks



Ultimate opening

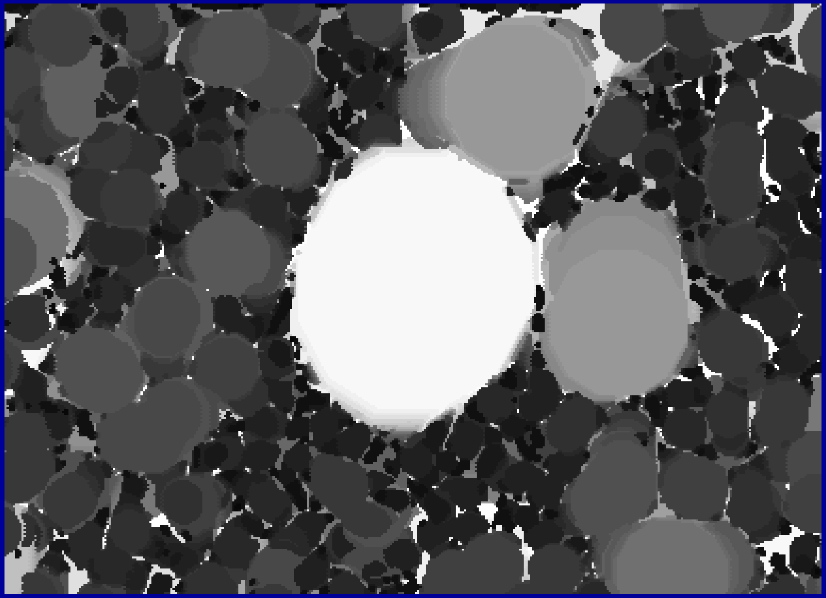
At every point x, q(x) is equal (up to the unit value) to the radius of the greatest significant cylinder covering x



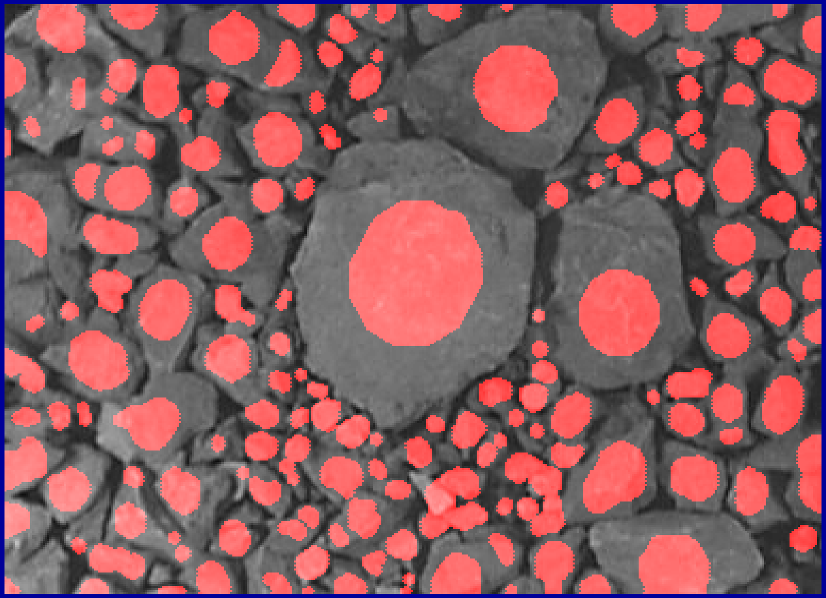
Granulometric function

MARKERS GENERATION

Each threshold λ of the granulometric function q is eroded by a disk of size kλ (k<1)

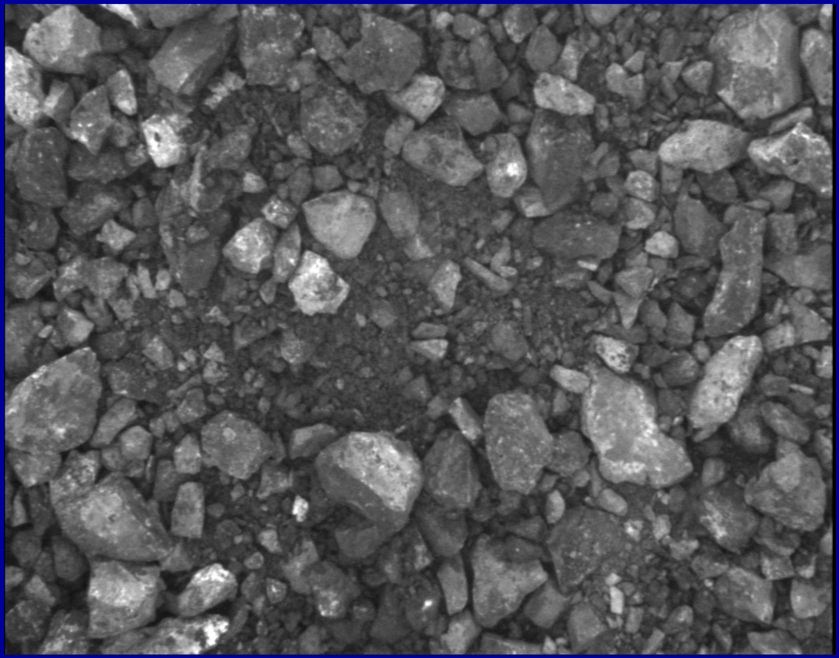


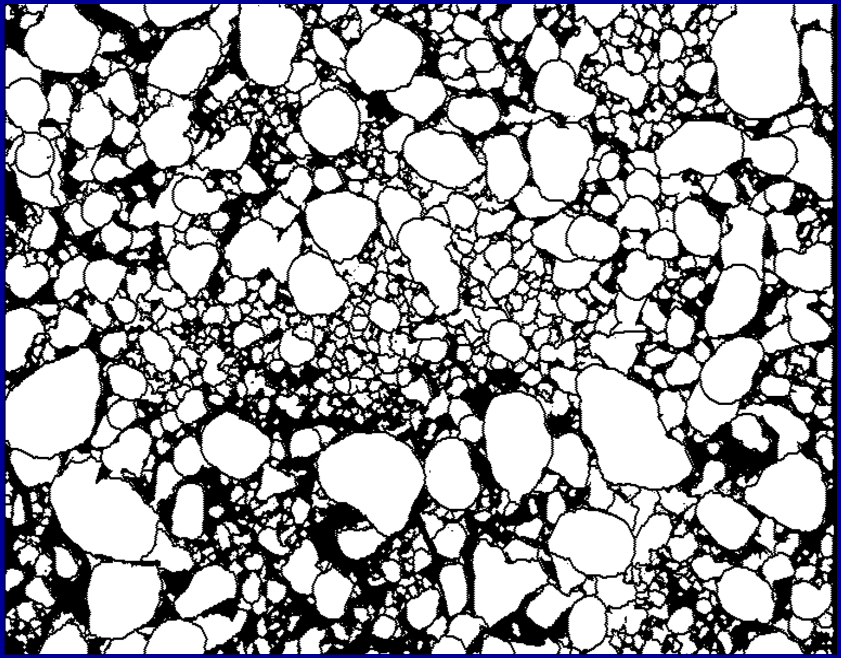
This operation produces markers of blocks whose size is proportional to the size of the block



ROCKS SEGMENTATION

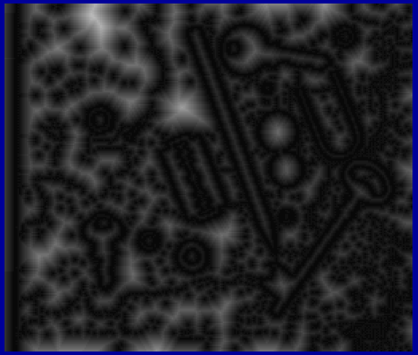
Markers extracted from the granulometric function provide valuable seeds for a marker-controlled watershed segmentation (size and contrast criteria can be mixed).



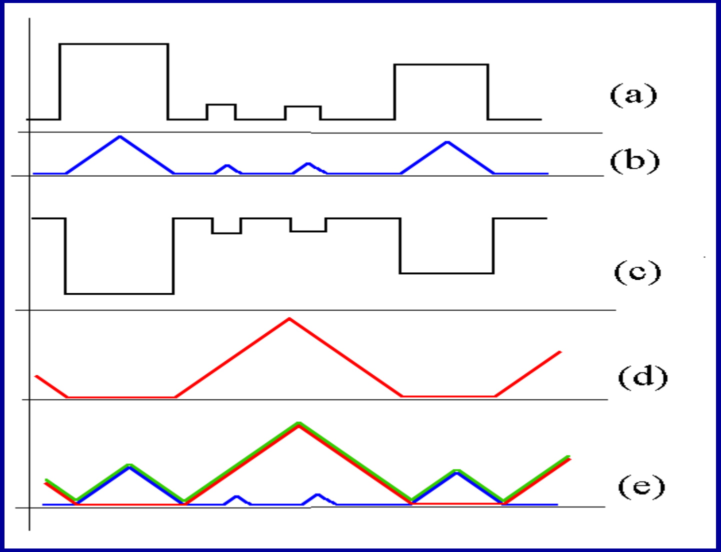


QUASI-DISTANCES

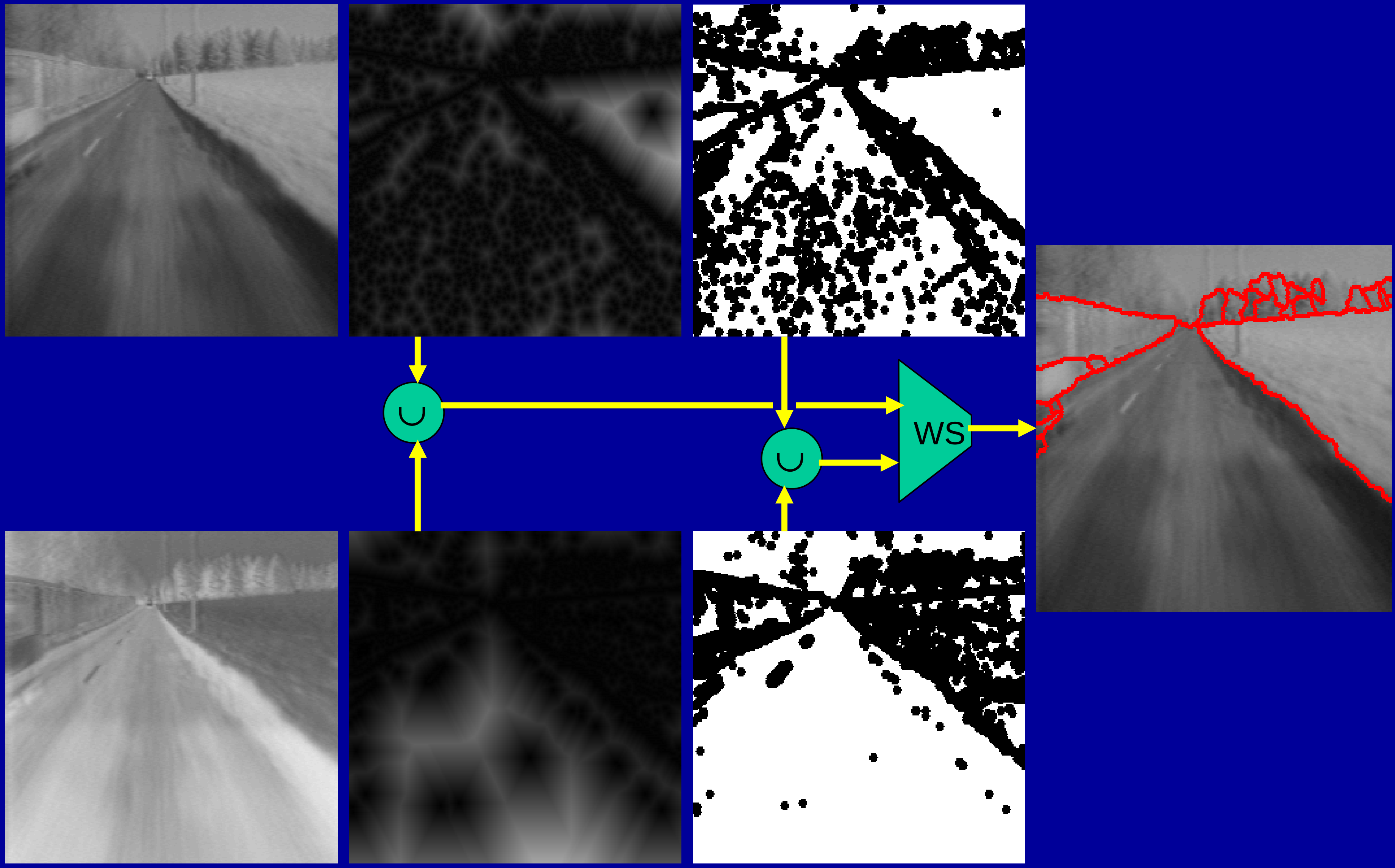
A corrected quasi-distance computed on a greytone image provides the sizes of the flat (homogeneous) regions → Markers for a segmentation based on size and geometry.



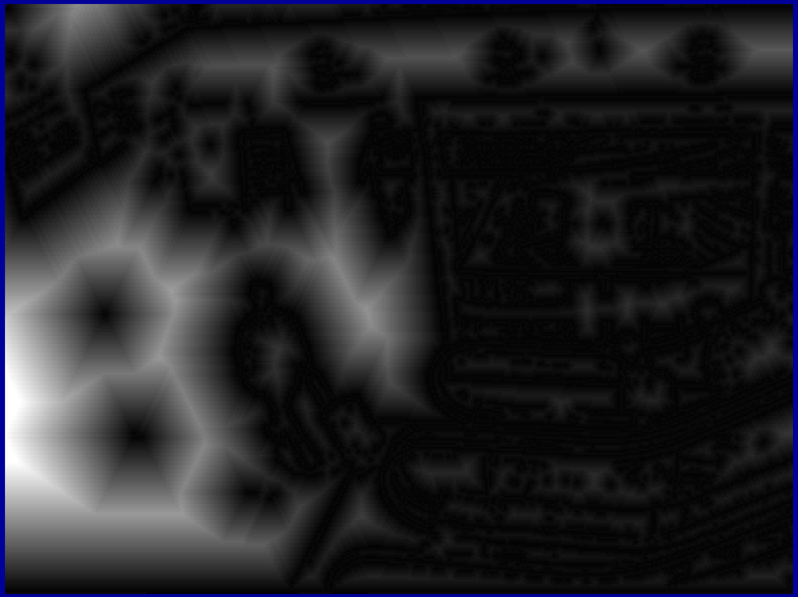
- **Quasi-distances performed both on the image and the complementary one → d, d'**
- **Sup of the results → h=sup(d,d')**
- **Markers extraction (maxima or threshold)**
- **Watershed of h**



SEGMENTATION WITH QUASI-DISTANCES

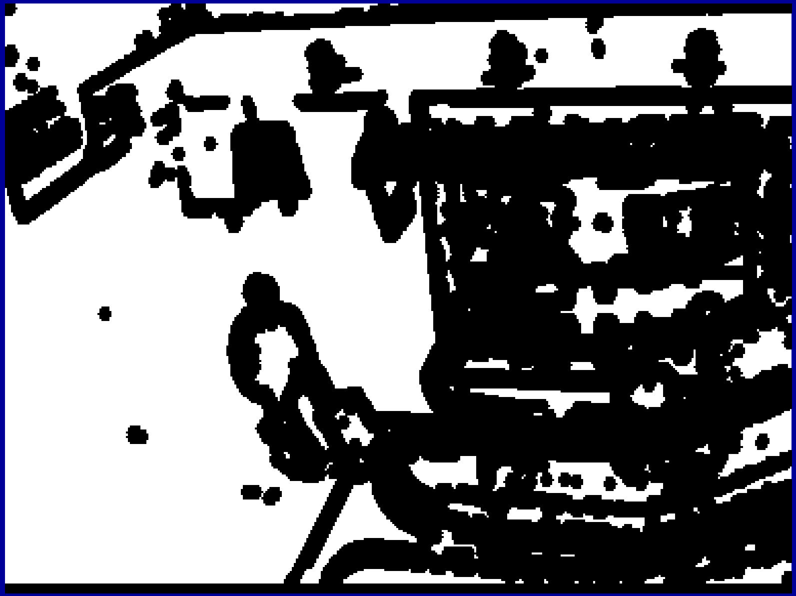


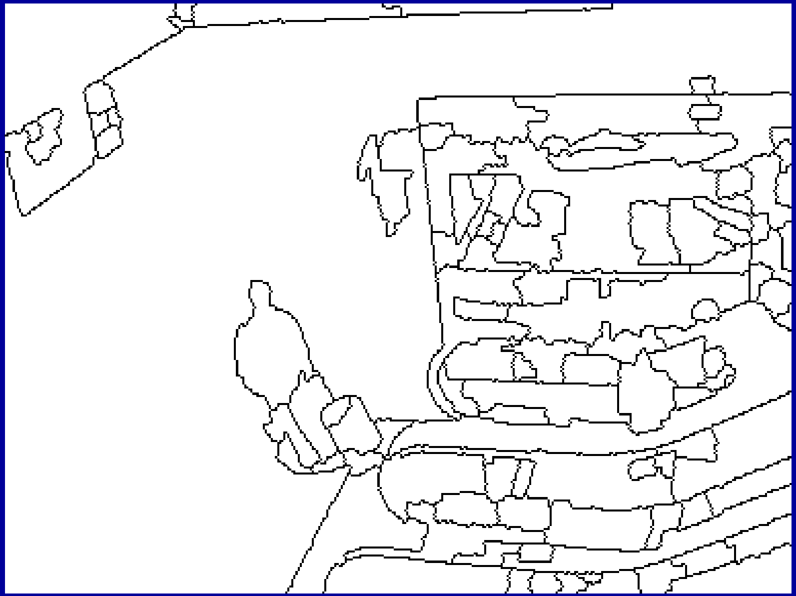
ANOTHER EXAMPLE

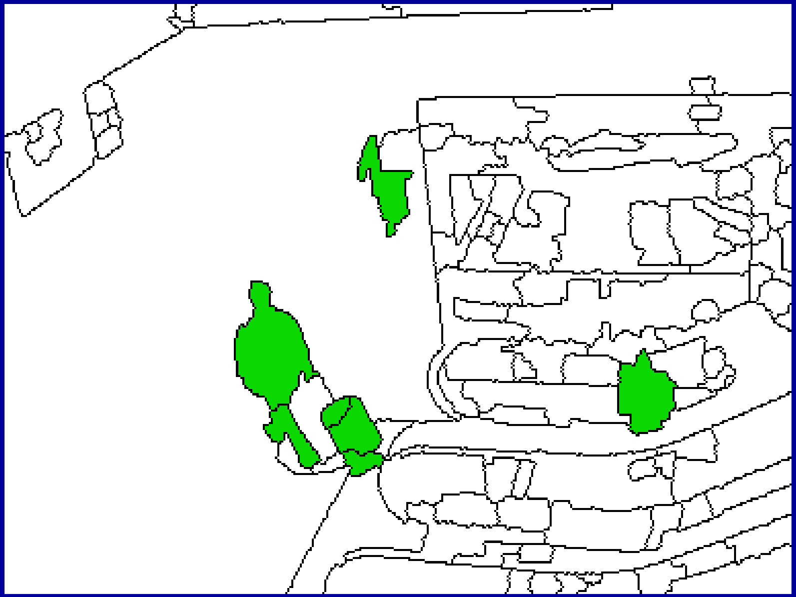


- Video surveillance scene (left)
- Quasi-distance (upper right)
- Quasi-distance of the inverted image (lower right)

ANOTHER EXAMPLE (2)



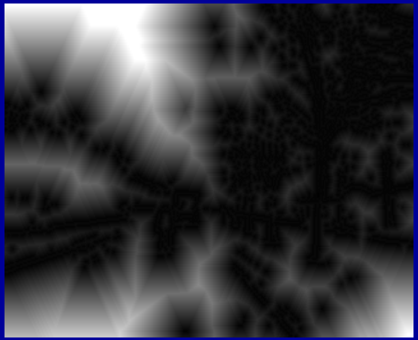




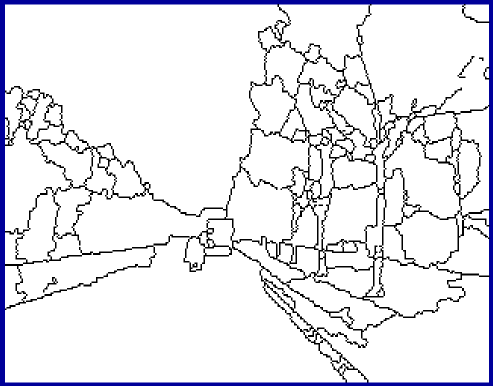
- **Markers of regions (left)**
- **Watershed of the sup of the quasi-distances (upper right)**
- **The moving regions are detected (lower right)**

GRADIENT AND QUASI-DISTANCE





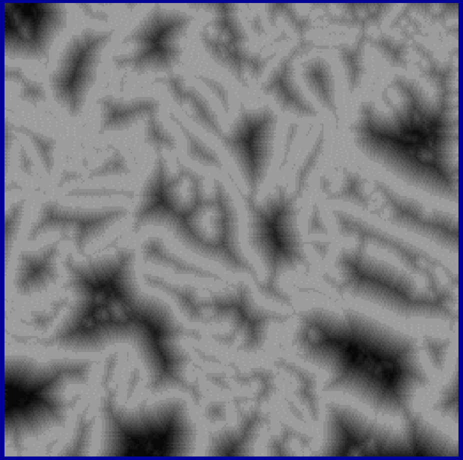


Quasi-distance can be computed on the inverted gradient function

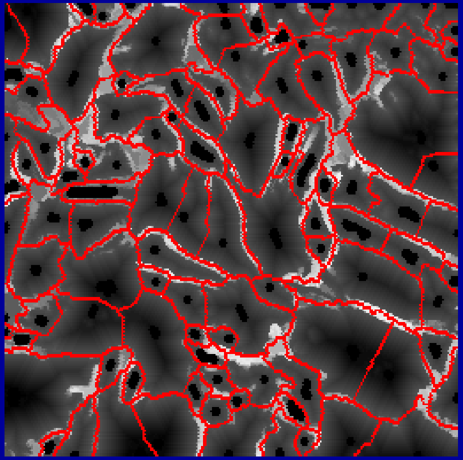
- **Only one quasi-distance is calculated**
- **Hierarchy of regions based on their relative contrast (similar to the waterfall)**
- **The shape of regions is taken into account (closure of imperfectly closed regions)**

DETAILED APPLICATIONS

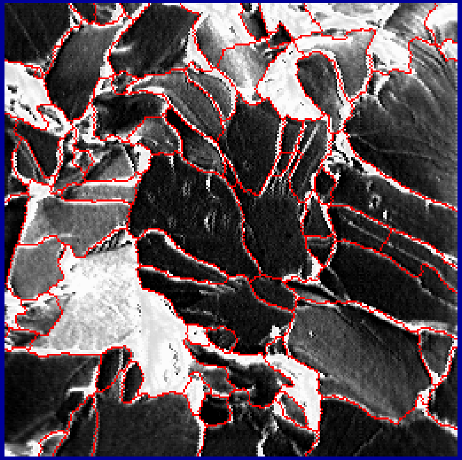
Cleavage fractures in SEM steel images



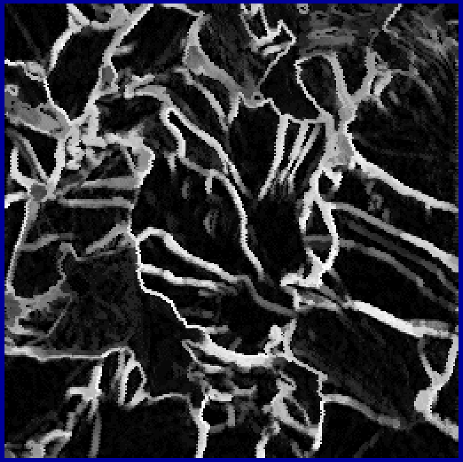
Distance function



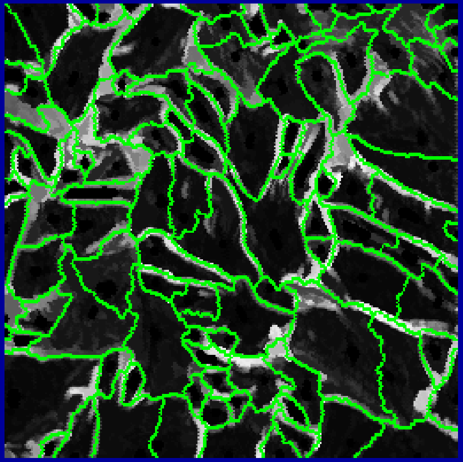
First watershed



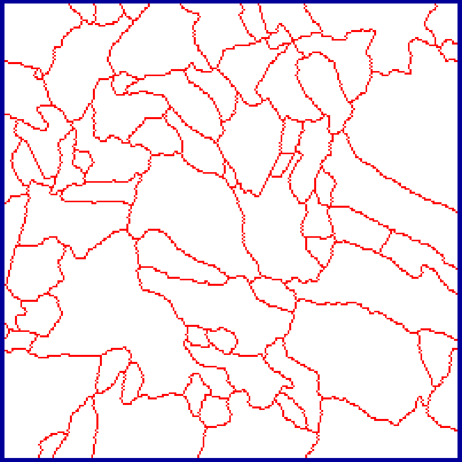
Common dams in both watersheds



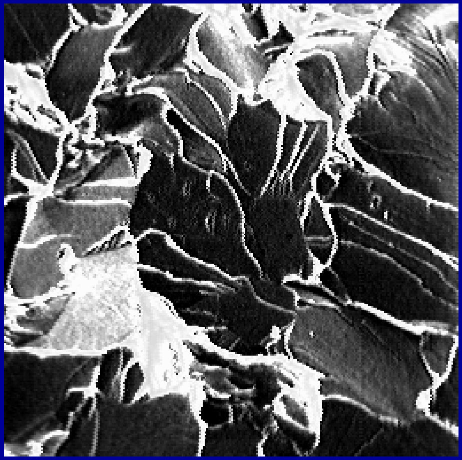
Contrast function



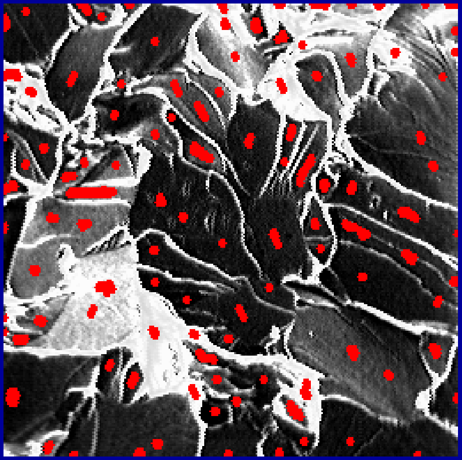
Second watershed



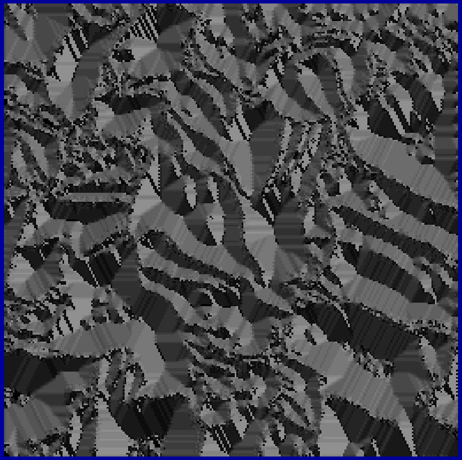
DETAILED APPLICATIONS (2)



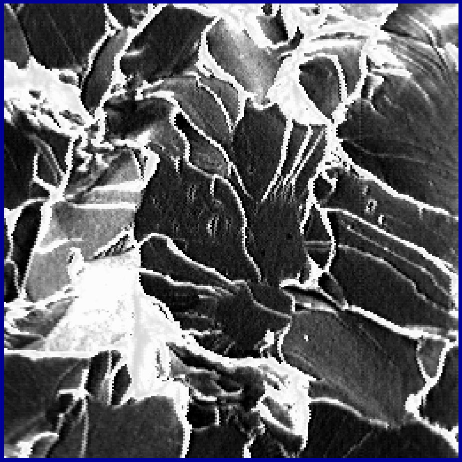
Stereoscopic pair

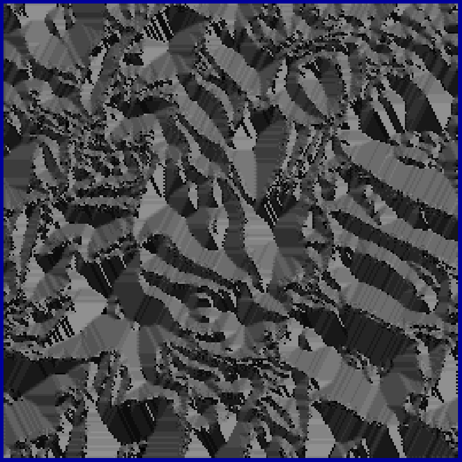


Markers of the first image

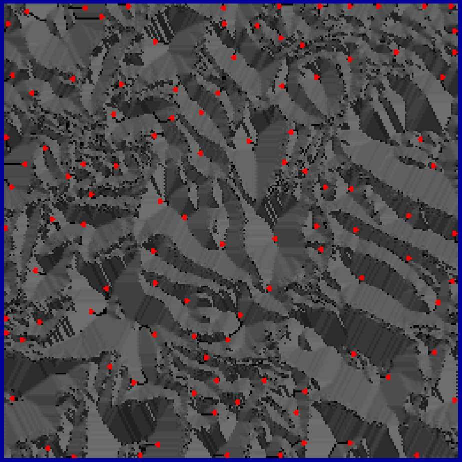


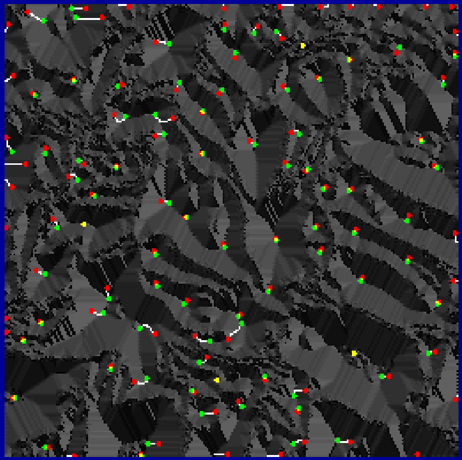
Azimuth of distance function





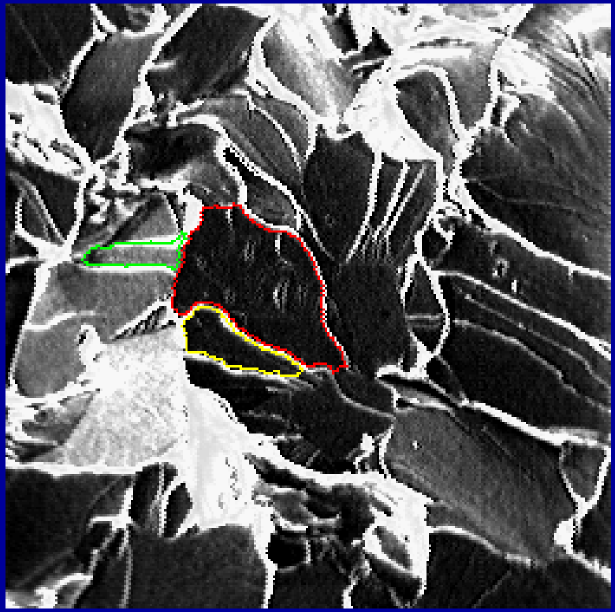
Azimuth (2nd image)



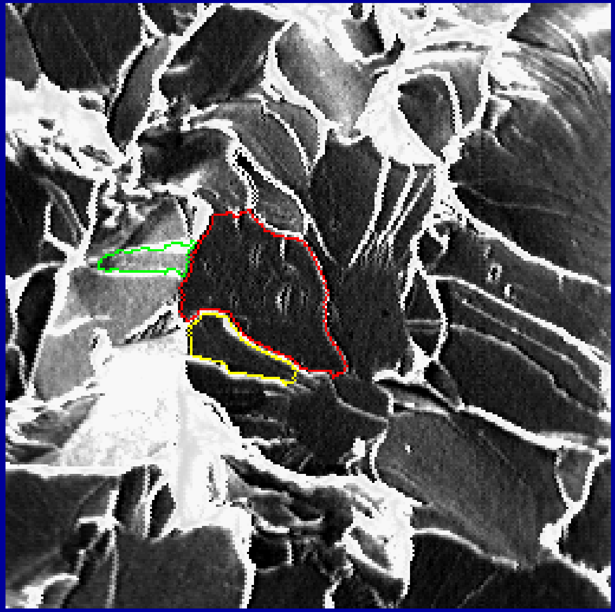


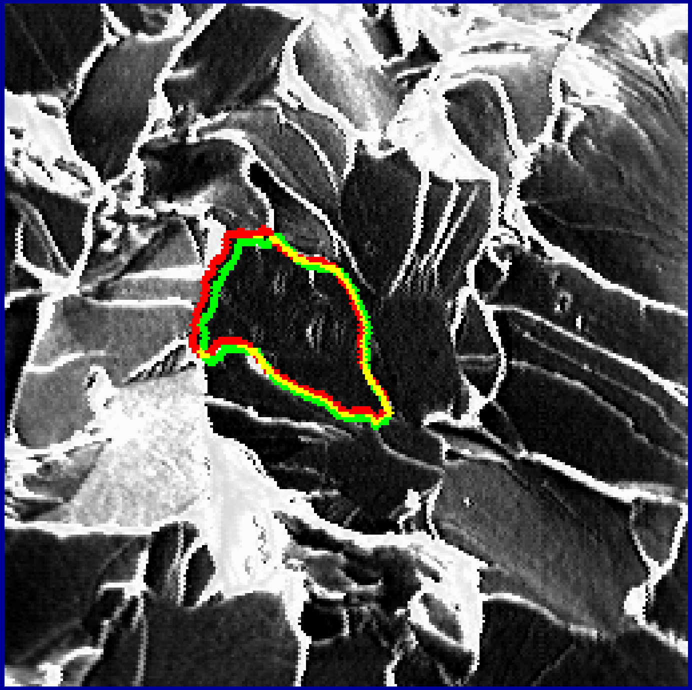
The markers of the first image are thrown on the second one…
and migrate along the steepest slope to give the new markers (in green).

DETAILED APPLICATIONS (3)



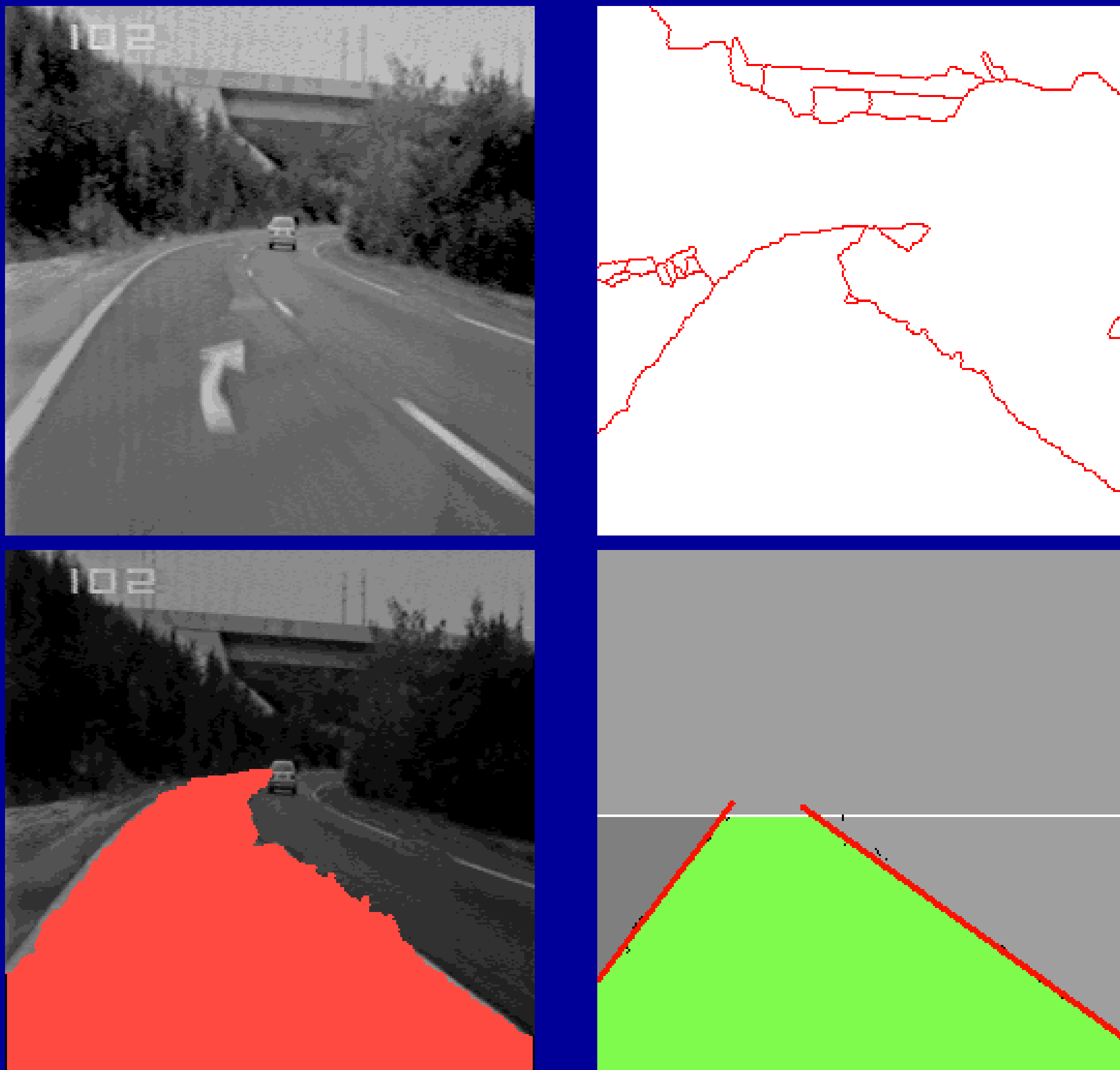
On the right, contour of facets on the first image and the homologous ones in the second image.
Below, displacement of a single facet which can be measured, allowing the computation of its altitude.





DETAILED APPLICATIONS (4)

The PROMETHEUS project



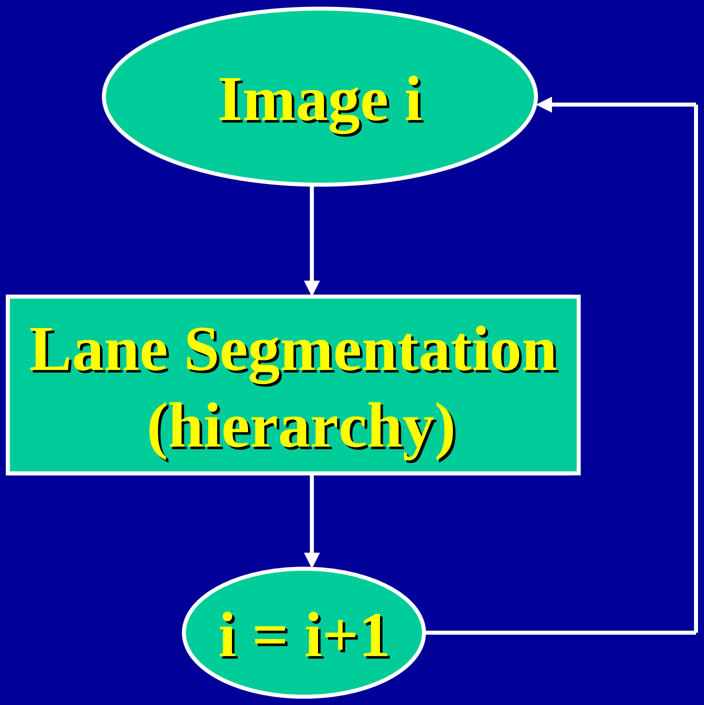
Road segmentation and obstacle detection

Two phases:

• primary road or lane segmentation (hierarchical watershed). No information is shared between pictures in the sequence
• Definition of a road/lane model (sometimes very basic) and use of this model to build the markers which will be used for the segmentation of the next picture.

DETAILED APPLICATIONS (5)

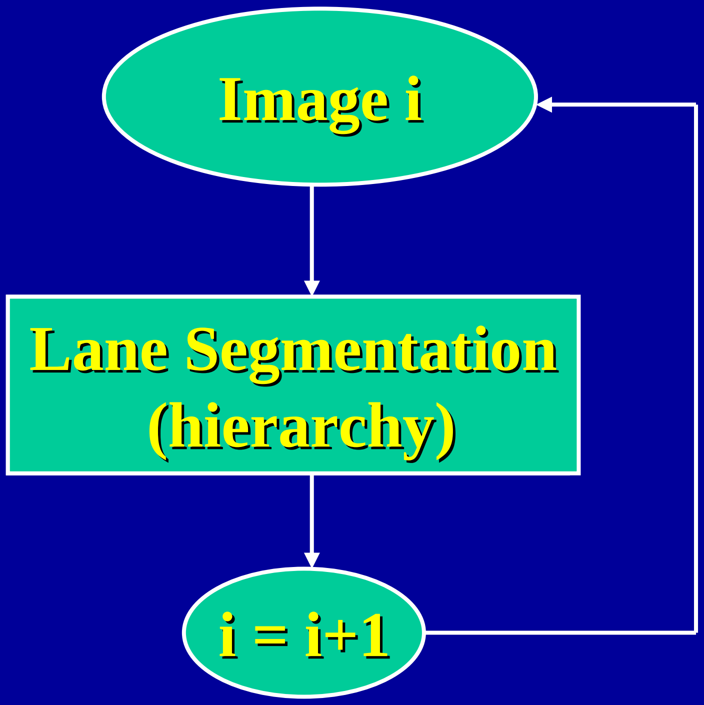
Phase I

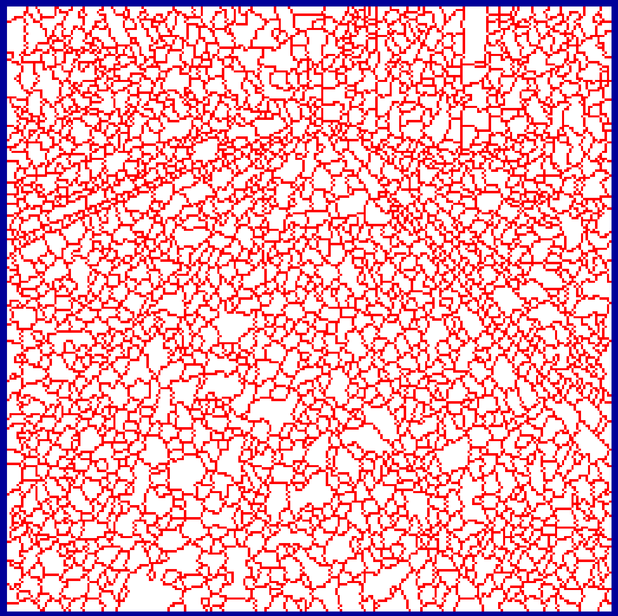


Initial image

DETAILED APPLICATIONS (5)

Phase I

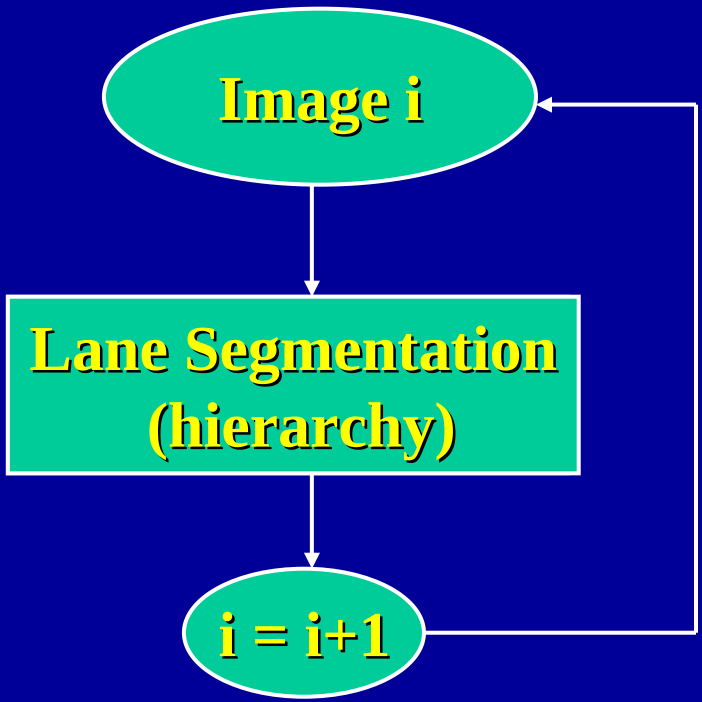


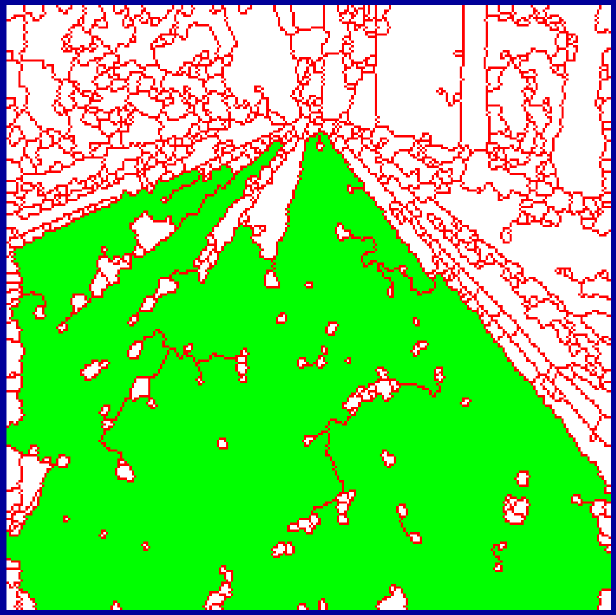


First segmentation

DETAILED APPLICATIONS (5)

Phase I

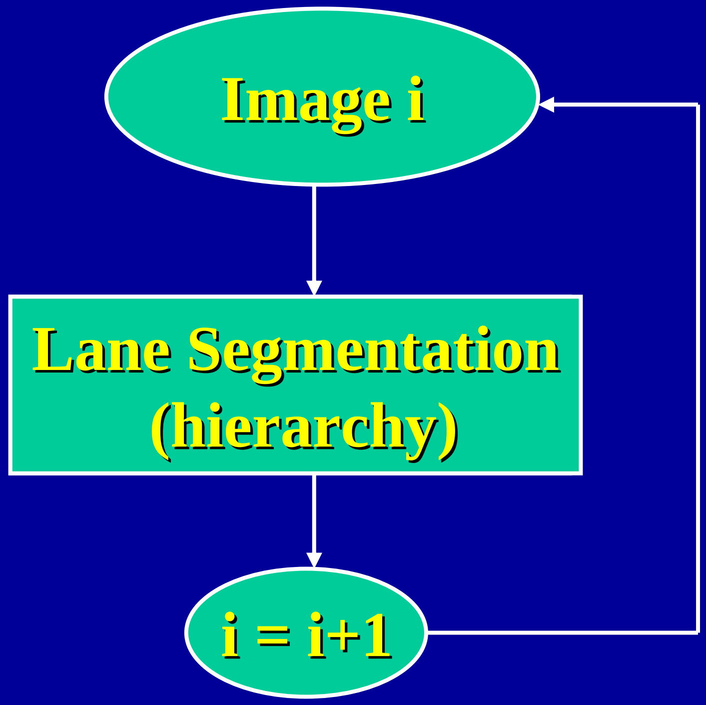




Second level of hierarchy and marker extraction

DETAILED APPLICATIONS (5)

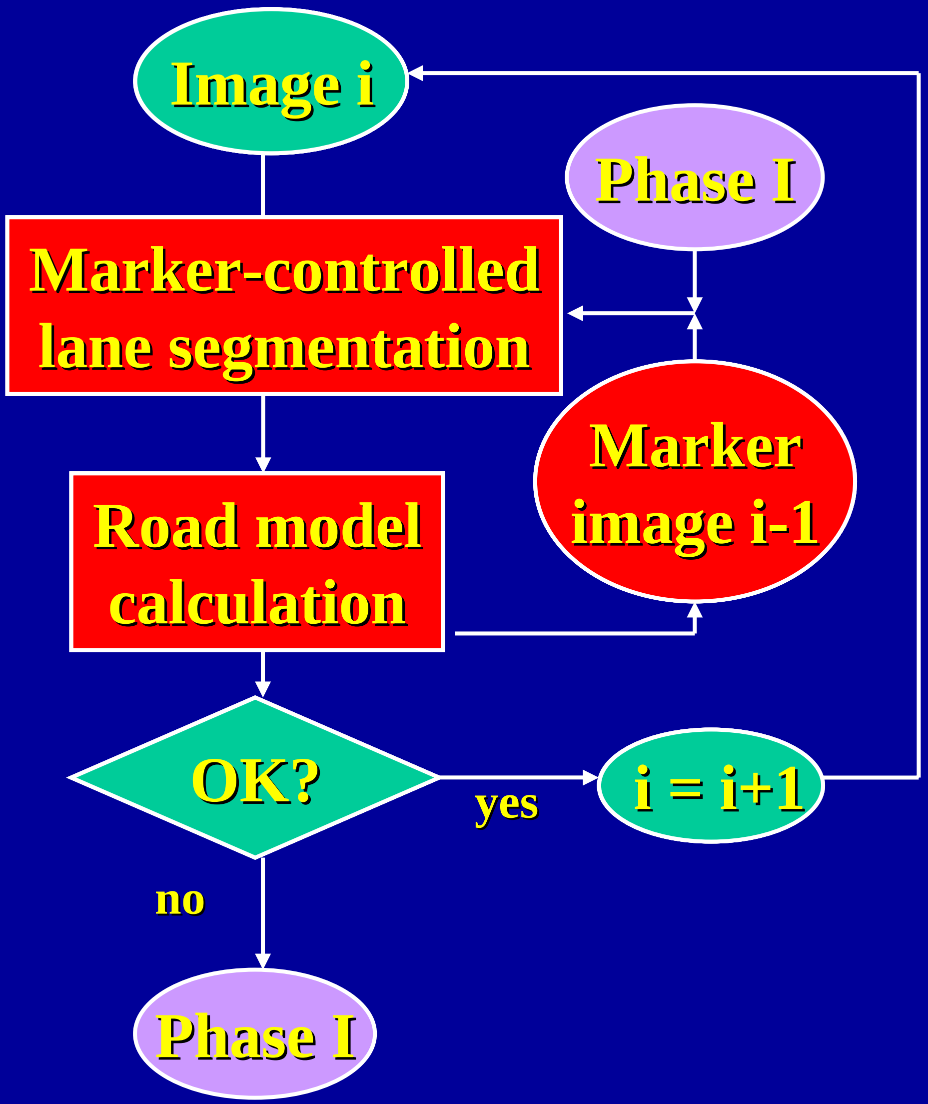
Phase I



Final segmentation

DETAILED APPLICATIONS (6)

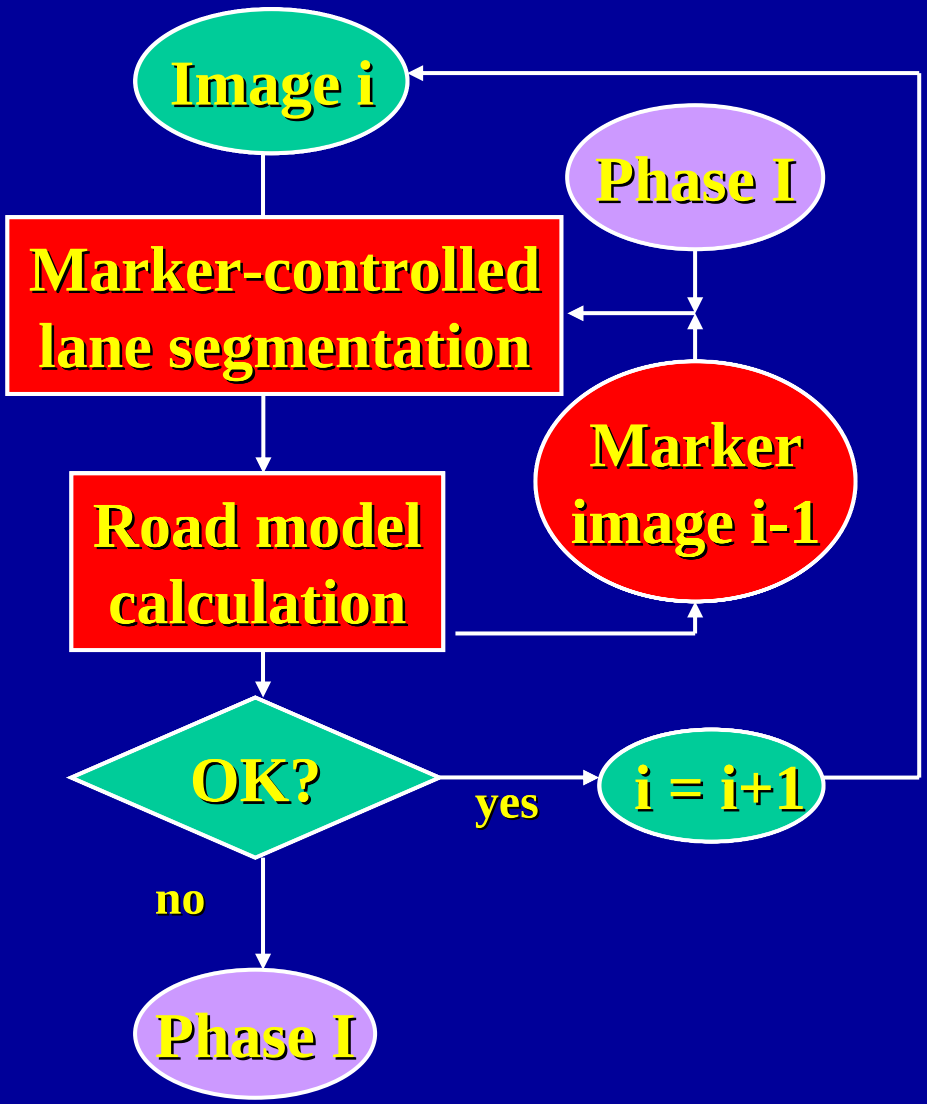
Phase II

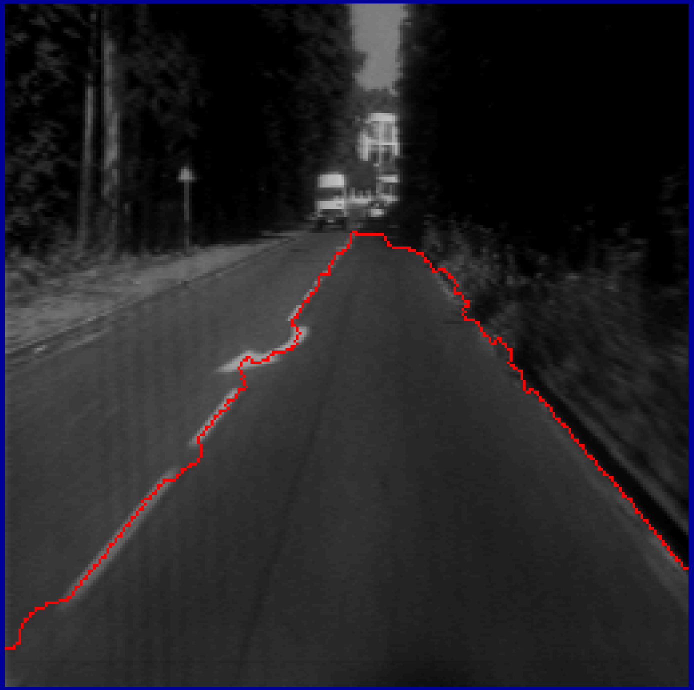


Sequential image at time i

DETAILED APPLICATIONS (6)

Phase II

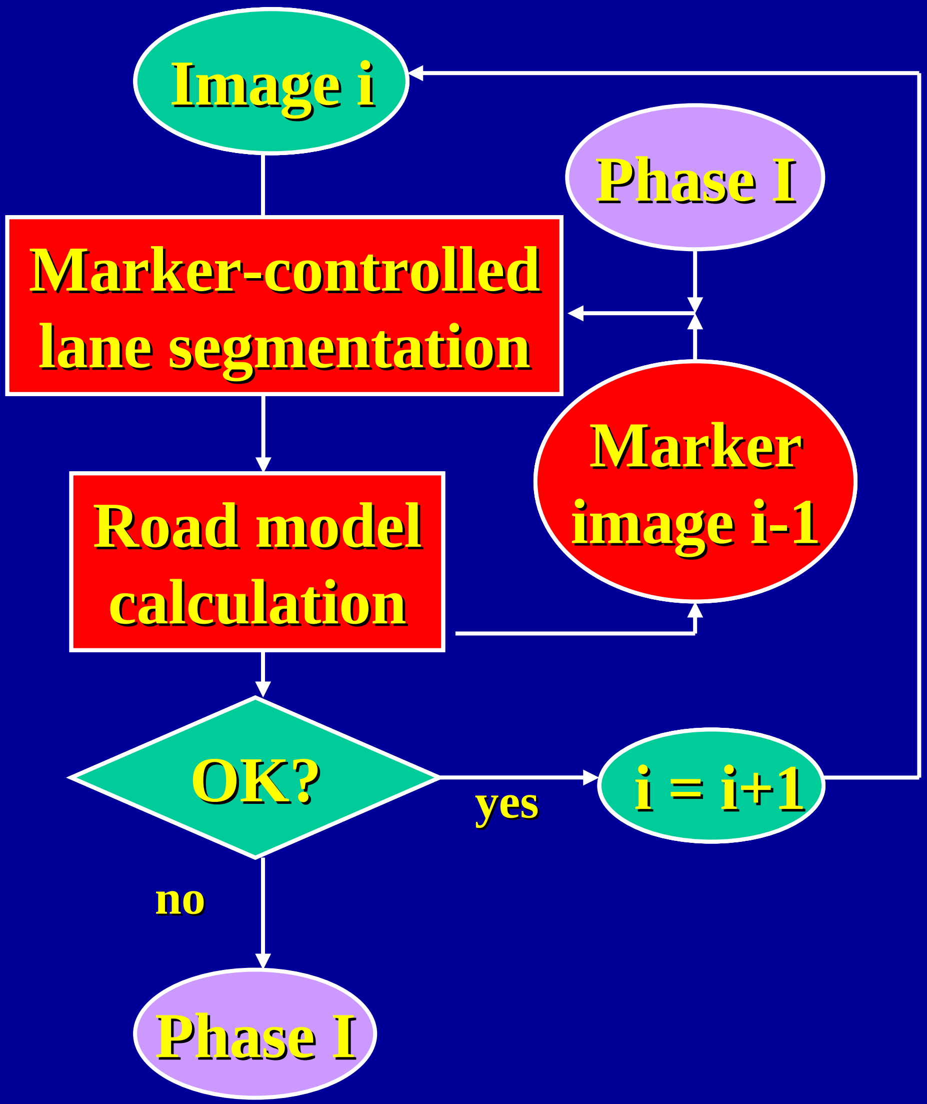


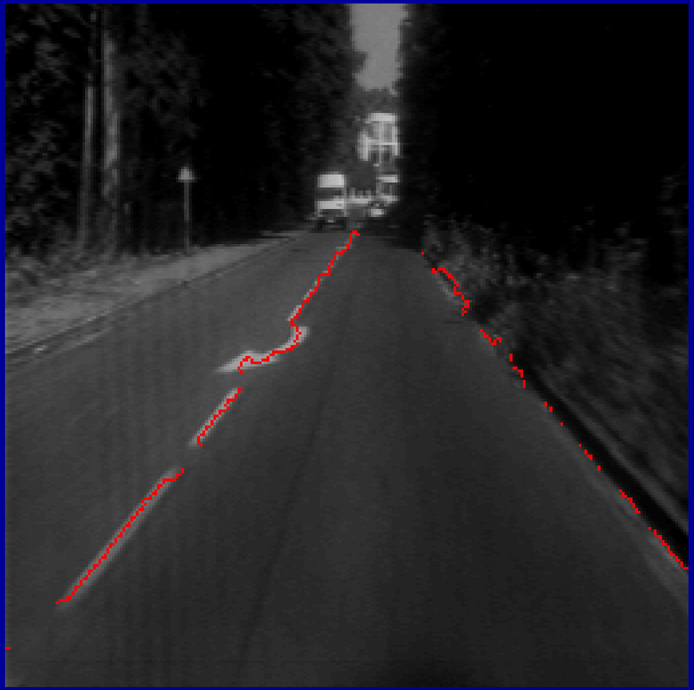


Marker-controlled segmentation of the lane (marker generated by the previous image)

DETAILED APPLICATIONS (6)

Phase II

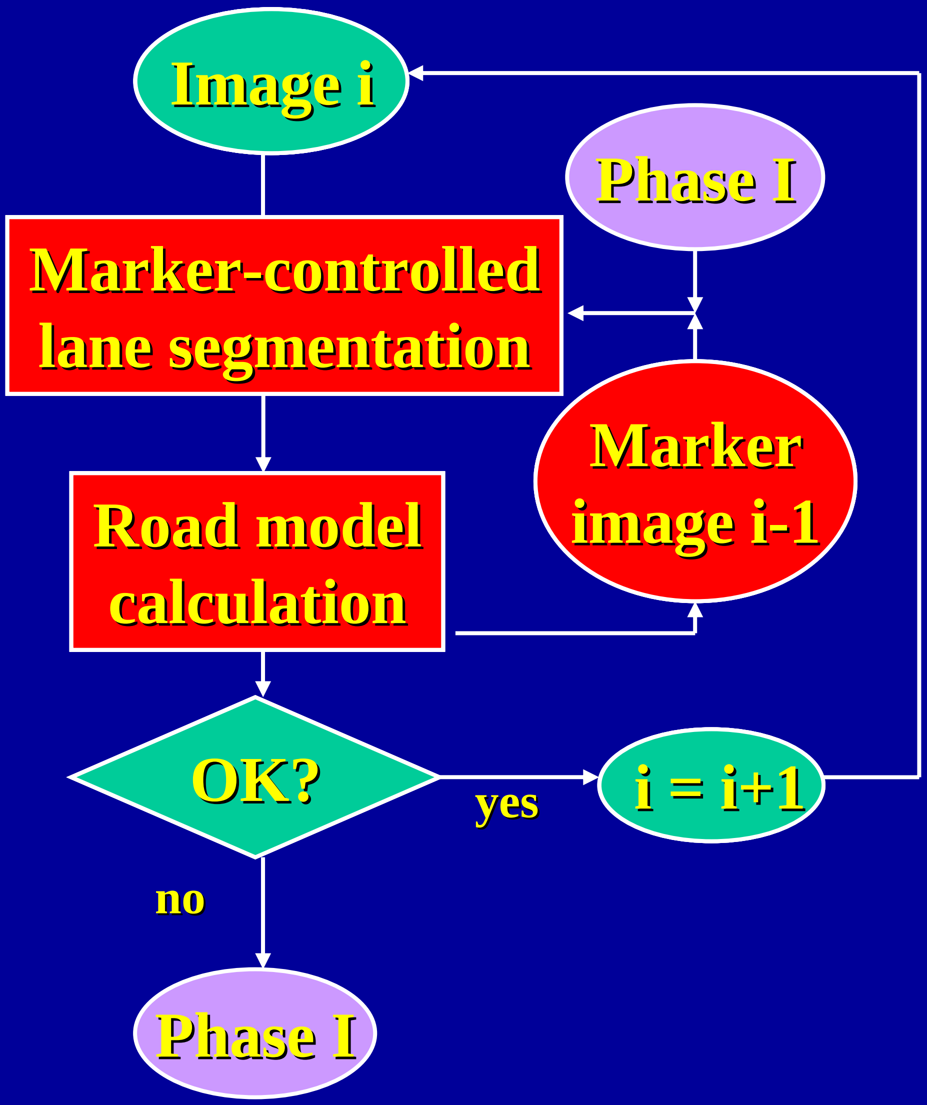


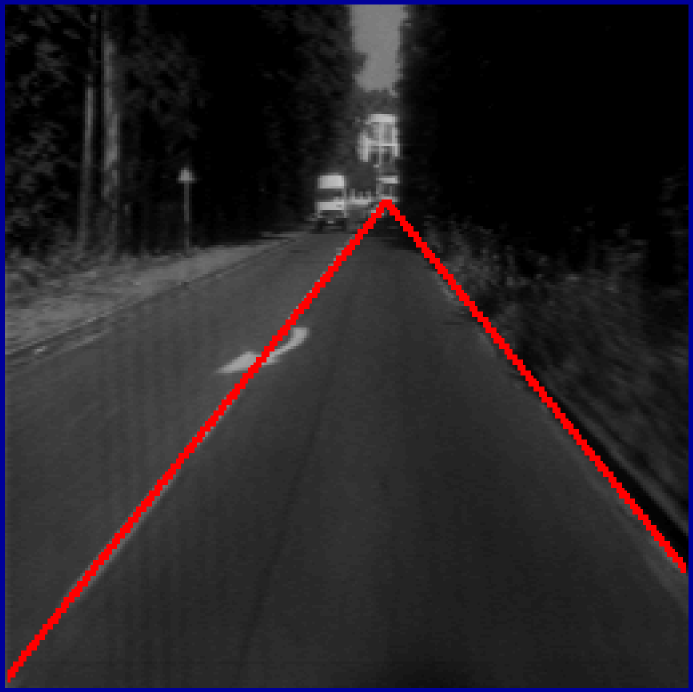


Those pixels belonging to the contours of the lane are selected...

DETAILED APPLICATIONS (6)

Phase II

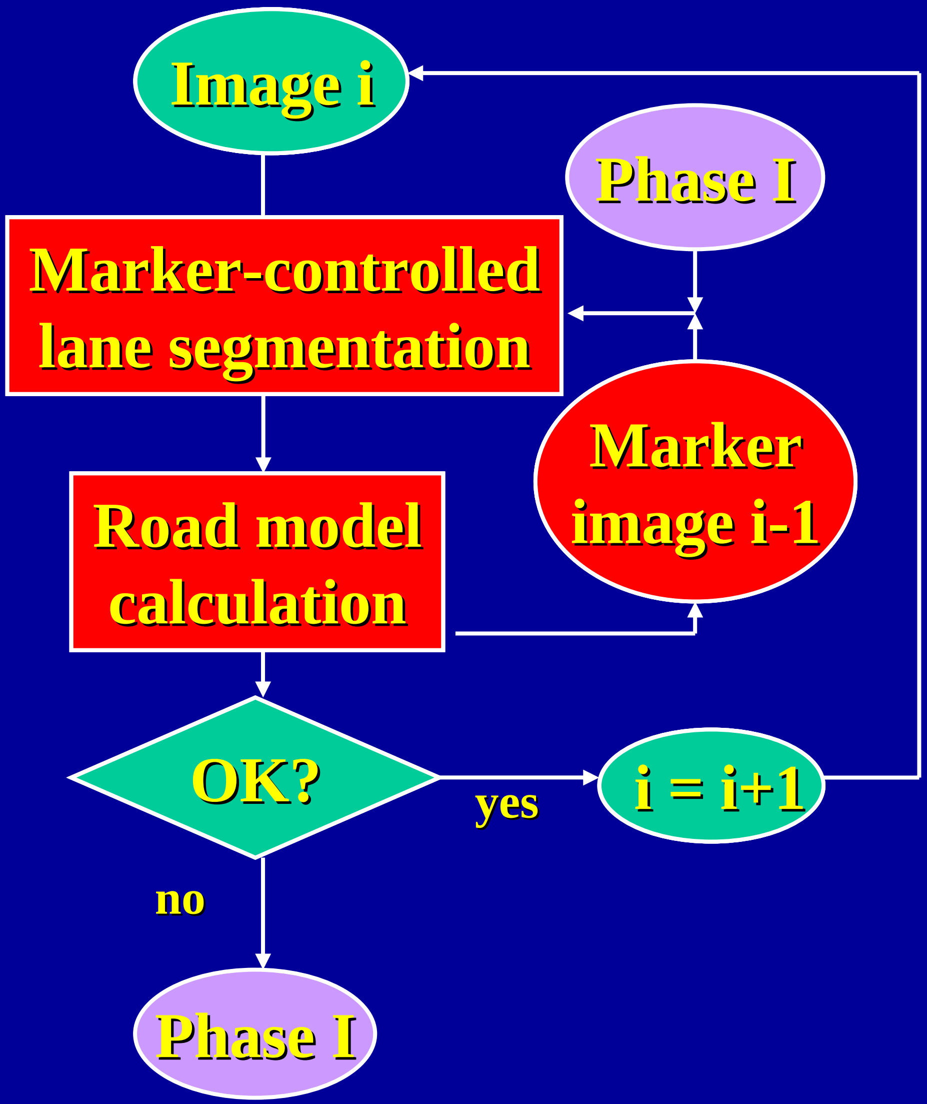


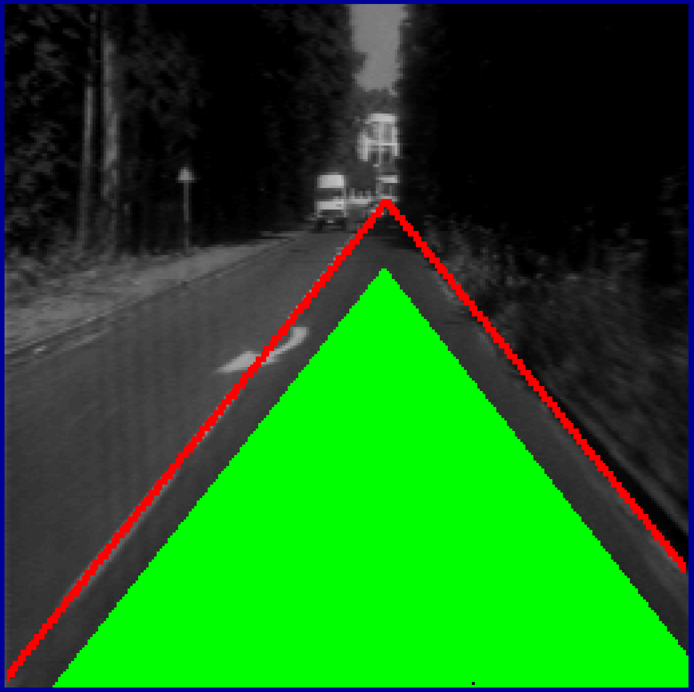


…and used to adjust a lane/road model

DETAILED APPLICATIONS (6)

Phase II

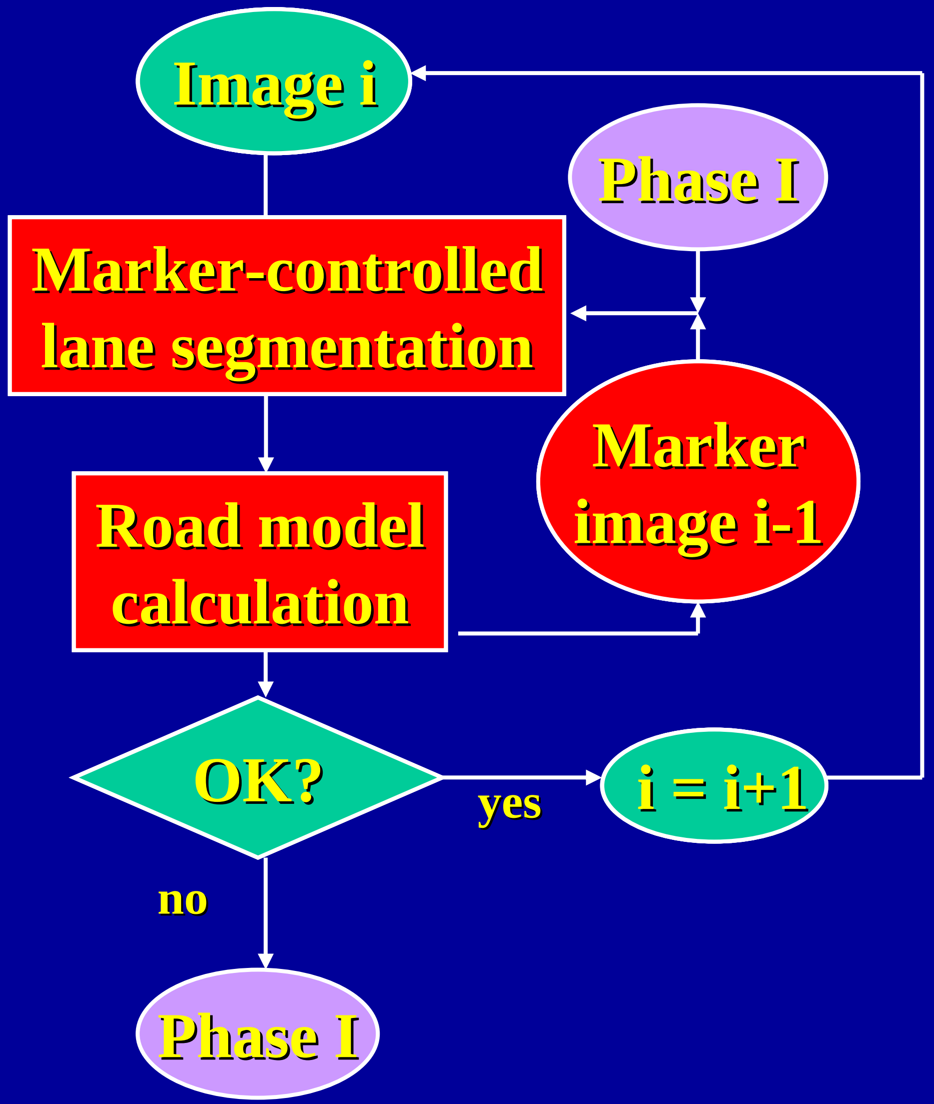


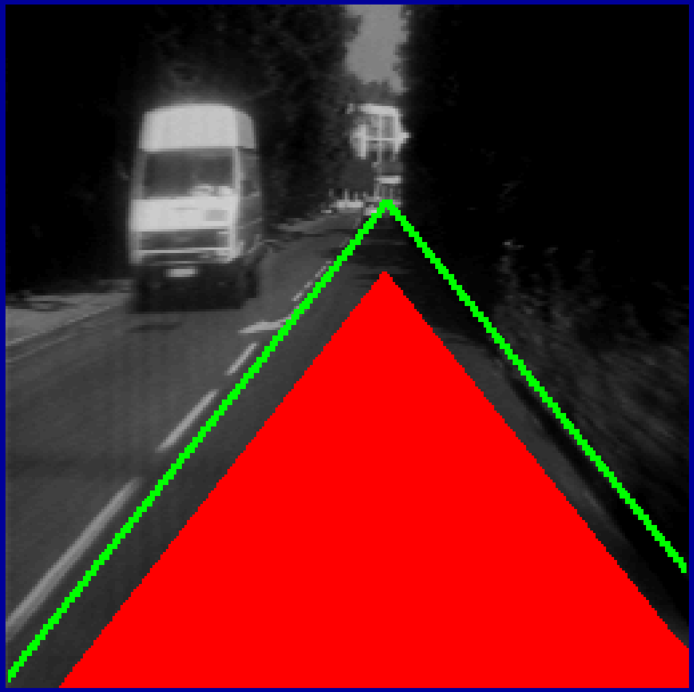


The road/lane model leads to the generation of a new marker

DETAILED APPLICATIONS (6)

Phase II

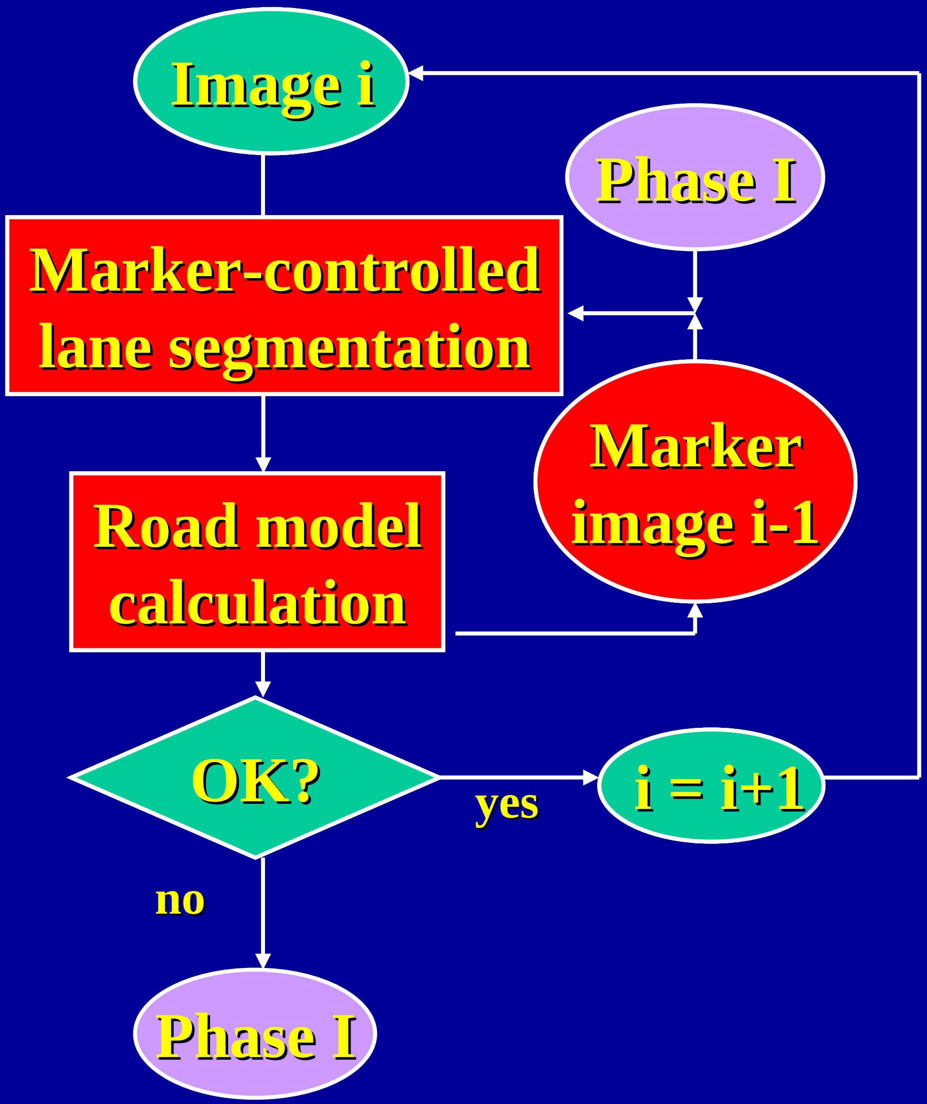


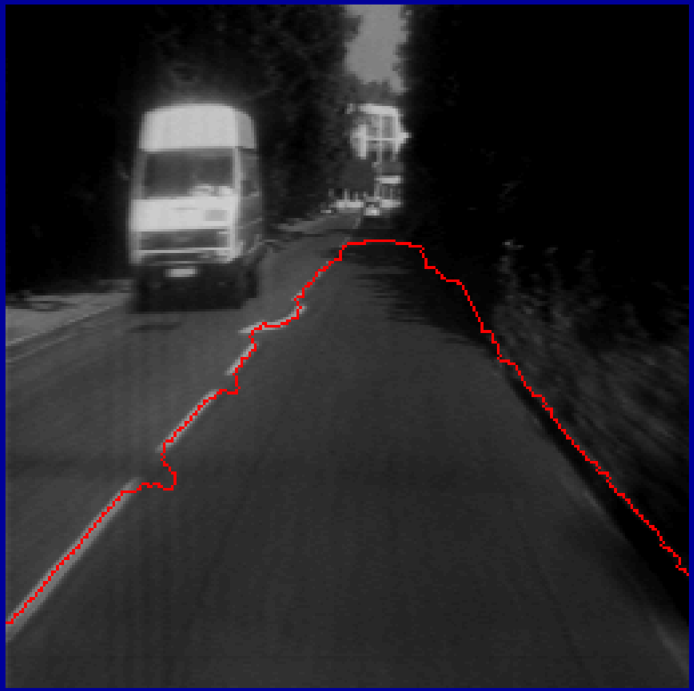


The previous marker is used to segment the current image

DETAILED APPLICATIONS (6)

Phase II

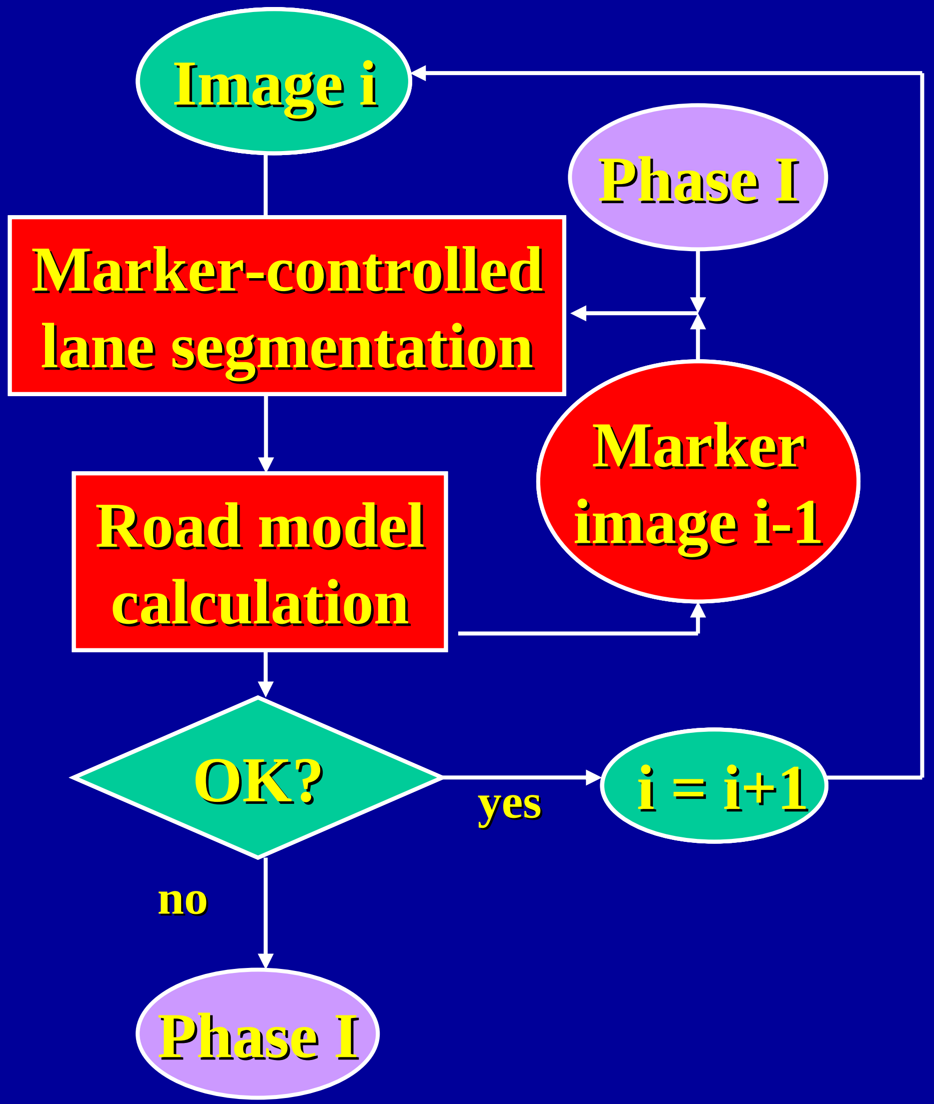


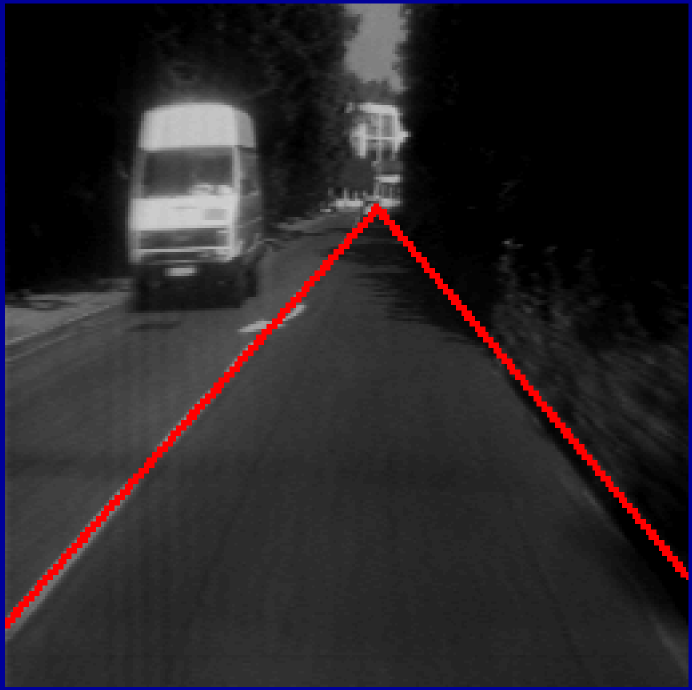


The previous marker is used to segment the current image

DETAILED APPLICATIONS (6)

Phase II



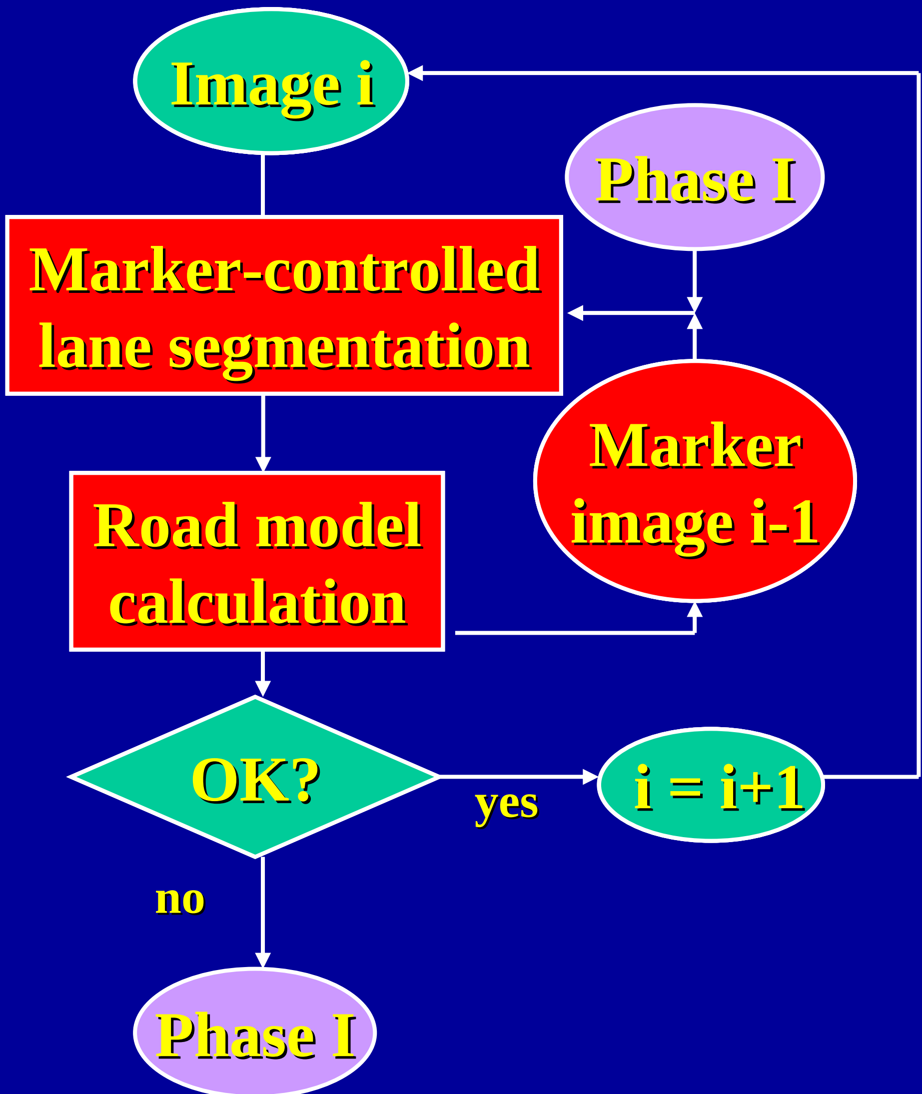


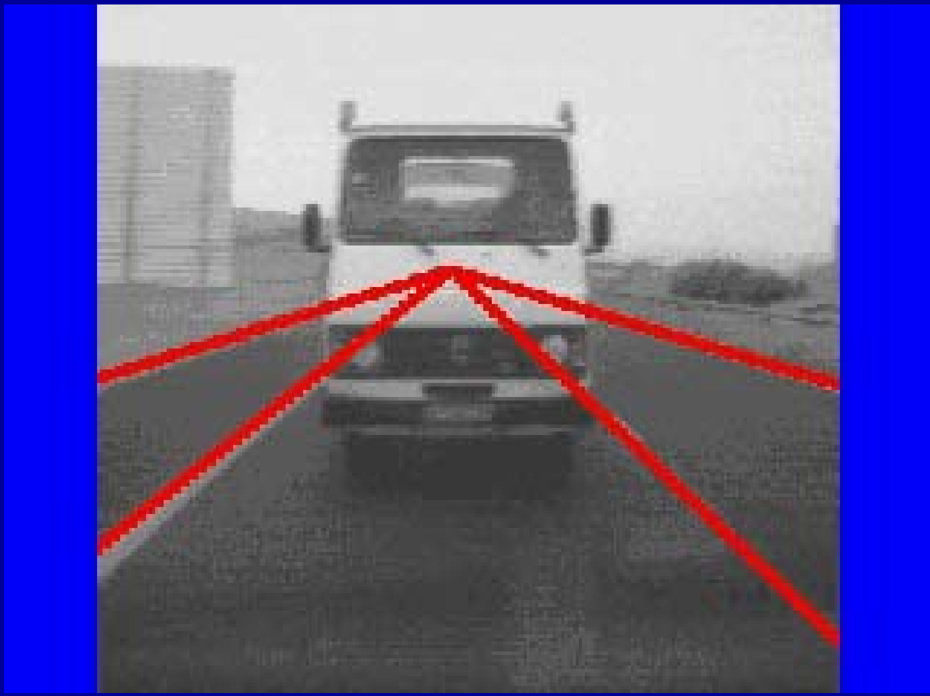
And a new adjustment of the road/lane model is performed

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II





Demonstration of the process on a complete sequence (three lanes road model)

HIERARCHICAL SEGMENTATION, WATERFALLS

It is not always possible to prevent over-segmentation by marker-controlled watershed because it is not always possible to find good markers and/or segmentation criteria.

Therefore, another approaches of the segmentation which are not based on the a priori selection of markers may be useful.

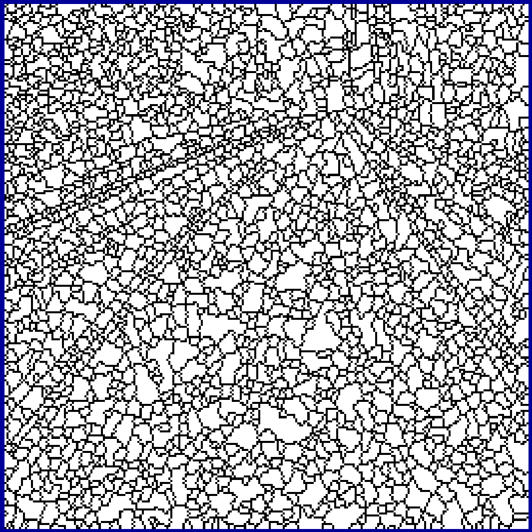
Different algorithms exist. They aim at defining a hierarchy of segmentations:

- **Hierarchy based on extinction values**
- **Hierarchy based on waterfalls**
- **Hierarchy based on pilings**

VALUED WATERSHED

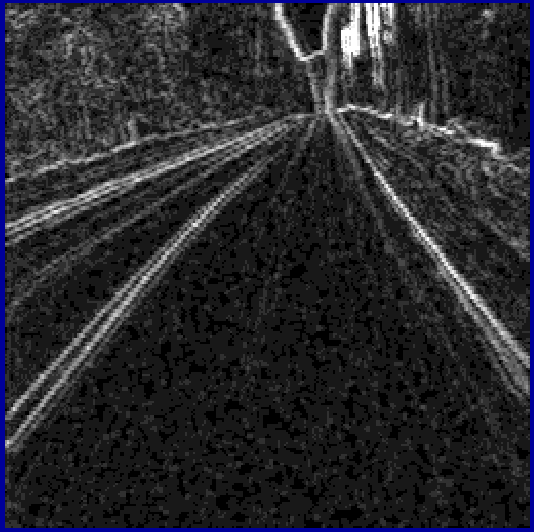
<u>**Valued watershed**</u>

The watershed of a function g is a set W(g)
The valued watershed is the function w(g) defined on W and equal to g on each point of W

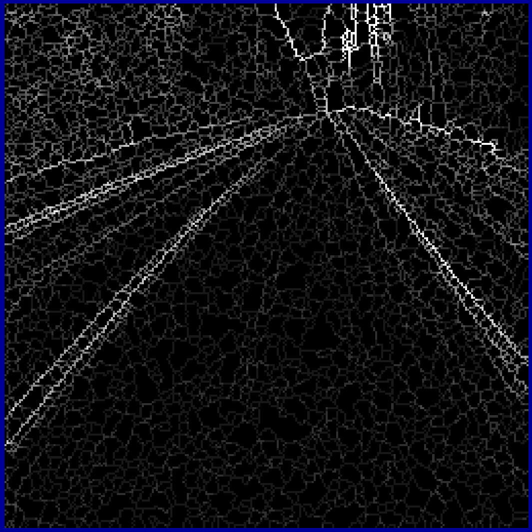


W

Initial image



Gradient g

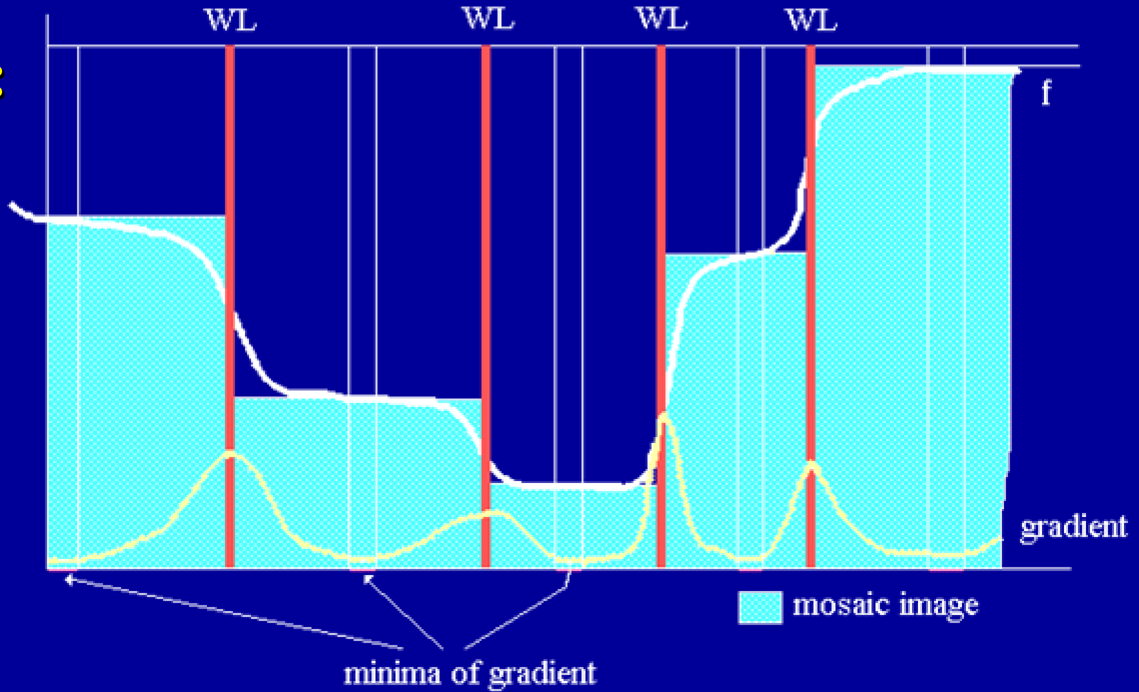


w(g)

MOSAIC IMAGE AND ITS GRADIENT

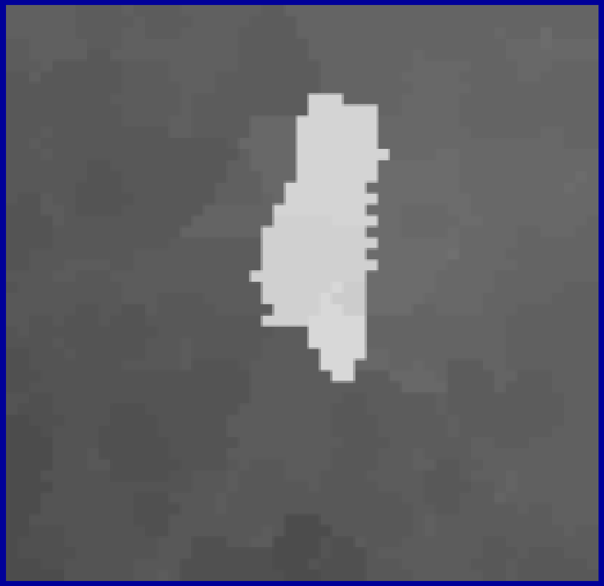
Building the mosaic image:

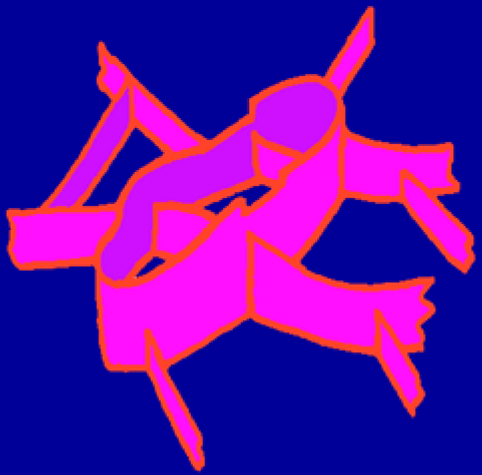
- **Watershed of gradient**
- **For each minimum of gradient, compute the corresponding grey value**
- **Fill in the catchment basin with this grey value**



OVER-SEGMENTATION AND PERCEPTION OF IMAGES

A simple illustration using a mosaic image

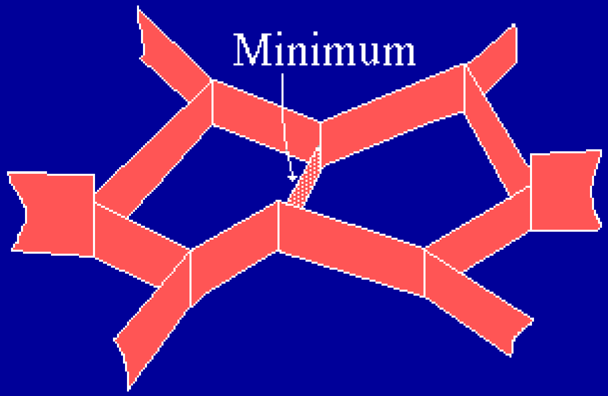


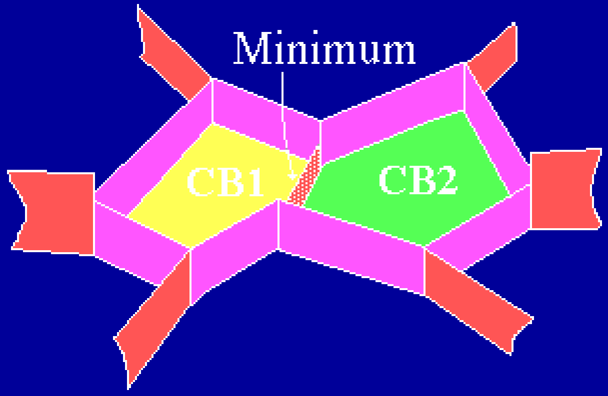


Despite the fact that the image is over-segmented, the white blob can be easily distinguished from the background because, at the same time, the boundaries between the regions inside the blobs and the boundaries inside the background are less contrasted than the boundaries wich separate the blob and the background. Both the blob and the background are <u>marked</u> by boundaries with a <u>minimal contrast</u>.

GRAPH DEFINITION

Arcs of minimal height





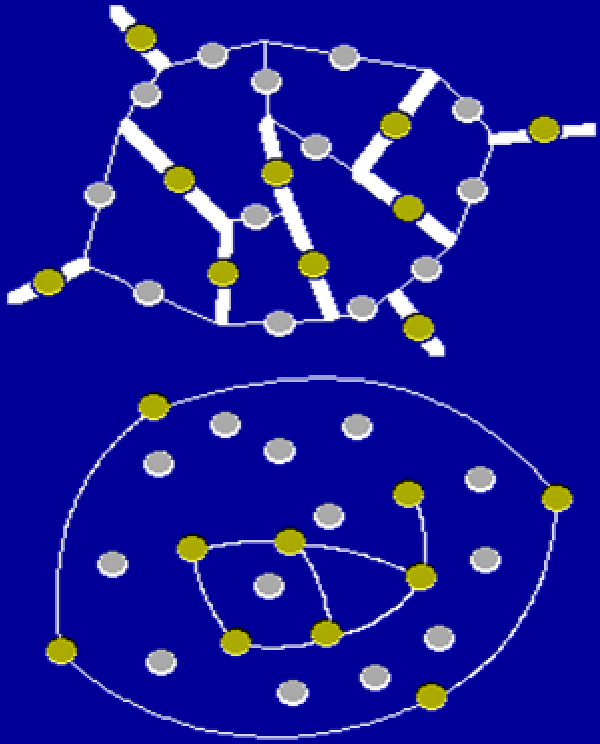
In the mosaic image, each arc c_{ij} separates two catchment basins CB_i and CB_j. The valuation v_{ij} of the arc is given by:

$$v_{ij} = | g_i - g_j |$$

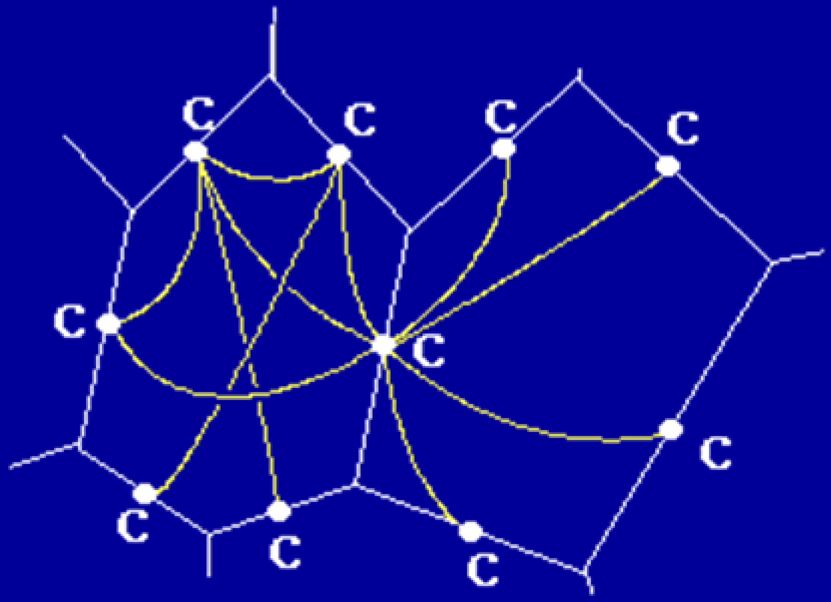
where g_i and g_j are the grey values in the catchment basins.
An arc c_{ij} is said to be minimal if its valuation is lower than those of all the other arcs surrounding CB_i and CB_j

GRAPH DEFINITION AND ASSOCIATED WATERSHED

Definition of a new graph

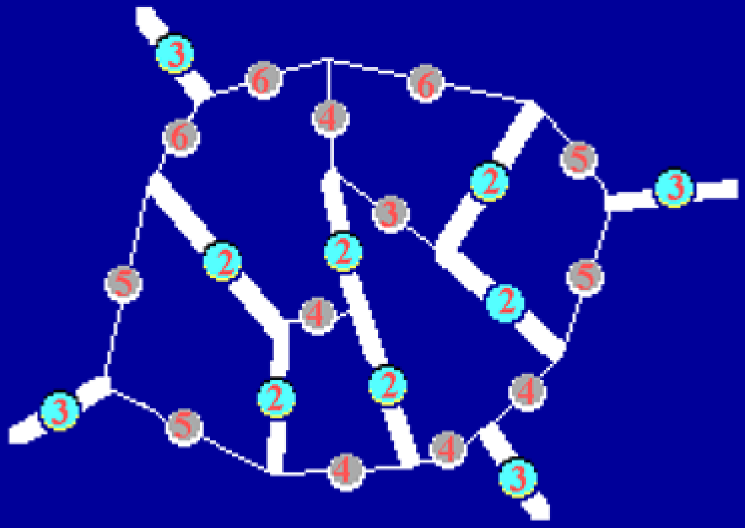


• its vertices correspond to the arcs of the gradient mosaic
• its edges link all arcs surrounding the same catchment basin
• each vertex is valued by the arc valuation as defined in the gradient mosaic

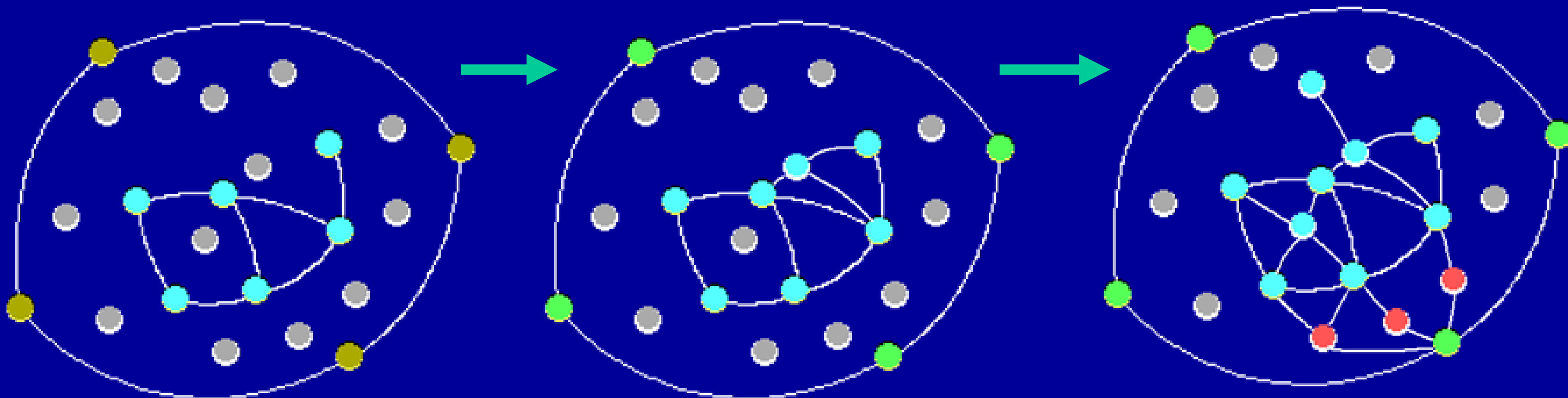


In this representation, the arcs surrounding the same catchment basin are adjacent. Therefore, minimal arcs can be connected although it is not the case in the gradient mosaic, as illustrated above (yellow summits correspond to minimal arcs).

GRAPH DEFINITION AND ASSOCIATED WATERSHED (2)

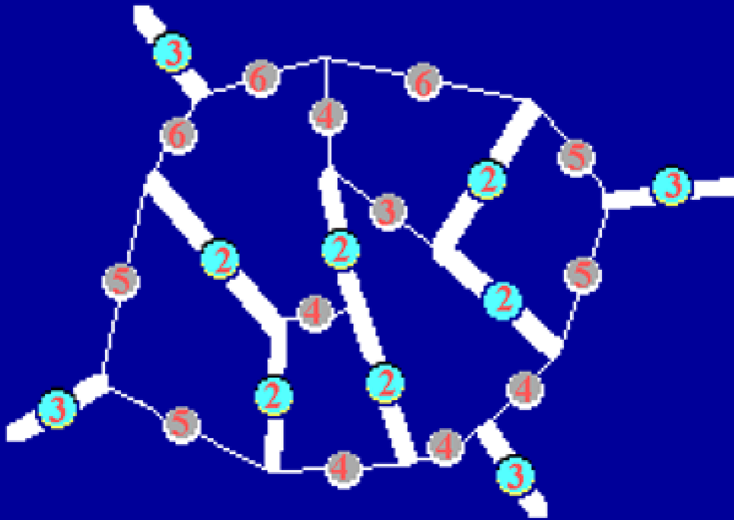


The most significant contours of the mosaic image correspond to those separating regions marked by minimal arcs. They are the watershed lines of the watershed transform defined on the previously defined graph.

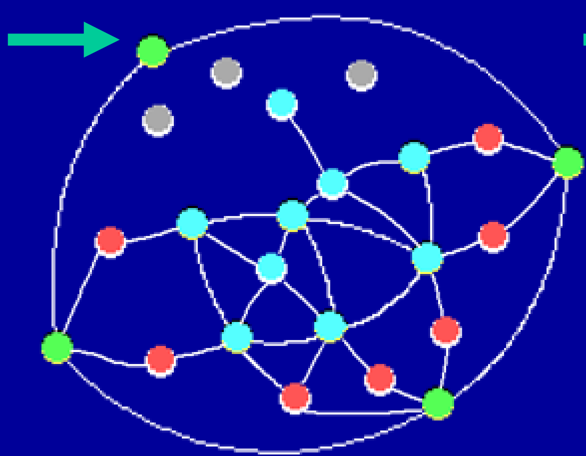


Flooding, step 1 (in blue) Step 2, two CBs, in blue & green Step 3, first dams in red

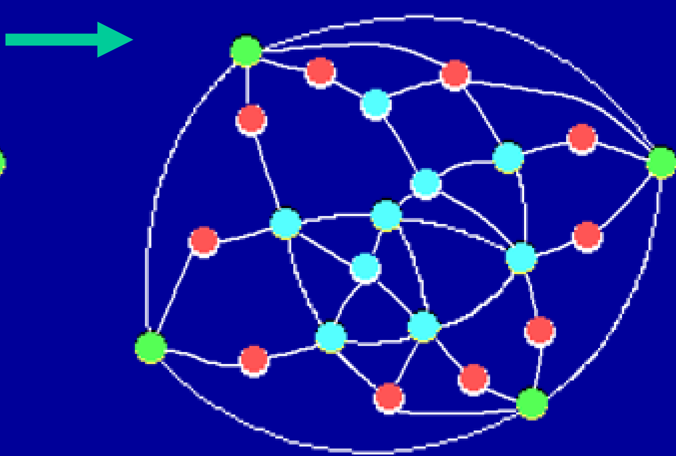
GRAPH DEFINITION AND ASSOCIATED WATERSHED (3)



Arcs of the gradient mosaic corresponding to the watershed lines.

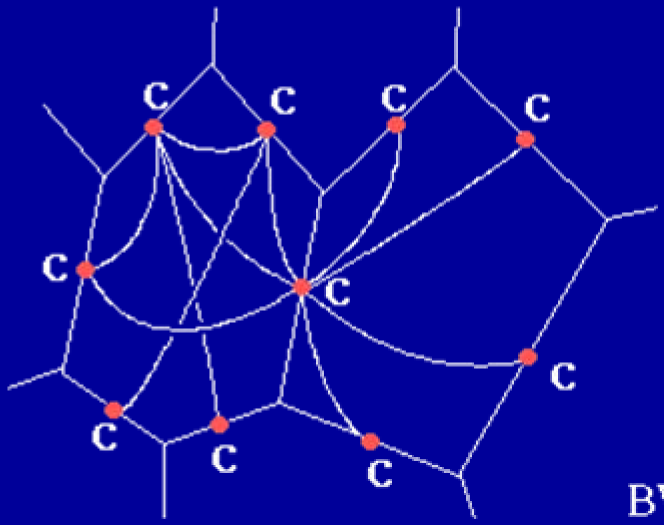


Step 4

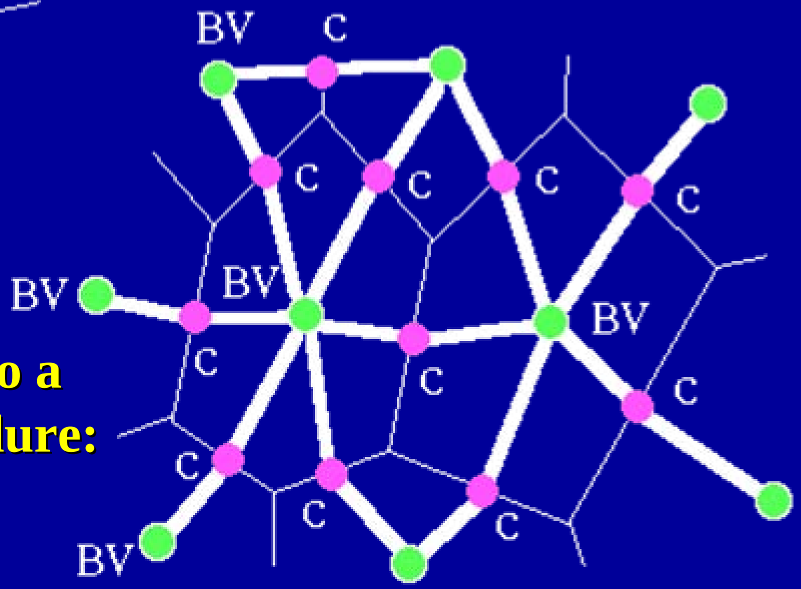


Final watershed

FROM A 3D to A PLANAR GRAPH



The previously defined graph is a 3D valued graph, which is not very handy.



This graph can be transformed into a planar one by the following procedure:

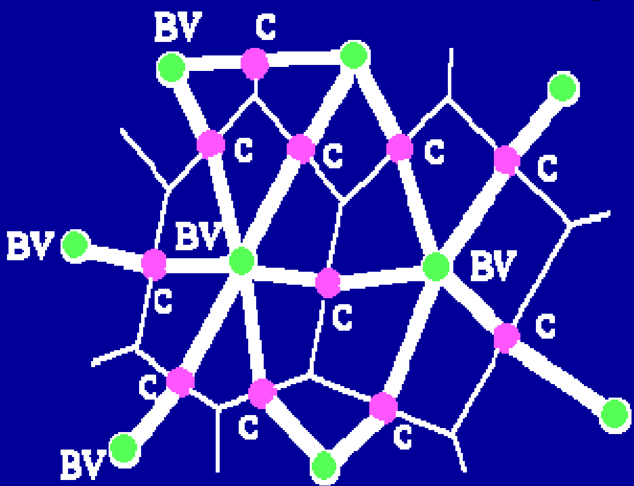
- **A new vertex is added in each catchment basin.**
- **The previous edges are replaced by two successive edges linking the original vertices through the new one.**
- **The valuation of the new vertex is given by:**

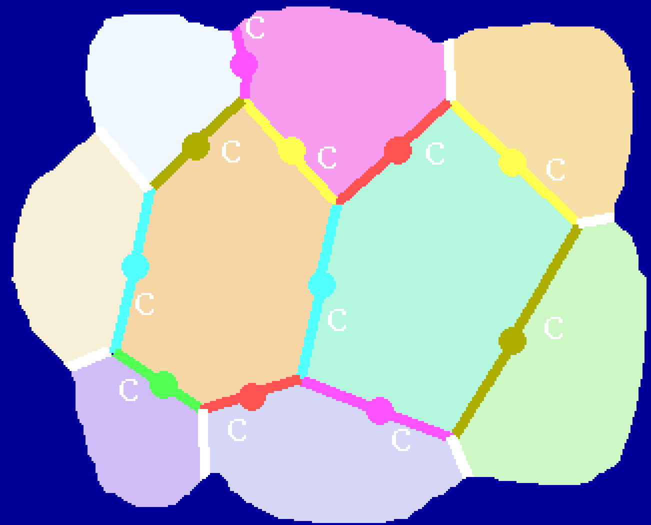
$$\min (v_{ij})$$

where v_{ij} are the valuations of the arcs surrounding the catchment basin.

IMAGE REPRESENTATION

The hierarchical image





mosaic

gradient mosaic

hierarchical image

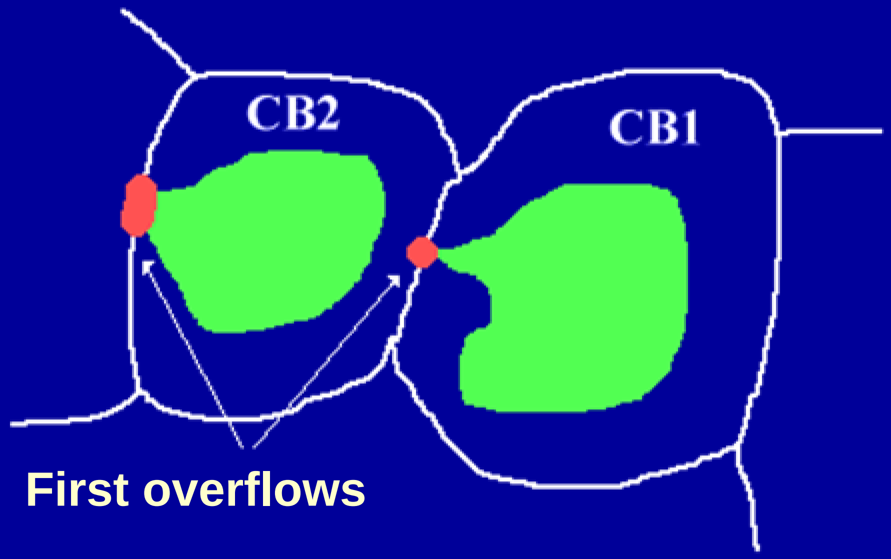
An image, named hierarchical image can be build from the planar graph. The catchment basins of the gradient mosaic are filled with grey values corresponding to the valuation of the new added vertices.

The watersheds of this hierarchical image give the higher level of hierarchy (with some restrictions).

FIRST OVERFLOW ZONES (FOZ)

Also called improperly saddle zones

(the FOZ notion has nothing to do with the classical saddle zones. It's not a local notion and, as the WTS, there is no way to know a priori if a given point of the image belongs or not to a FOZ)

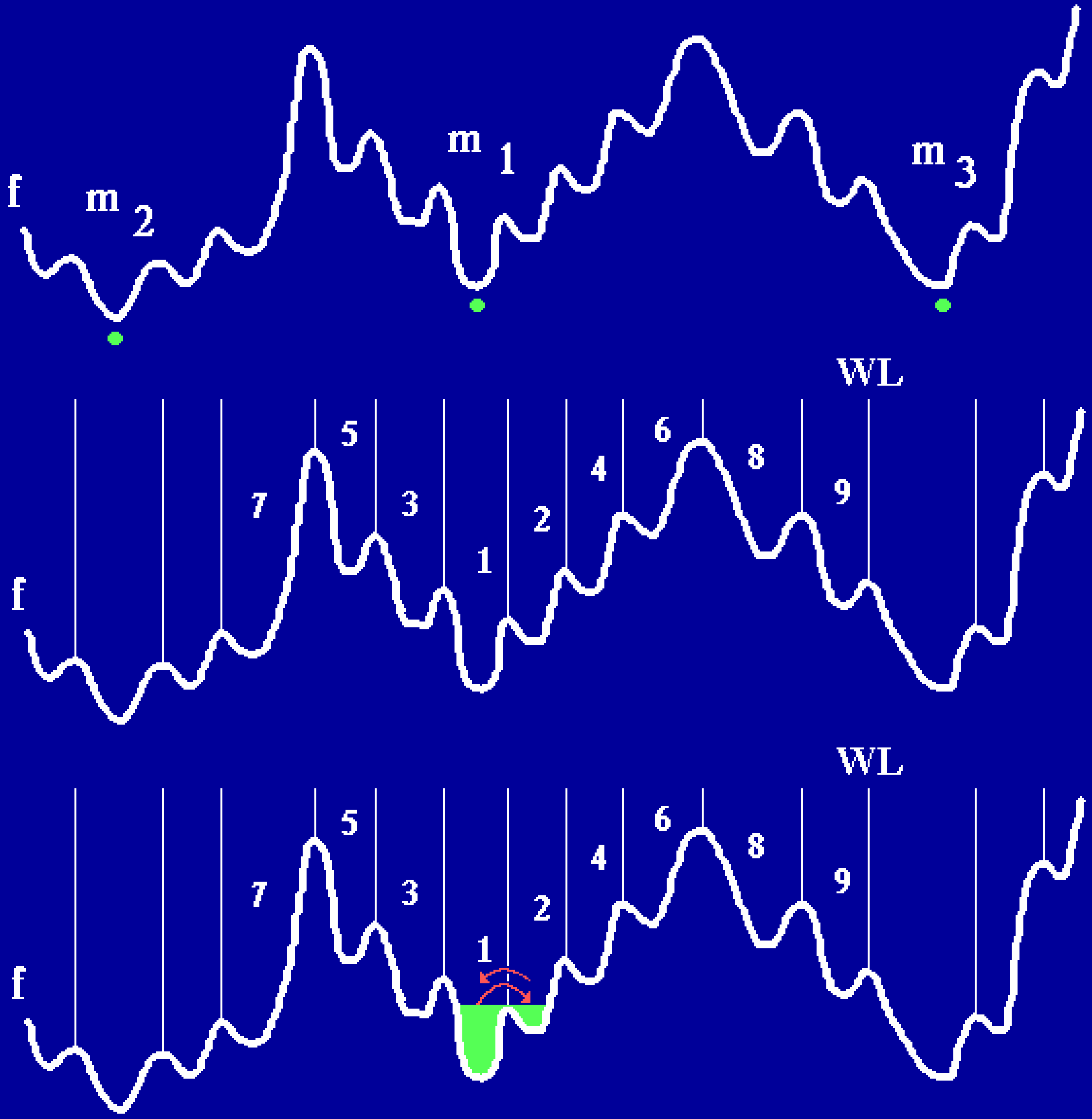


Lower catchment basin

It is the part of the catchment basin which is flooded before the first overflow (which occurs through the lowest FOZ)

WATERFALLS

Introduction



Consider the function f and its watershed. Various catchment basins are numbered from 1 to 9. Consider the flooding from the minimum m_1 .

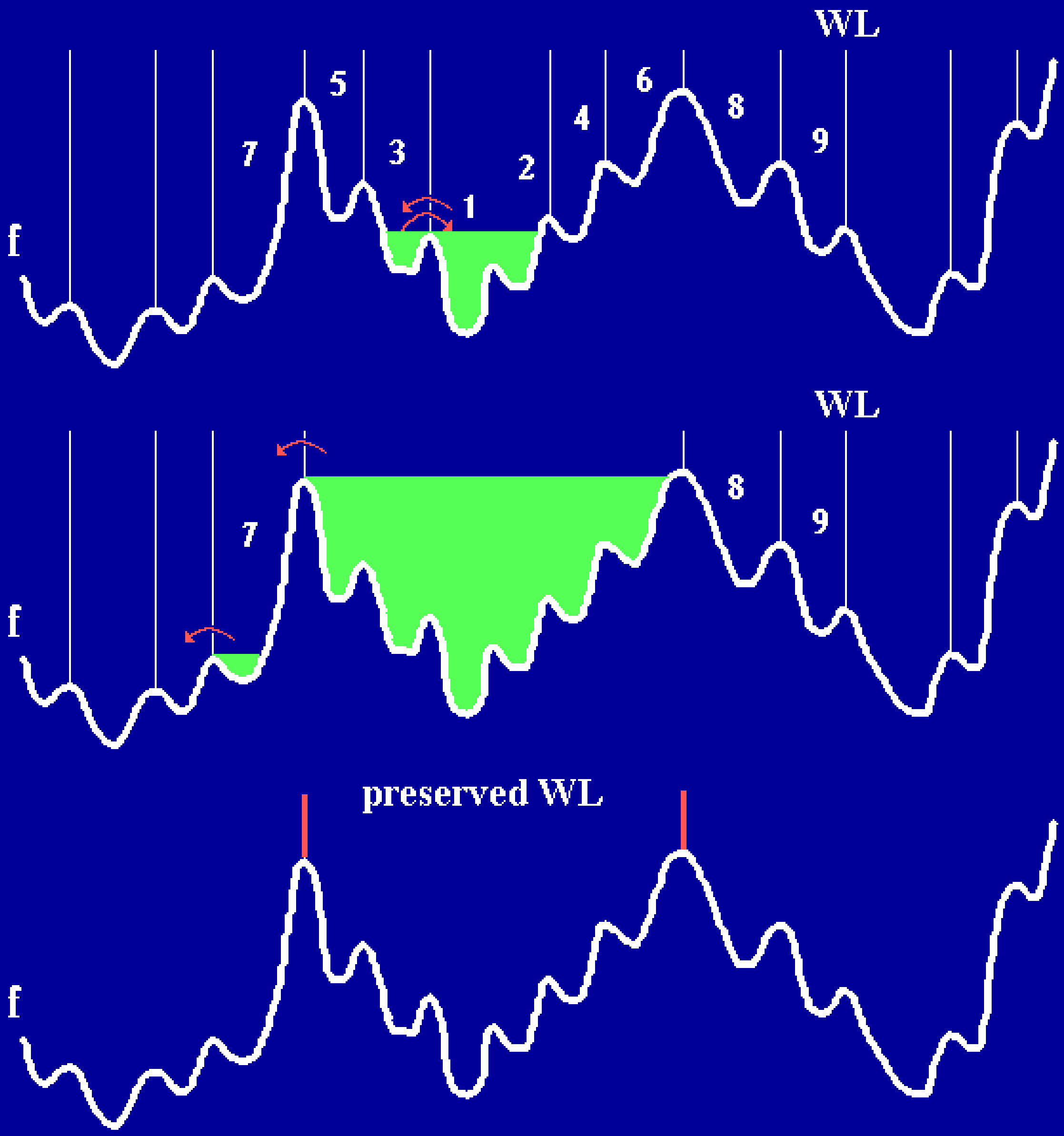
When filling CB1, an overflow occurs towards CB2.

Now, if we fill in CB2, the first overflow occurs towards CB1.

In this case, overflows (waterfalls) are symmetrical.

Therefore, the part of the watershed line separating CB1 from CB2 can be removed and the floods in CB1 and CB2 can be merged.

WATERFALLS (2)

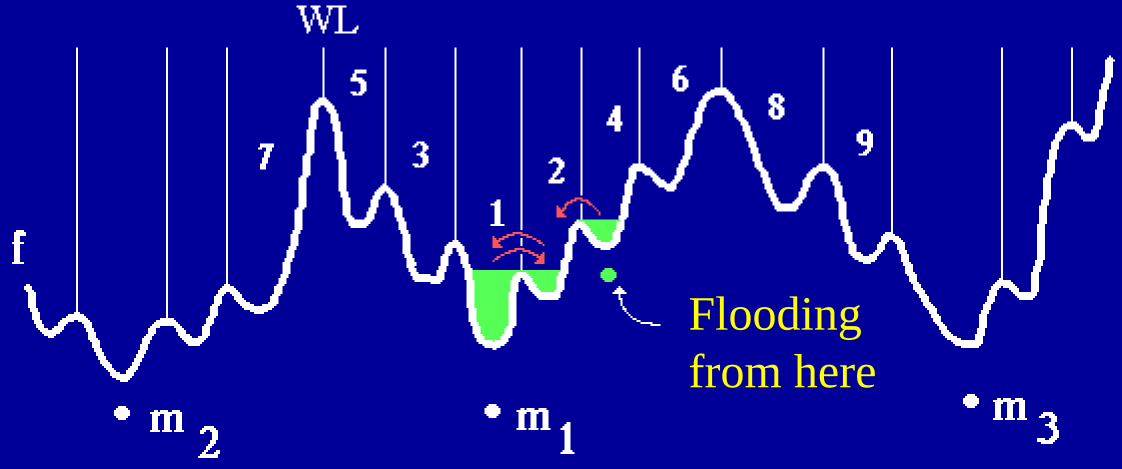


If this flooding process is iterated, the flood invades CB3 which in return, when flooded, pours into the merged basins CB1 and CB2. Here again the waterfalls being symmetrical, CB3 is merged to the flood.

Step by step, because, in each case, waterfalls are symmetrical, all the catchment basins from 1 to 6 are merged.

But, when the flood pours into CB7, the situation changes. Now, if we flood CB7, the waterfall is no longer symmetrical. Therefore, the watershed line between CB7 and the merged basins must be preserved.

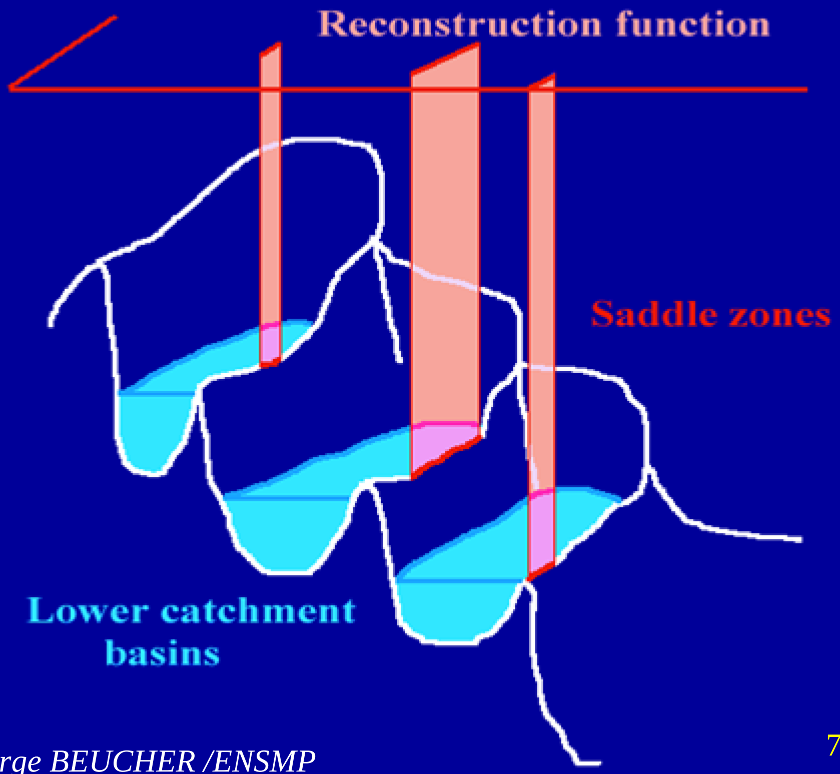
SIGNIFICANT CB/ARCS AND RECONSTRUCTION



The previous process does not work if we start from any basin. However, the flooding in the end reaches the significant CBs.

The successive floods generate the lower catchment basins associated with each CB (flood just before the overflow through the lower saddle zone).

This can be achieved directly by a dual reconstruction of the initial function by the lower saddle zones.



RECONSTRUCTION AND HIERARCHICAL IMAGE

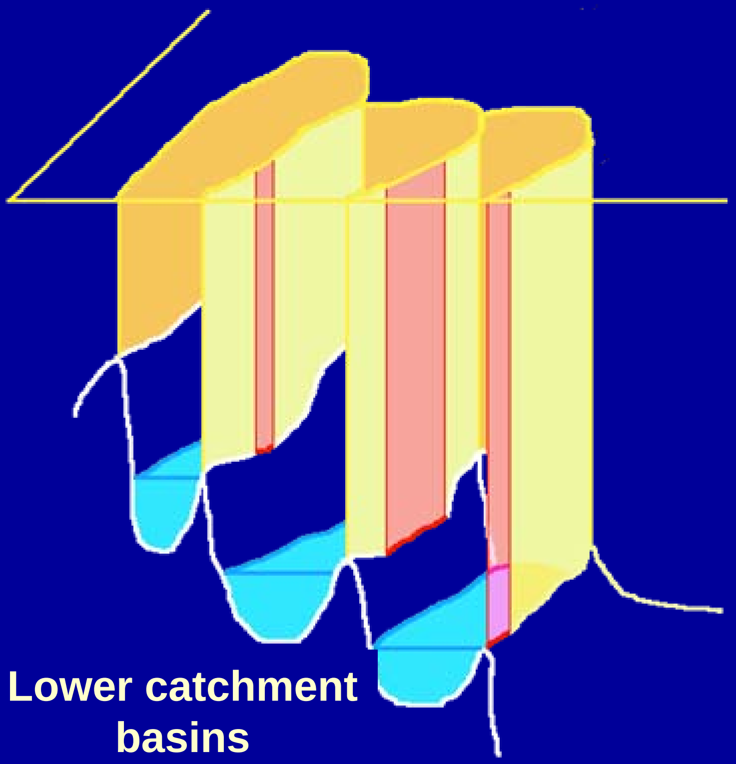
Instead of using FOZ (not easy to detect them), the whole set of watershed lines may be used. The result will be the same because the FOZ is the region at the lowest altitude bordering the catchment basin.

- **f, initial function**
- **let us define g:**

g (x) = f (x) if and only if x belongs to the watersheds of f

g (x) = max if not

- **h = R^*_f (g) , result of the dual reconstruction of f by g, is also called hierarchical image**
- **W(h), watershed of h, produces the hierarchical segmentation of higher level**

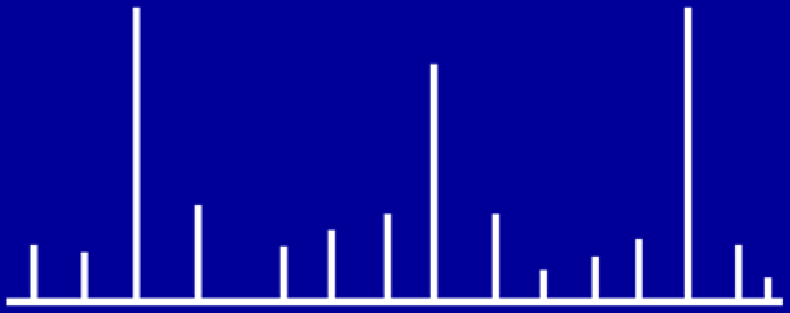


When f is a valued WTS, this hierarchical image is identical to the previous definition.

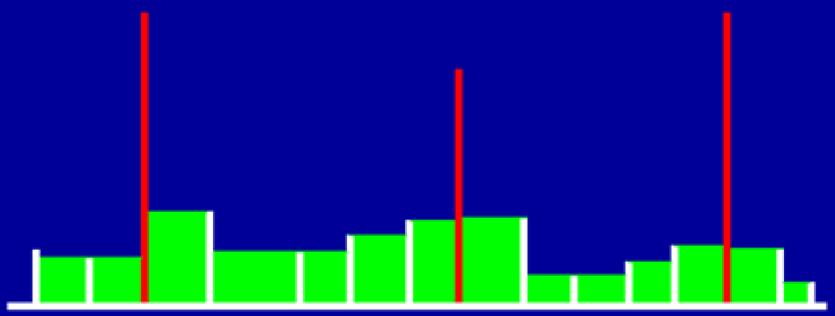
WATERFALLS AND MOSAIC IMAGES

In this case, the hierarchical approach and the waterfall approach are identical. The waterfall transformation is the generalisation for any function of the hierarchical approach.

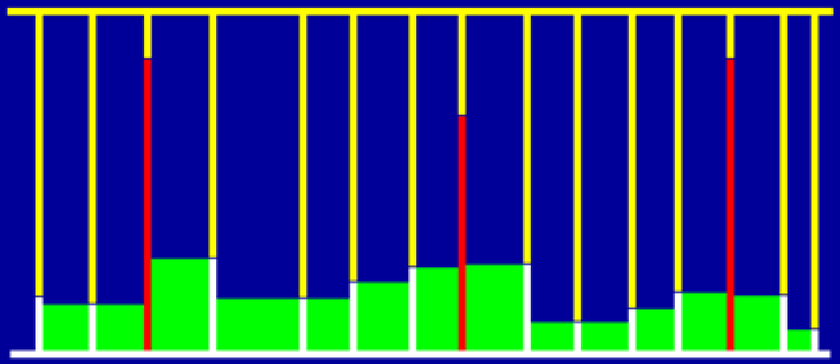
The minimal valuation of the catchment basin corresponds to the height of the lower FOZ. This valuation produces the same result as the reconstruction of the gradient mosaic function by the lower FOZ.



Gradient mosaic image

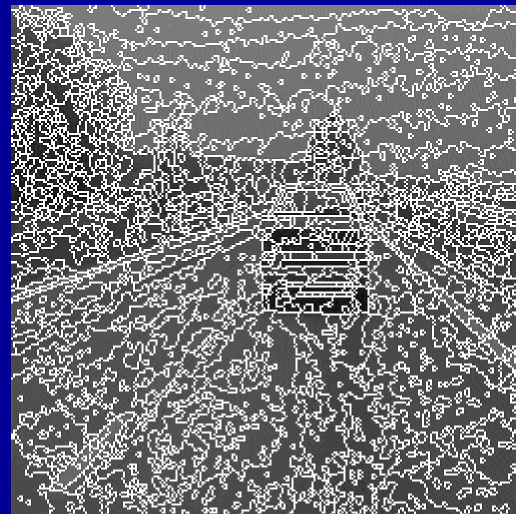


Hierarchical image



Waterfall image

HIERARCHICAL SEGMENTATION: EXAMPLE



Initial watershed

Mosaic image

Original image

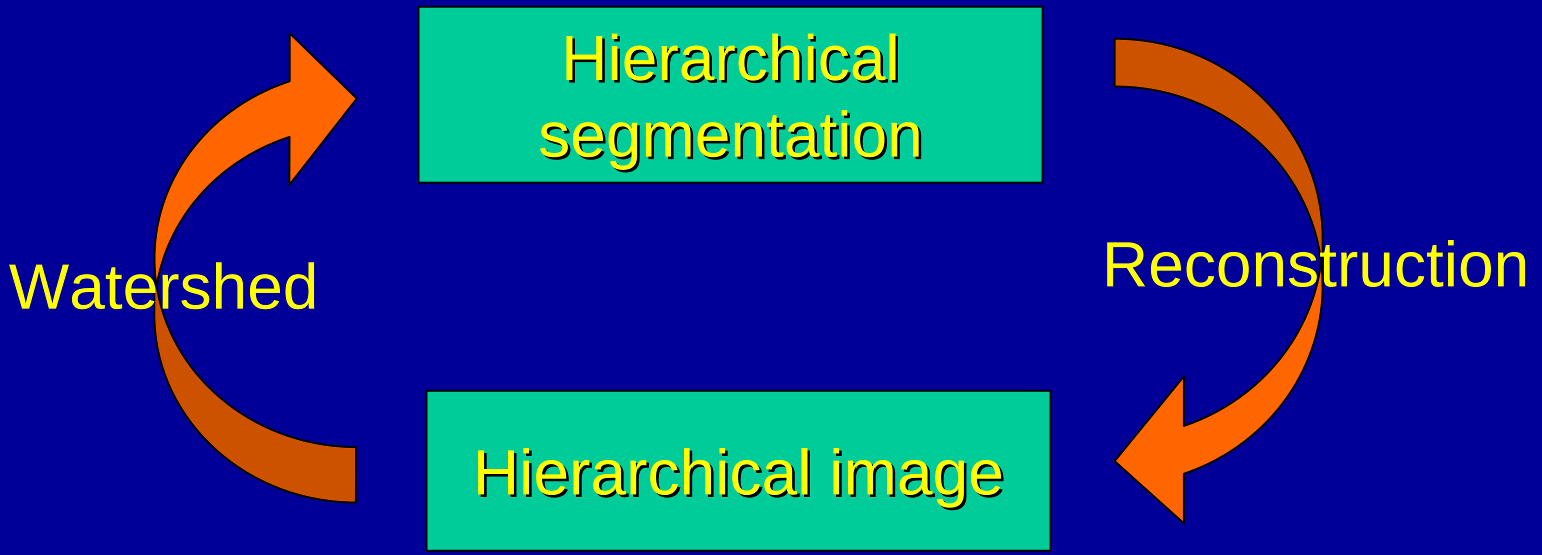
First level of hierarchy

USE OF WATERFALLS

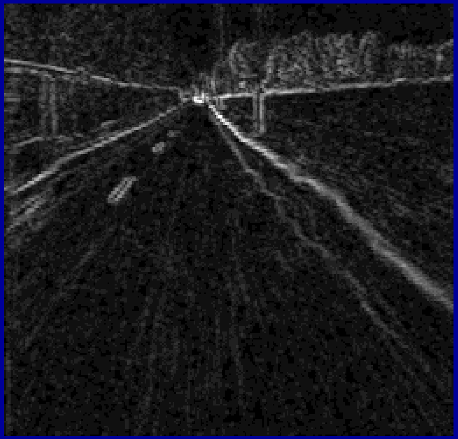
Protocol

- **One starts from an initial valued wateshed s_0**
- **An iterative process produces successive hierarchical segmentations s_i:**

$s_i = w(h_{i-1})$ where h_{i-1} is the hierarchical image associated to the segmentation s_{i-1}

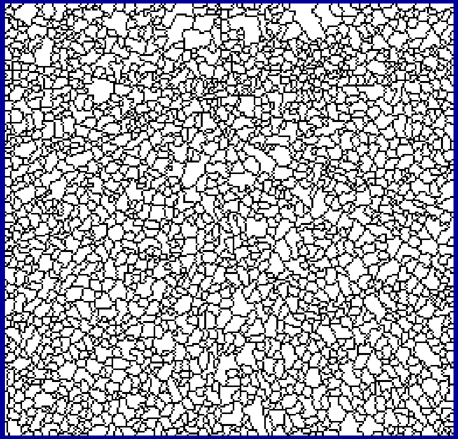


USE OF WATERFALLS (2)

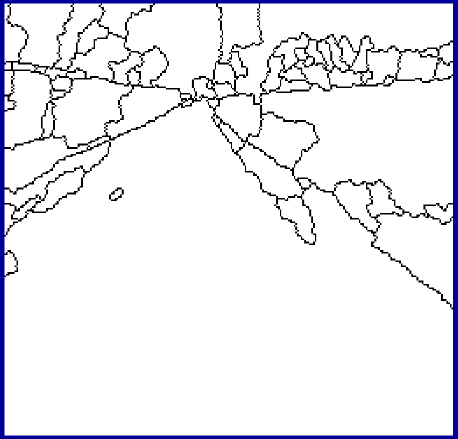


Initial image f and its gradient g

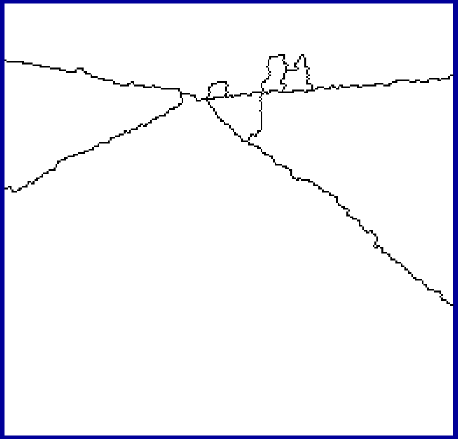
$S_0 = W(g)$



S_0



S_1



S_2

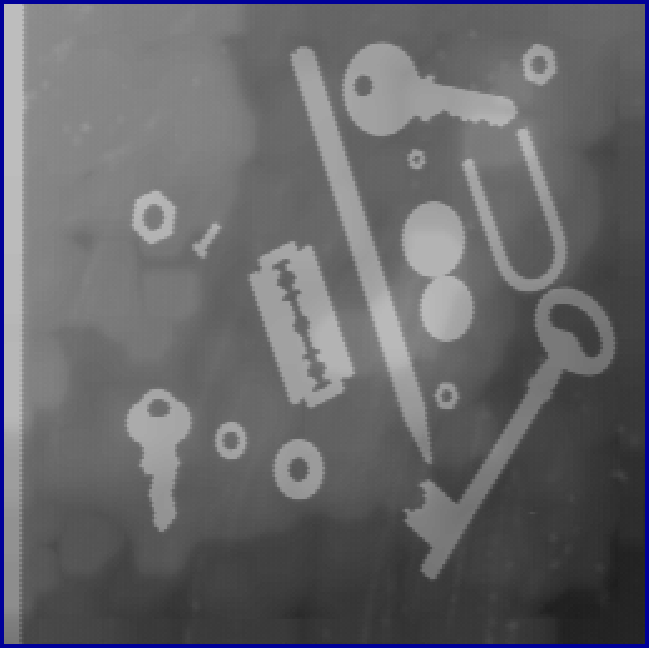
….

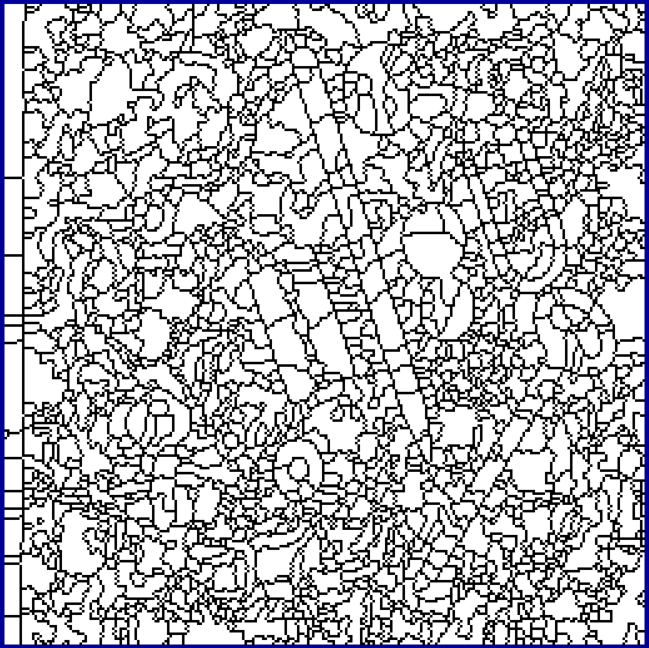
- **N hierarchcal levels, with $S_N = \varnothing$**
- **It is difficult to select a « good » hierarchical level**
- **Other crucial problems appear…**

PROBLEMS WITH THE WATERFALLS

- It's a non parametric approach
- The waterfall can be iterated, leading to possible higher levels of hierarchy, but…
- Stop criterion not available
- The successive hierarchical levels are far from being relevant.

« Myopia » for hierarchies of different levels (classification errors)





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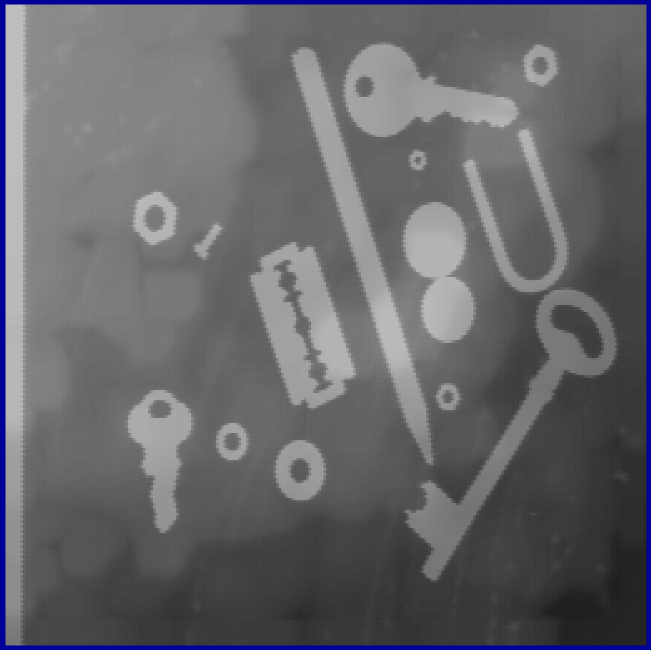

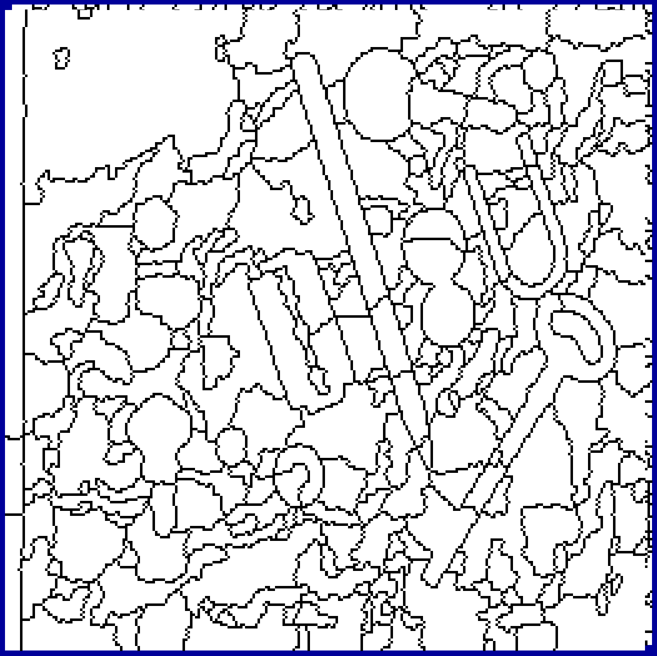

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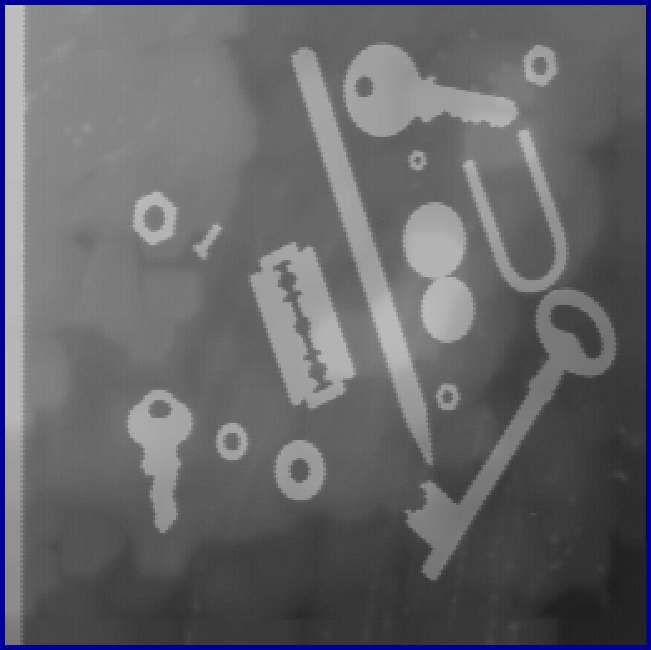

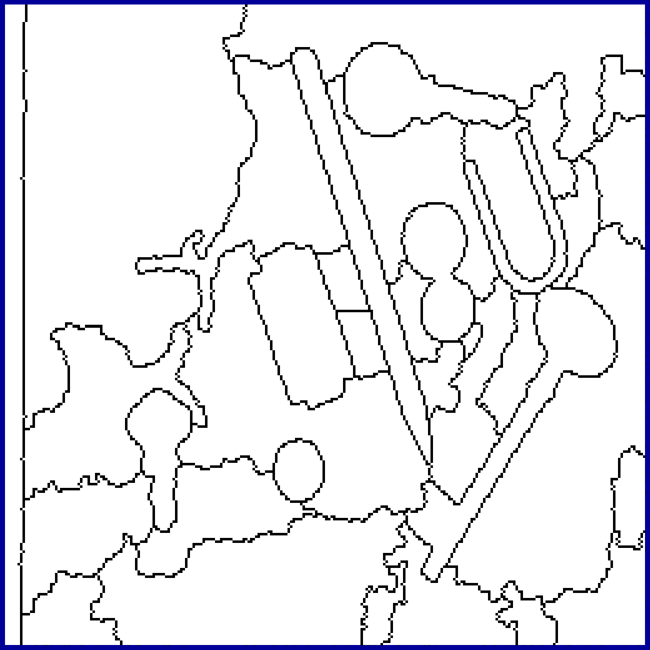

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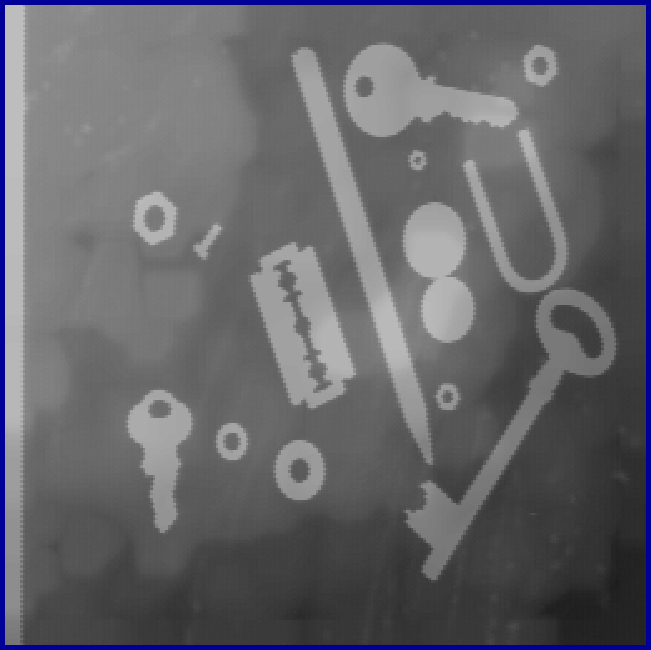

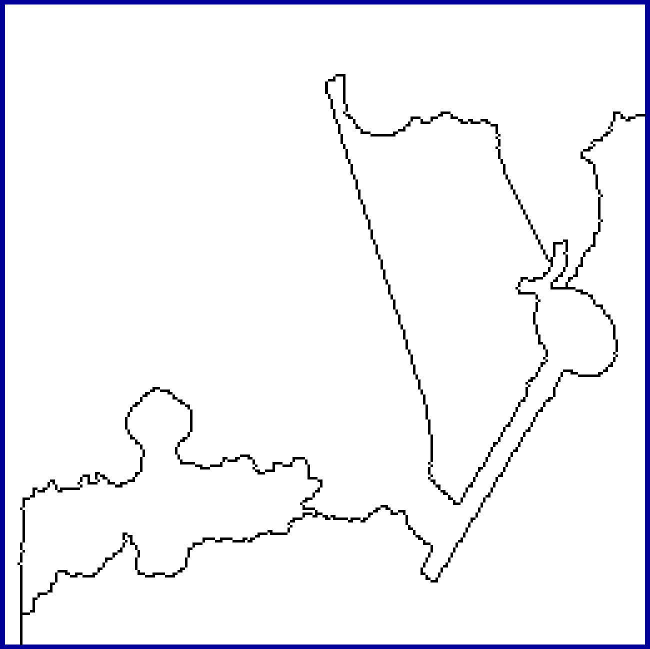

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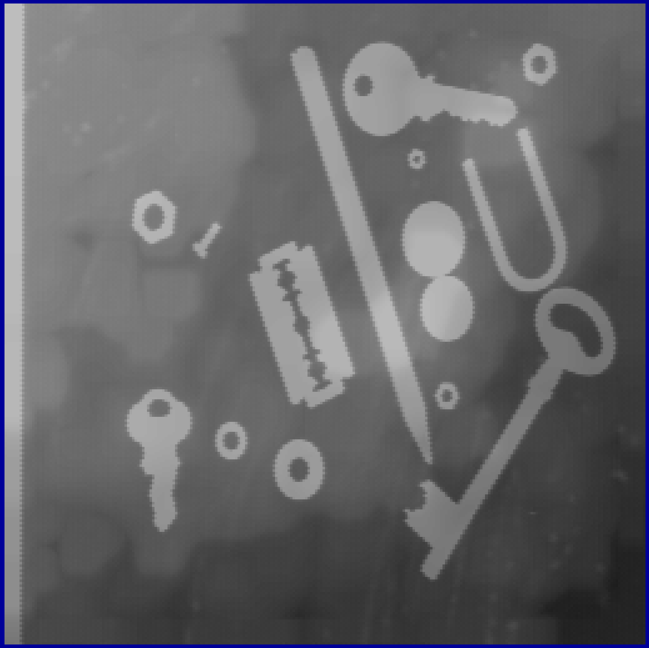
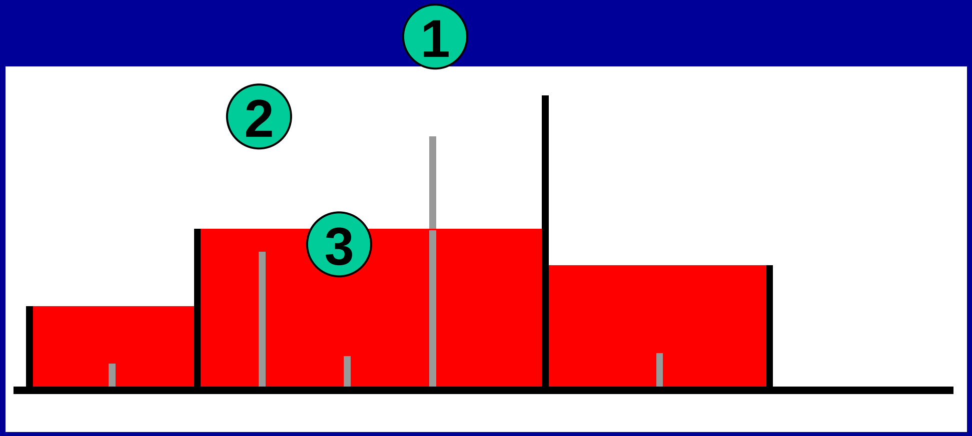


WATERFALLS SHORTSIGHTNESS



Hierarchical image h_{i+1} (in red)

In grey, the contours which are removed by the waterfall transformation

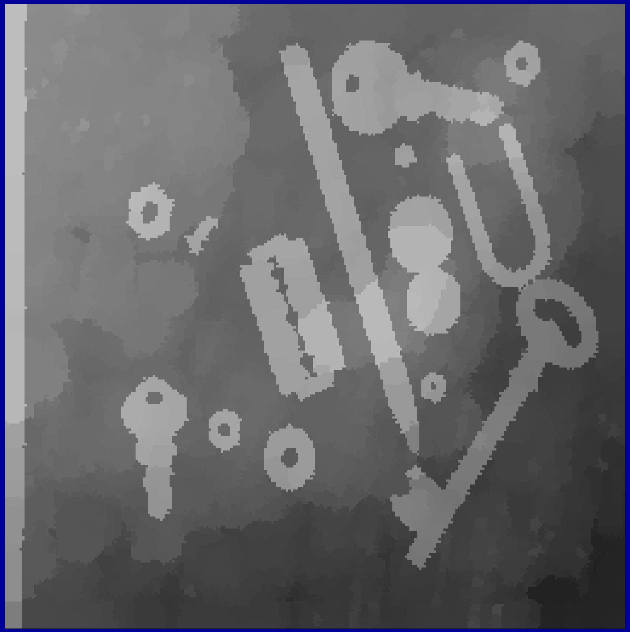
Three different kinds of removed contours appear:

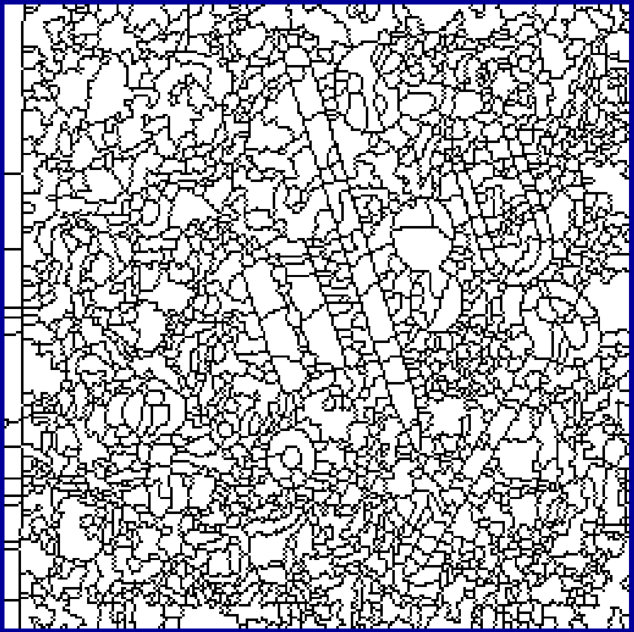
1. **Contours whose altitude is higher or equal to h_{i+1}**
2. **Contours whose altitude is lower than h_{i+1} but closer to the hierarchical image h_{i+1} than to 0**
3. **Contours whose altitude is close to 0**

Only the removal of the last kind of contours is justified

PROPERTIES AND EXAMPLE

- A new hierarchy level appears only if at least one contour is removed
- The last level is NEVER empty. It's a self-blocking process





4 levels of hierarchy only…

This result is due to the fact that the watershed is not an homotopic transform

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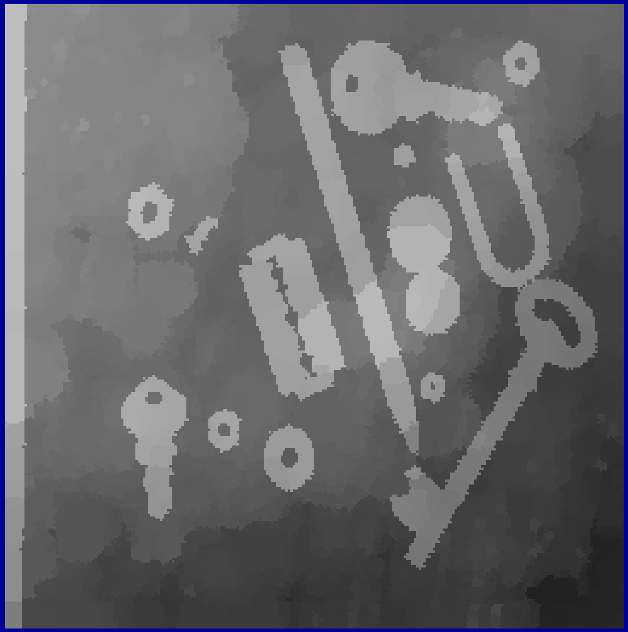

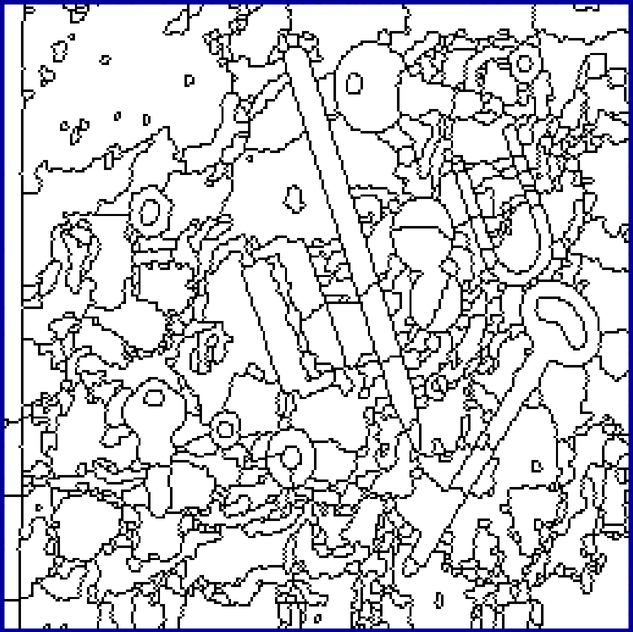

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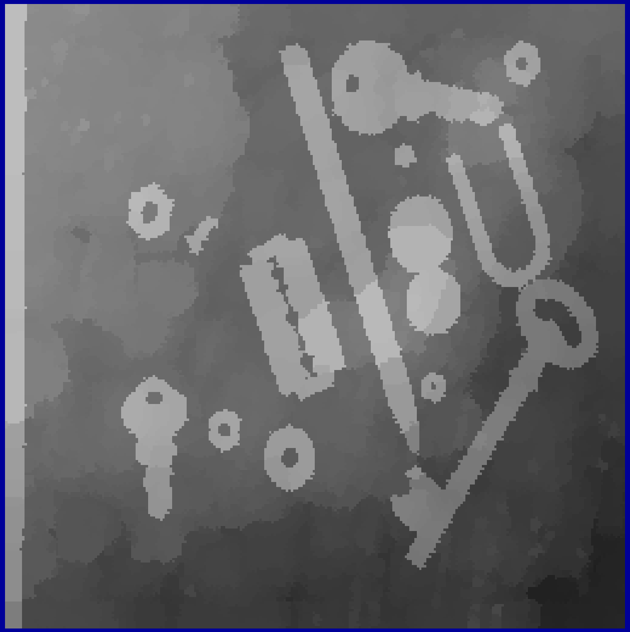
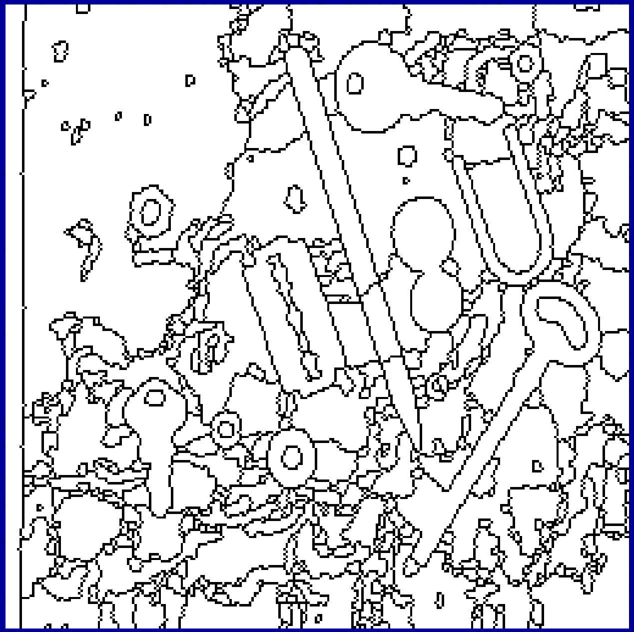


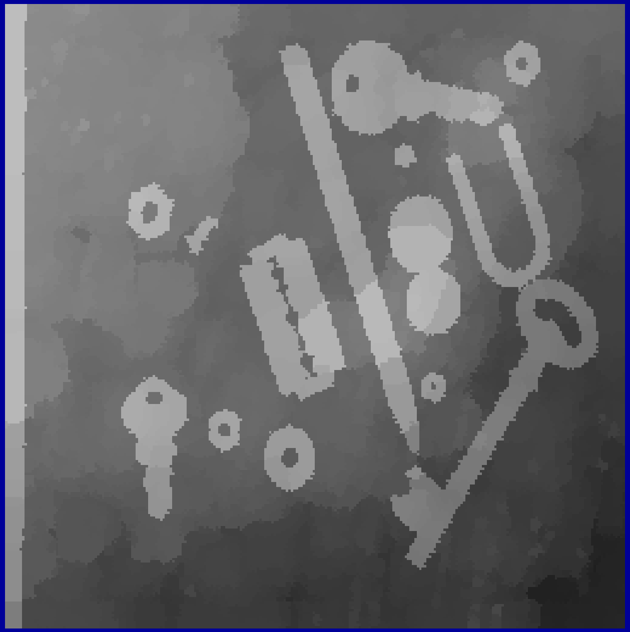


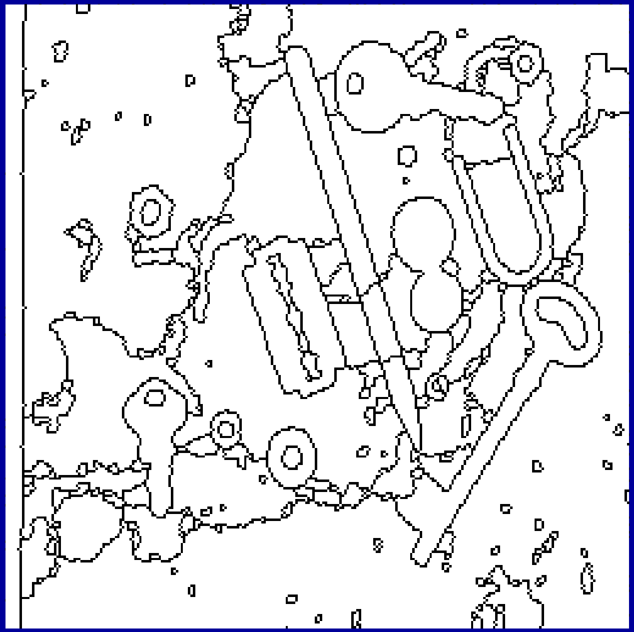
4 levels of hierarchy only…

This result is due to the fact that the watershed is not an homotopic transform

PROPERTIES AND EXAMPLE

- A new hierarchy level appears only if at least one contour is removed
- The last level is NEVER empty. It's a self-blocking process

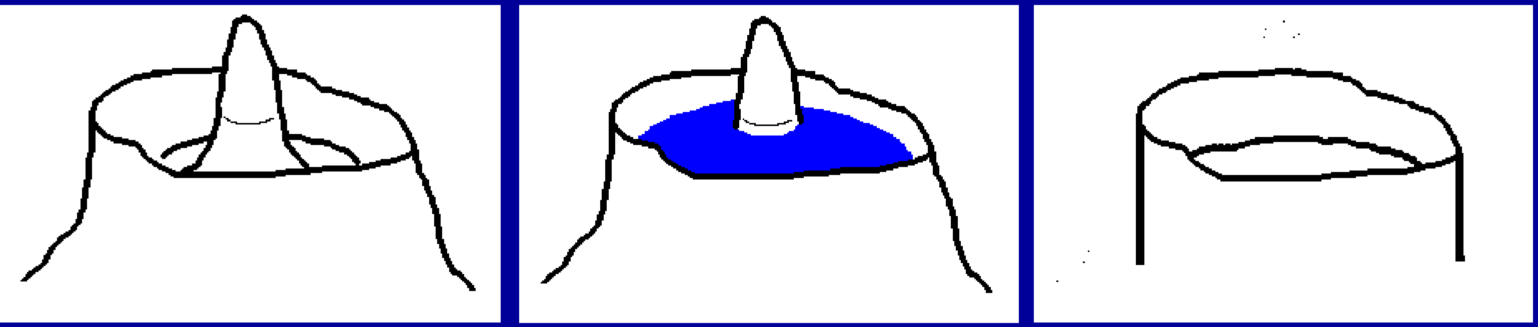




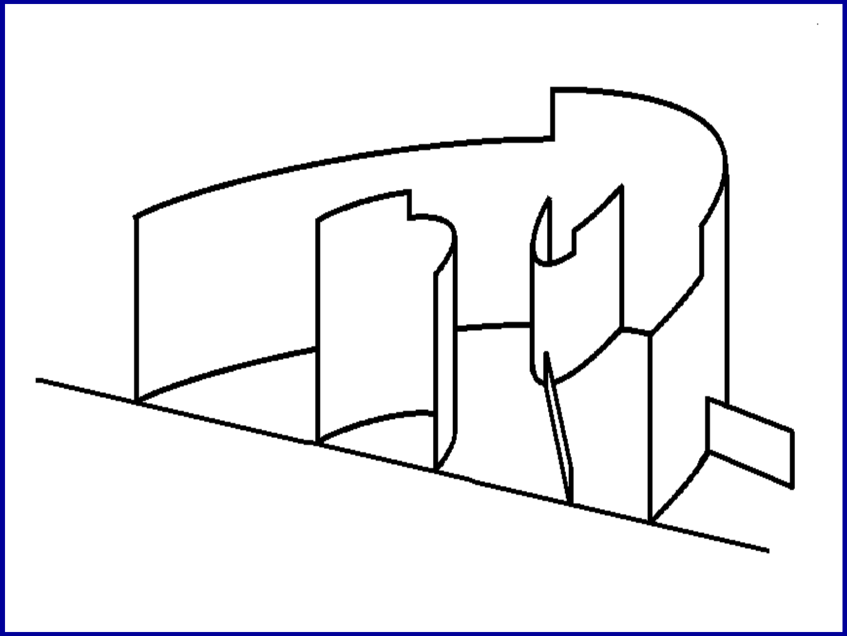
4 levels of hierarchy only…

This result is due to the fact that the watershed is not an homotopic transform

SEMI-HOMOTOPY OF THE WATERSHED

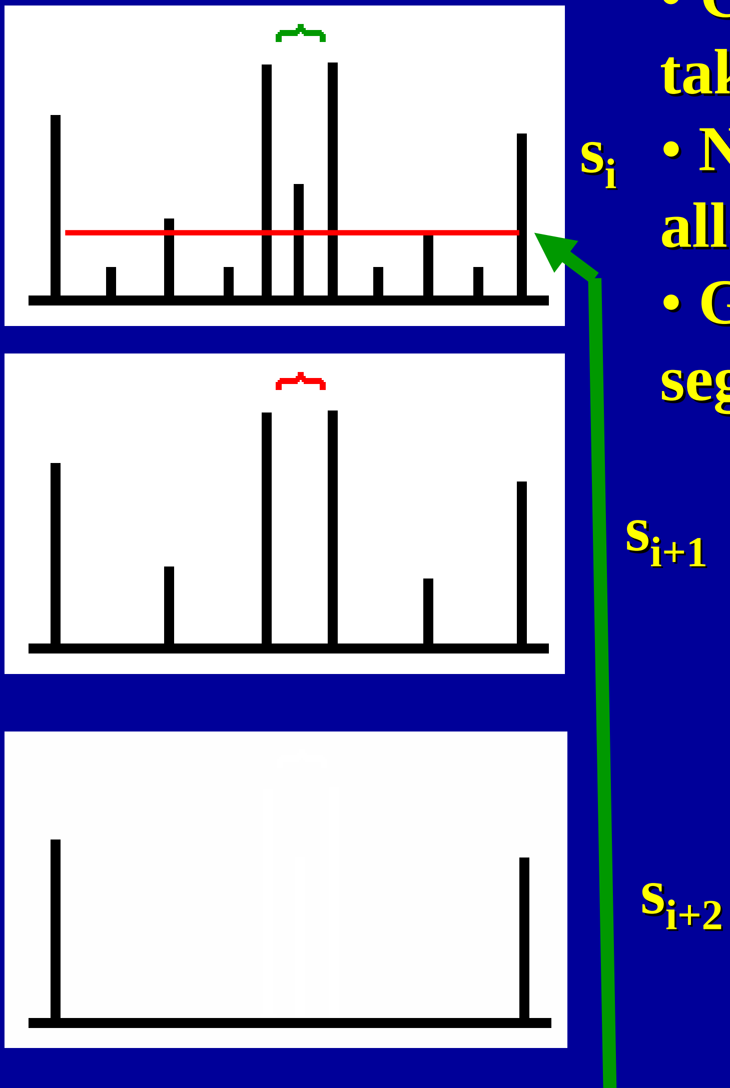


Watershed

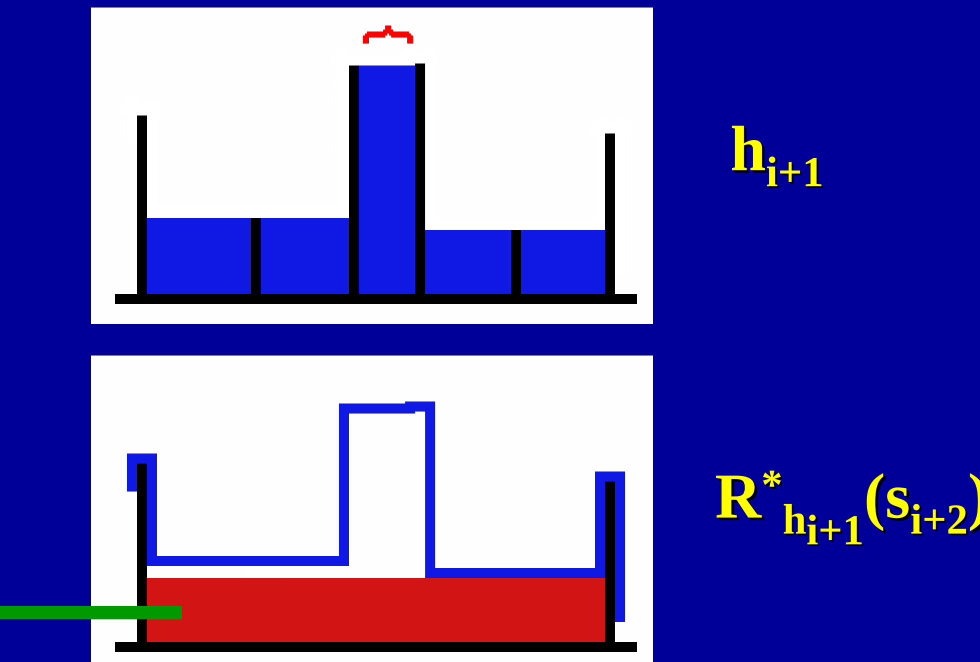


- **Maxima which are inside the catchment basins are not taken into account.**
- **The islands appearing during the flooding process are always submerged**

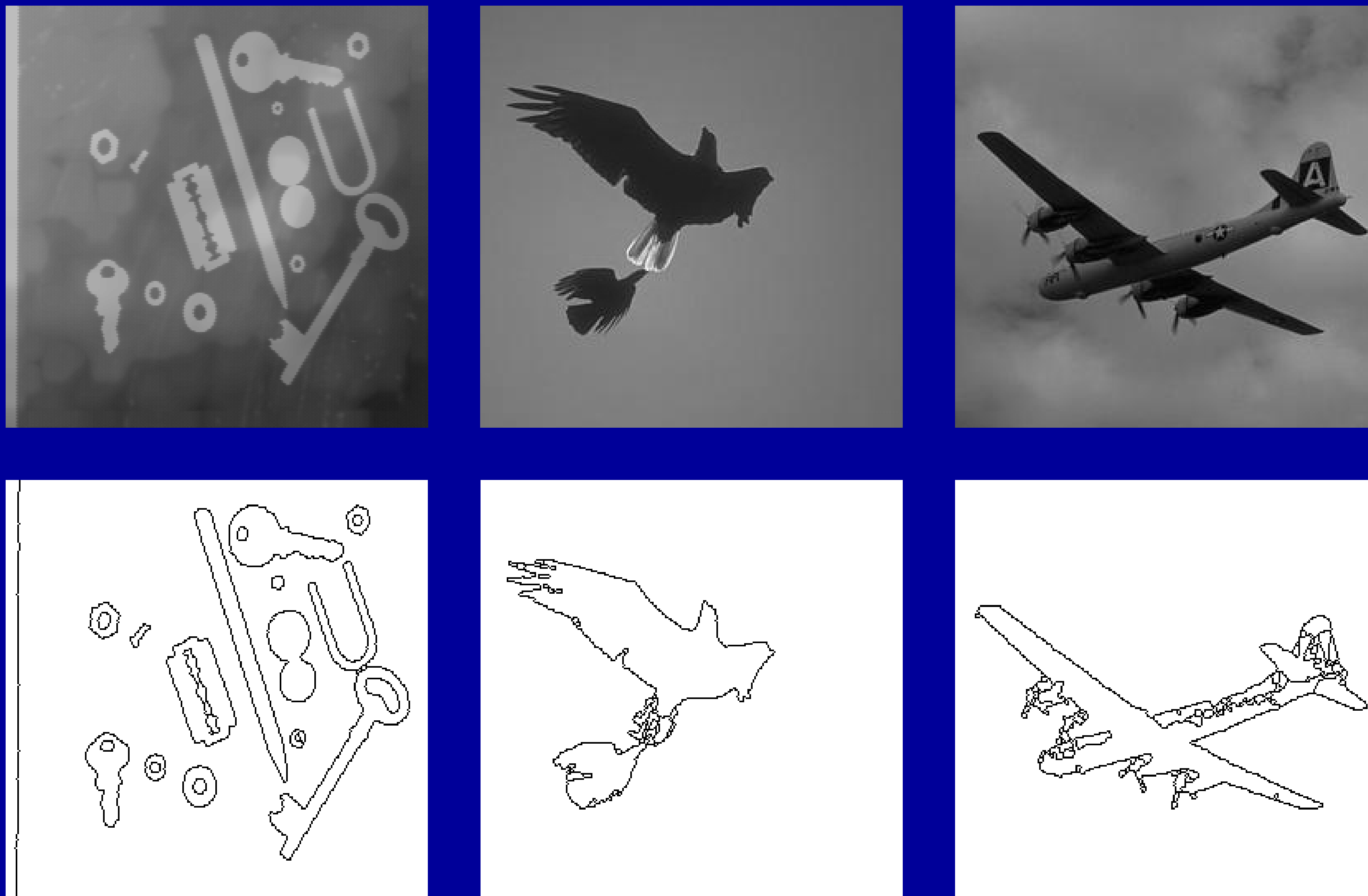
ENHANCED WATERFALL ALGORITHM



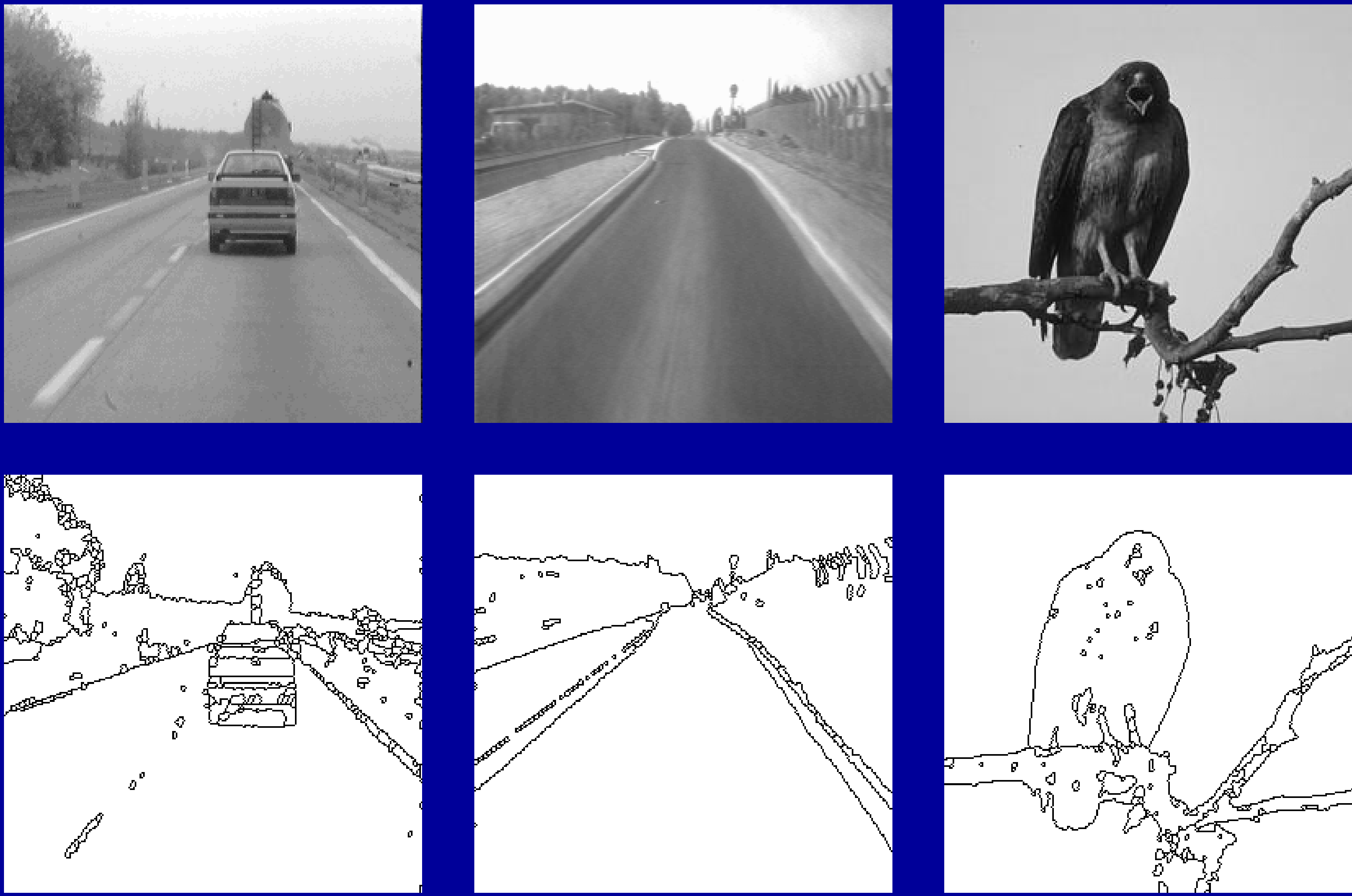
- Contours inside the « islands » must be taken into account
- No « interference » between isles is allowed
- Geodesic dual reconstruction of segmentation s_{i+2} by hierarchy h_{i+1}



EXAMPLES OF SEGMENTATION



EXAMPLES OF SEGMENTATION (2)

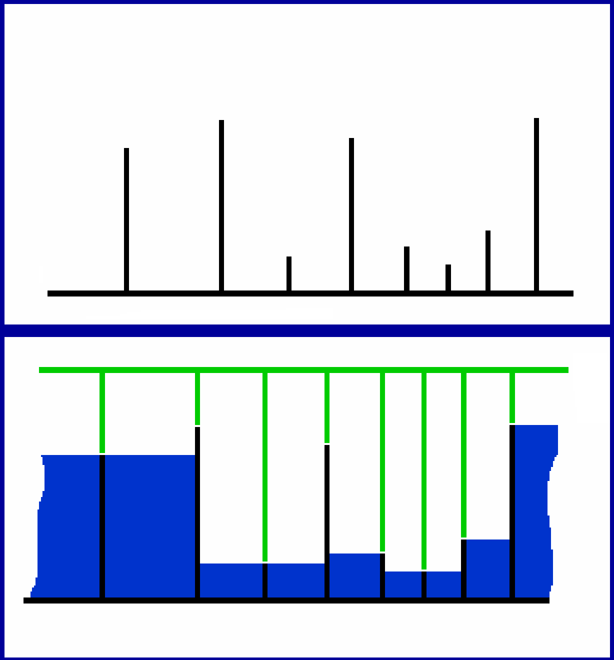


INTRODUCTION TO PILINGS

- Not the same approach as waterfalls but with identical premises)

- Definition of a residual transform
  (works only on valued WTS – CB and SCB have same support)

$w_0 = \psi_0$



ψ_1

We define a function:

$\xi_0 = \psi_0$ on $Min^c(\psi_0)$

$\xi_0 =$ max on $Min(\psi_0)$

We define then:

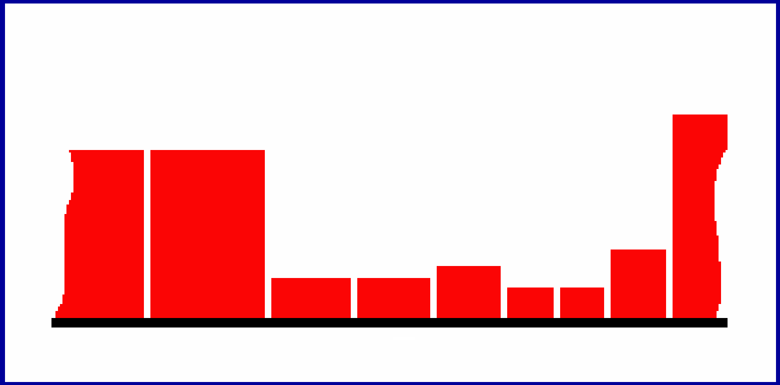
$\psi_1 = R^*_{\xi_0}(\psi_0)$

At the first step, ψ_1 is the hierarchical image because W_0, the WTS is the complementary set of the minima of ψ_0

RESIDUES OF PILINGS

We can then define a first residue r_1 as the difference between ψ_1 and ψ_0:

$$r_1 = \psi_1 - \psi_0$$

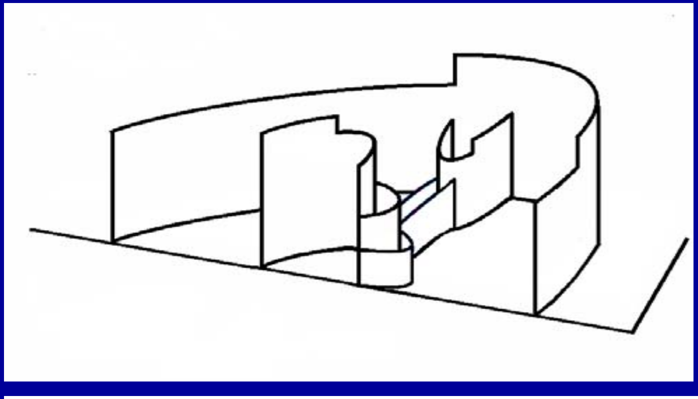


This residue corresponds to the pikings which are necessary to fill in the minima of ψ_0

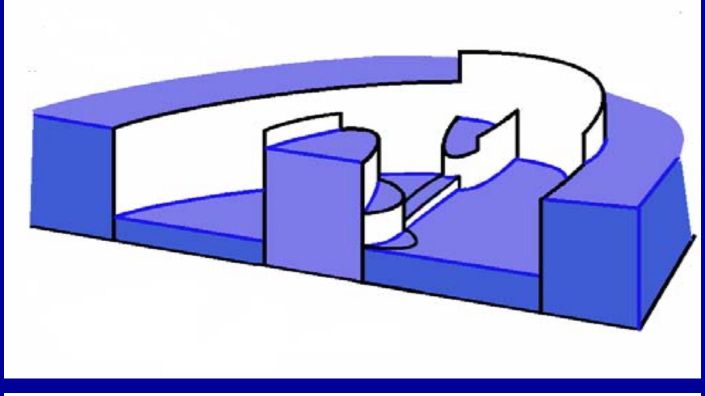
At the next step, the same transformation is defined by using the function ξ_1, itself defined from the minima of ψ_1

RESIDUES OF PILINGS (2)

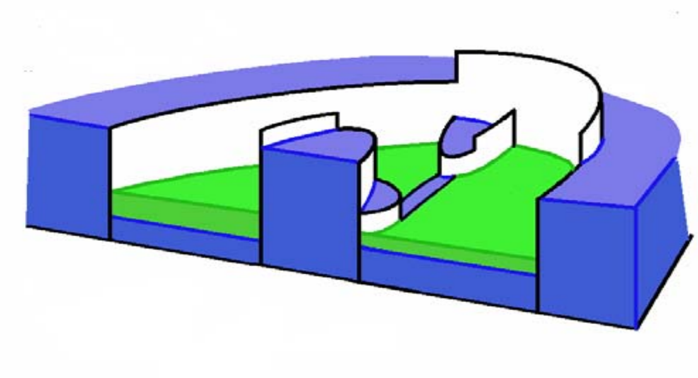
ψ_0



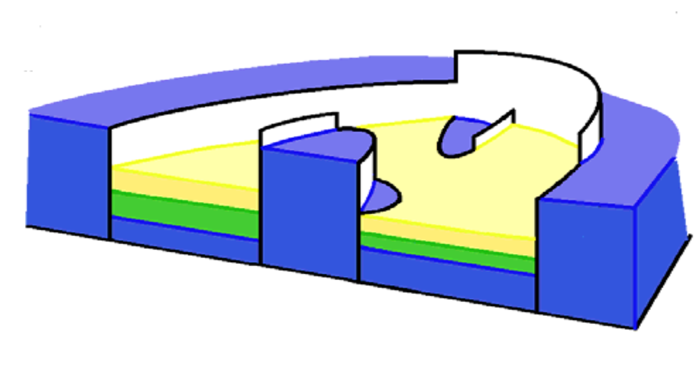
ψ_1



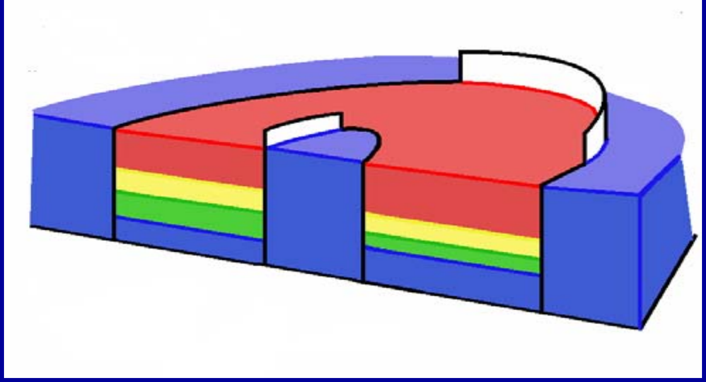
ψ_2



ψ_3



ψ_4



Transformations y_i and successive residues r_i (different colors)

PILING RESIDUAL TRANSFORM

Definition of a residual transform by iteration:

ψ_i is defined from ψ_{i-1}:

$$\psi_i = R^*_{\xi_{i-1}} (\psi_{i-1})$$

with:

$\xi_{i-1} = \psi_{i-1}$ on $Min^c(\psi_{i-1})$

ξ_{i-1} = max on $Min(\psi_{i-1})$

The residue r_i is equal to:

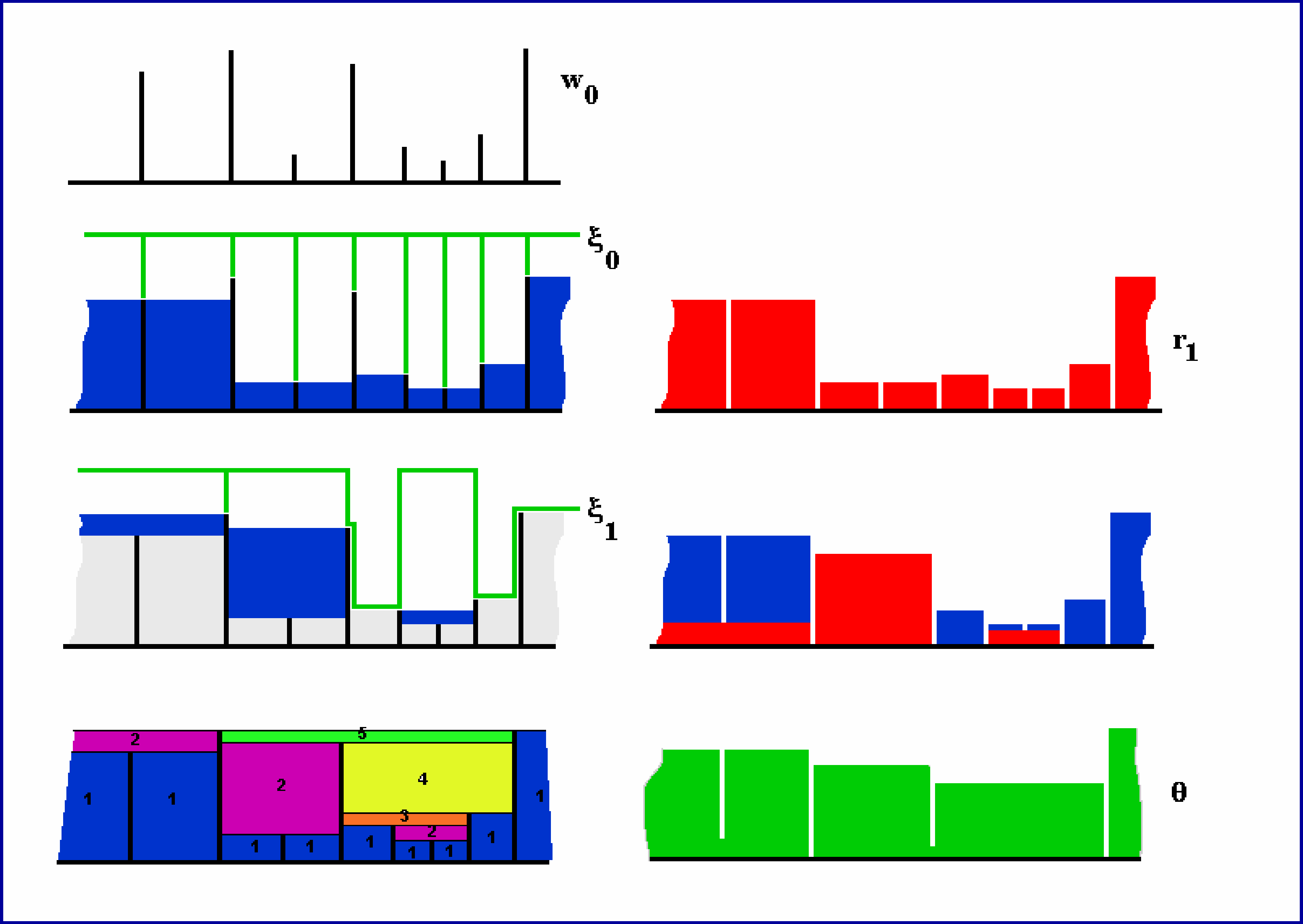
$$r_i = \psi_i - \psi_{i-1}$$

Finally, two functions θ and q are defined:

$$\theta = \sup_{i \in I} (r_i) = \sup_{i \in I}(\psi_i - \psi_{i-1})$$

$$q = \arg\max(r_i) = \arg\max(\psi_i - \psi_{i-1})$$

PILING RESIDUAL TRANSFORM (2)



Successive steps of the construction of function θ

PILINGS AND HIERARCHY

- Pilings cover some contours when some others are preserved (function θ)
- The preserved contours remain in $sup(w_0,\theta)$. So, they can be extracted by a top-hat transform

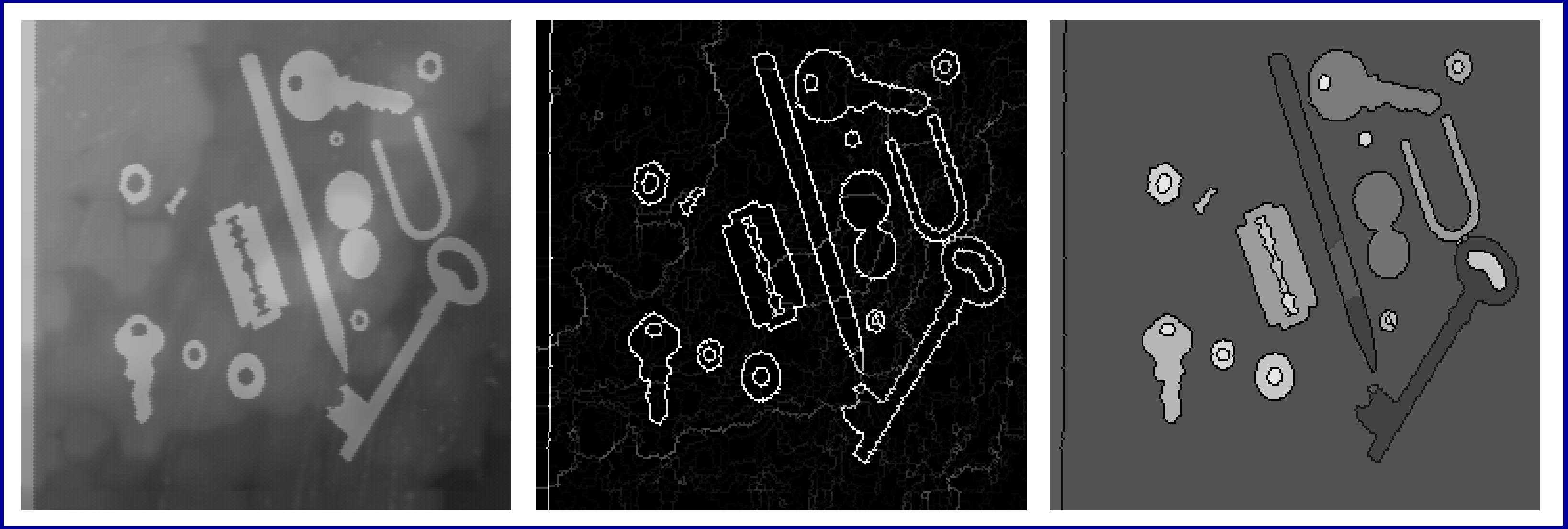
What is the criterion for selecting the preserved contours?

- Waterfall algorithm (primitive one):

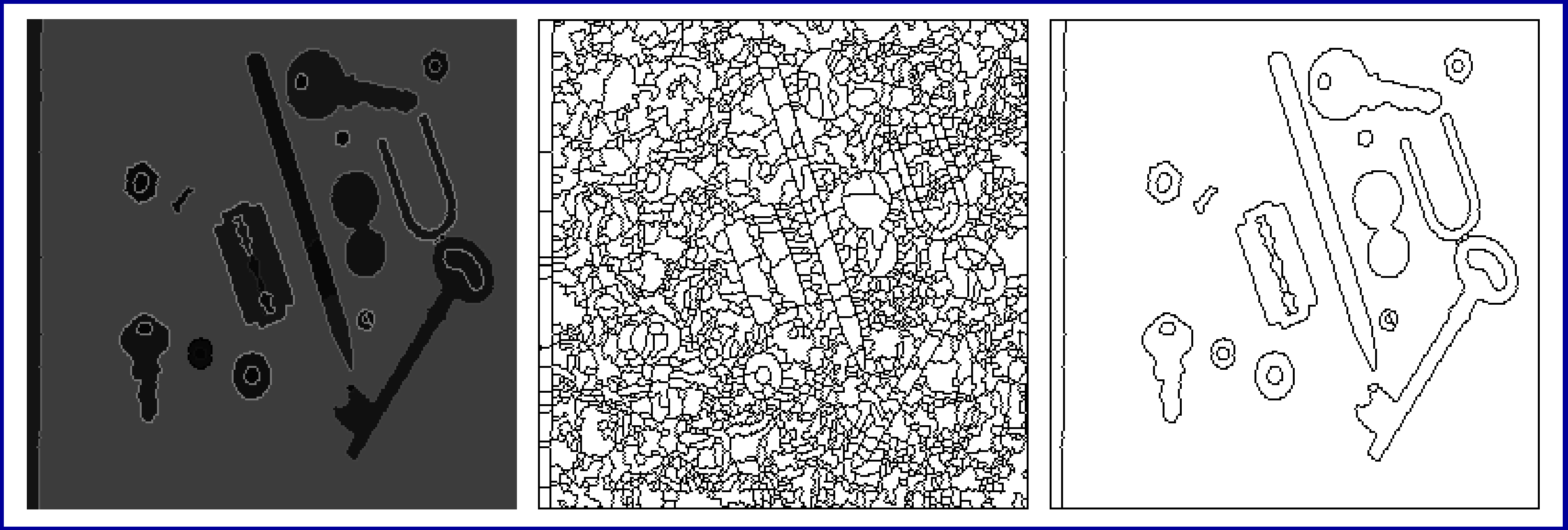
  Contours separating regions where (inside) contours heights are below the maximal height of the preserved contours
- Pilings residues:

  Contours separating regions where (inside) contours heights (if any) are at least twice lower than the minimal height of the preserved contours

EXAMPLE



Original image ψ_0 θ



q *Initial contours* θ *contours*

A TEMPORARY CONCLUSION…

It remains a fruitful methodology

- Extension to the 3D world
- Extension to graphs
- New developments (residues)

These tools are (quite) easily understandable and handy

- There is usually no change of space (image space)
- The toolbox is enriched with affordable transformations

The performances are improving in a dramatic way

- New algorithms, especially for hierarchical segmentation
- Speed of algorithms allowing real-time computation

New tools are available

Strong connexions with perceptive principles (Gestalt theory)

(Very) SHORT BIBLIOGRAPHY

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For a more complete bibliography, see:

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