Controlling the Ultimate Opening Residues for a Robust Delineation of Fragmented Rocks

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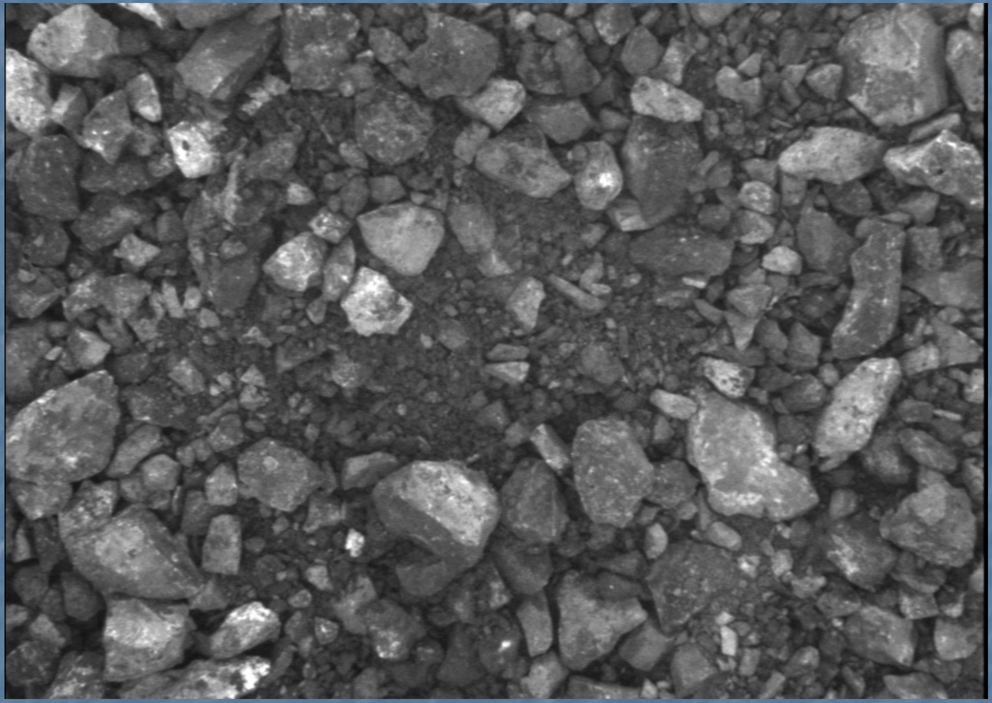
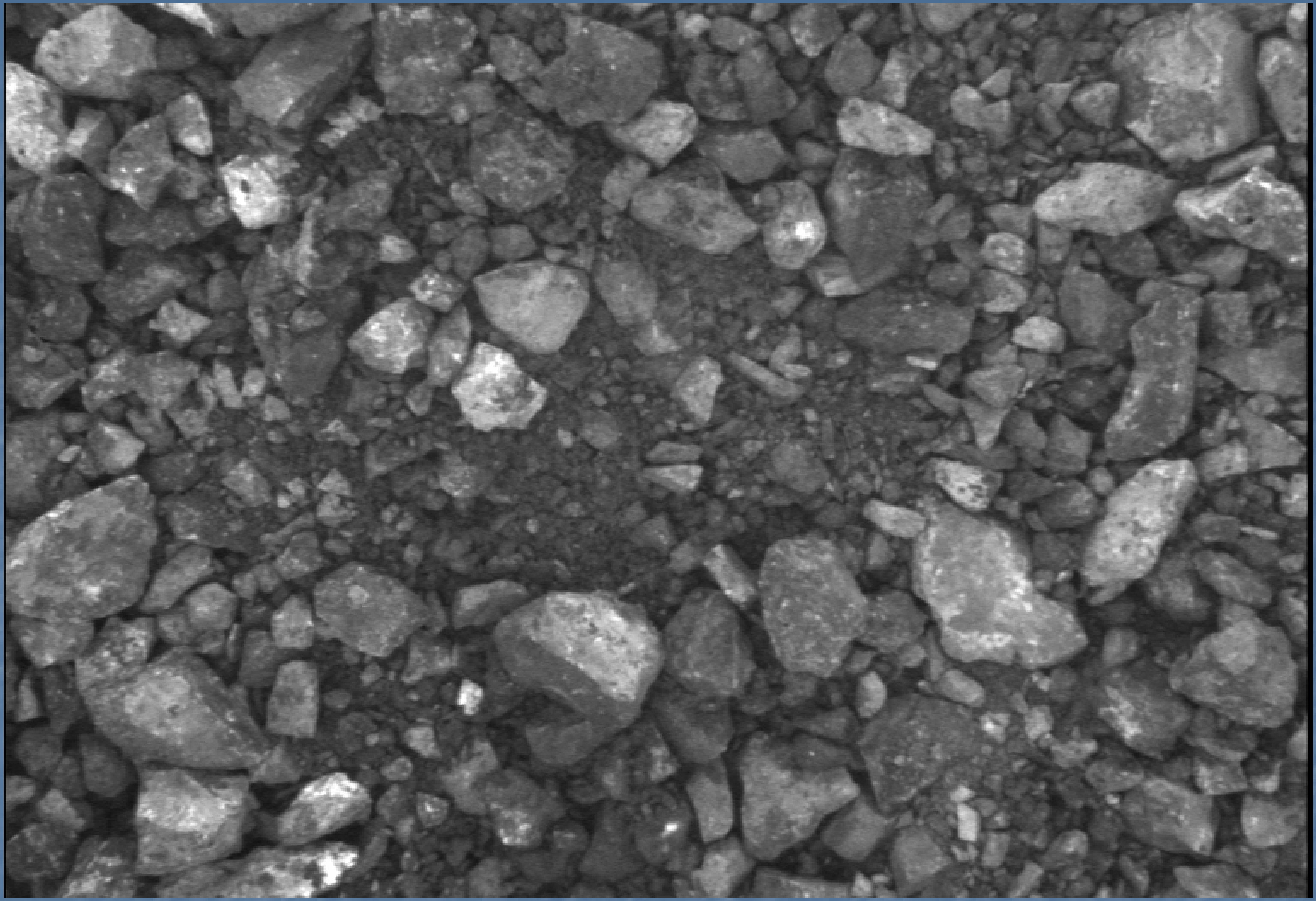
Problem Presentation

Size distribution measurement of rocks in heap in order to optimize the crushing process



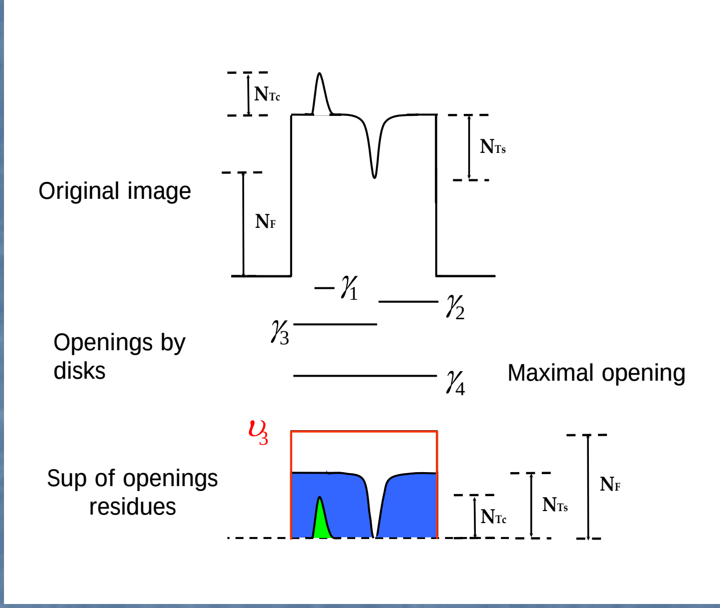
Images are acquired by cameras placed above dump trucks or conveyor belts



This is not a heap of stones

Ultimate Openings

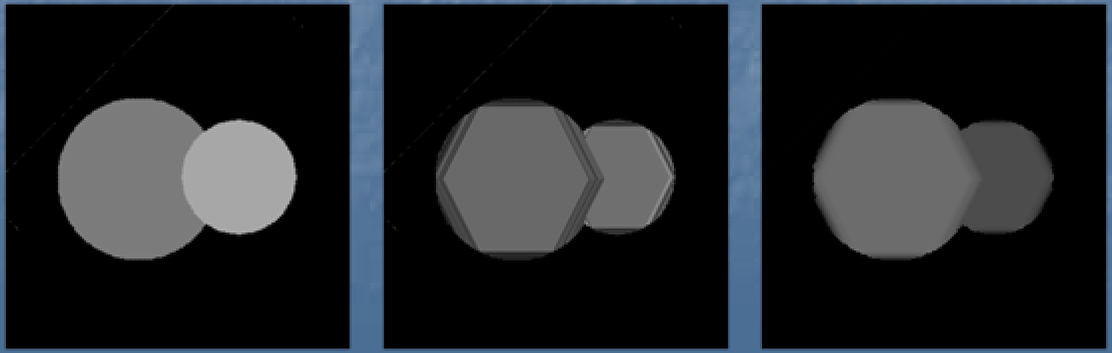
- Old algorithms based on thresholds and set openings (FRAGSCAN software)
- New approach based on ultimate openings (residual transform)



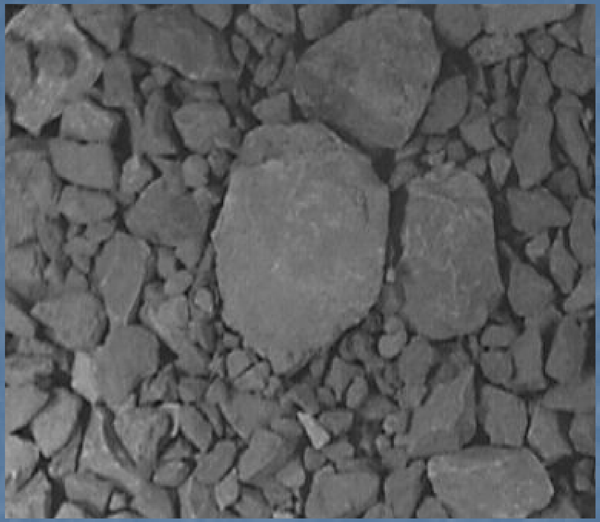
$\rho_i = \gamma_{i-1} - \gamma_i$, residue of openings

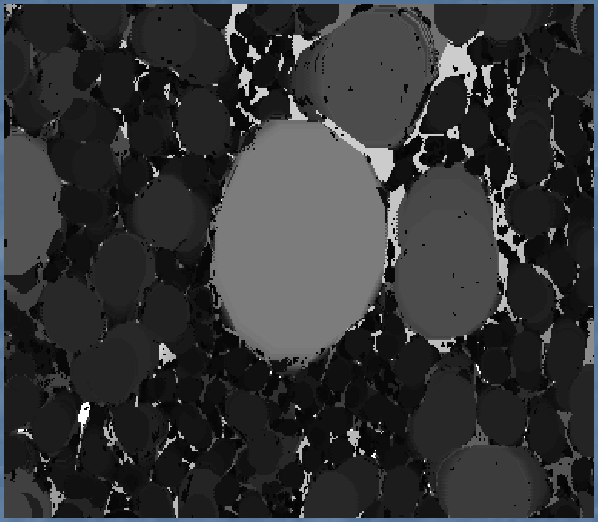
$\upsilon = \operatorname{Sup}_{i \in N} \left(\gamma_{i-1} - \gamma_i \right)$, ultimate opening

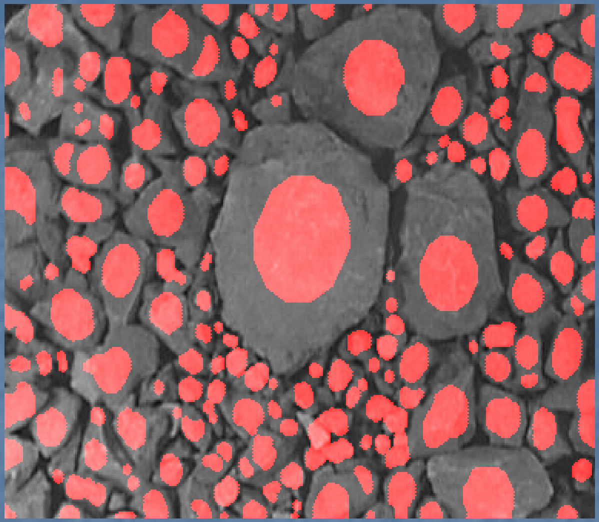
$\varsigma = \arg\max \left(\gamma_{i-1} - \gamma_i \right)$, granulometric function



(almost) Ideal Case

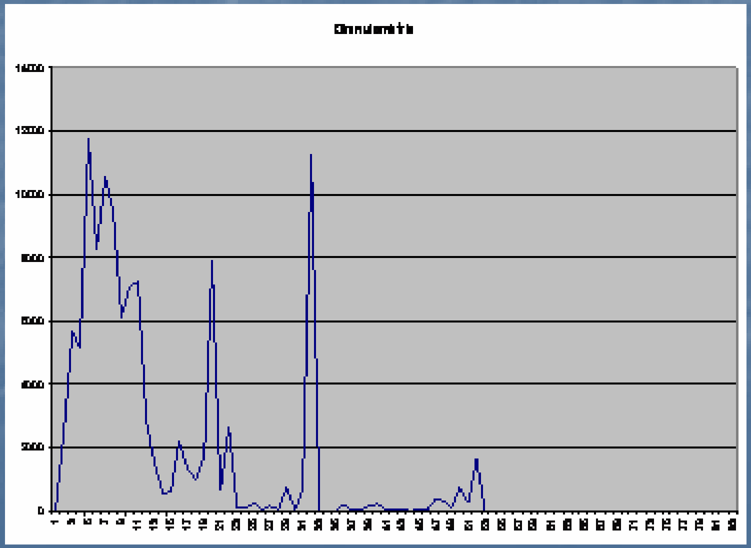




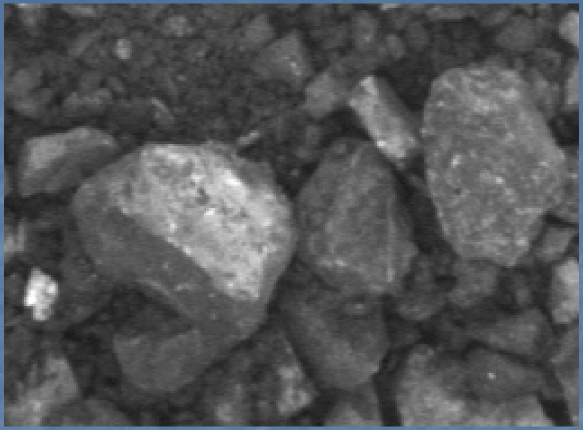


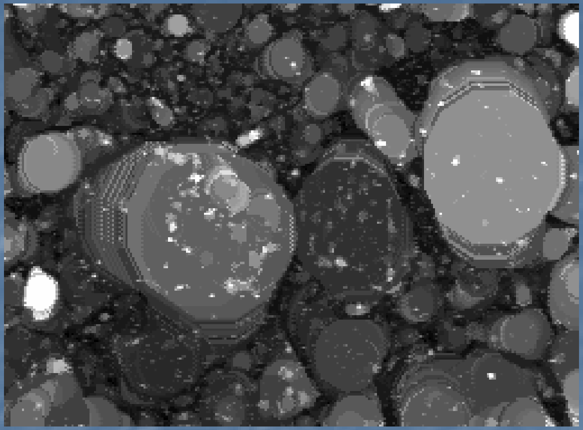
- The granulometric function defines the size distribution of more or less homogeneous regions of the image BEFORE their segmentation

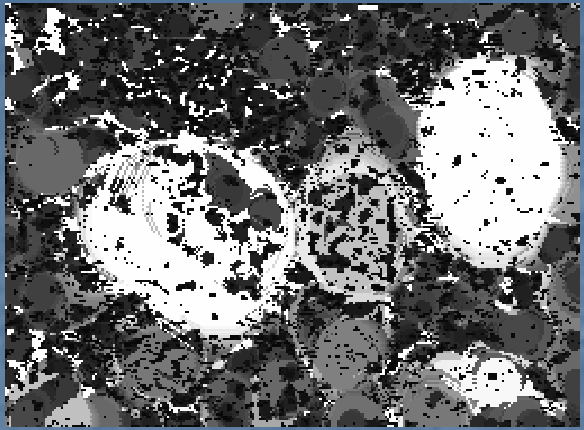
- This function allows also to define markers which can be used for a subsequent segmentation



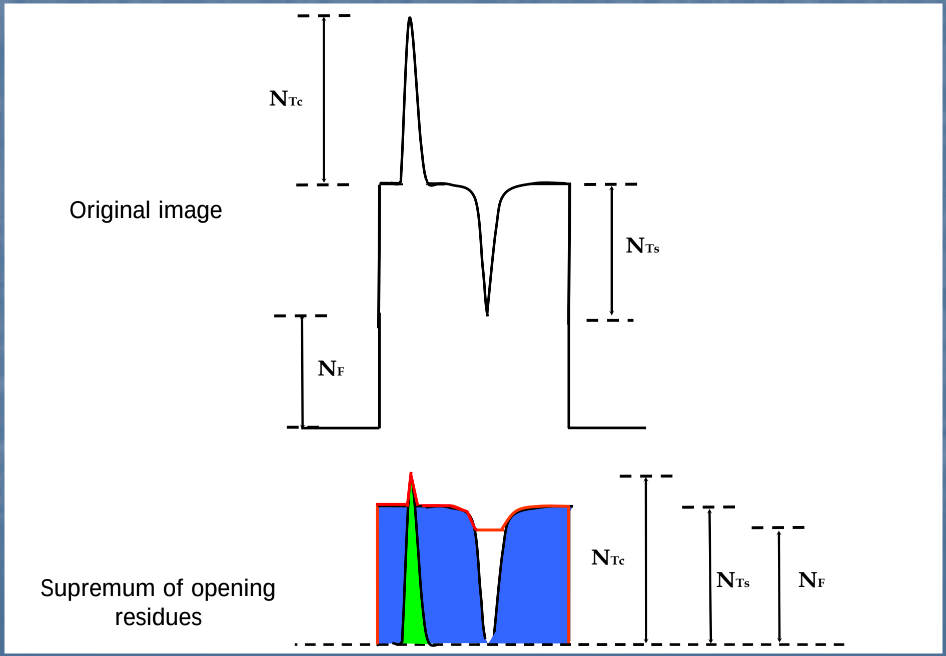
A Real Case







Various factors (fine particles, textured zones, crystalline materials, etc.) produce very noisy ultimate openings and granulometric functions



Building the Ultimate Opening

$$\upsilon_{k-1}(f) = \underset{i \in [1,k-1]}{\mathrm{Sup}} \left(\gamma_{i-1}(f) - \gamma_i(f)\right)$$, k-1 th step of the ultimate opening construction

$$\upsilon_k(f) = \underset{i \in [1,k]}{\mathrm{Sup}} \left(\gamma_{i-1}(f) - \gamma_i(f)\right) = \mathrm{Sup}\left[\left(\gamma_{k-1}(f) - \gamma_k(f)\right), \upsilon_{k-1}(f)\right]$$

At step k, the points x which are modified (for which $\upsilon_k\left(f(x)\right) \geq \upsilon_{k-1}\left(f(x)\right)$)

Correspond to the threshold $M_k = \left\{x : \varsigma_k(x) = k\right\}$ of the granulometric function

Modified values are equal to $\chi_k = \mathrm{Inf}\left(\gamma_{k-1} - \gamma_k, m_k\right)$

The ultimate opening can be written as $\upsilon = \underset{k \in N}{\mathrm{Sup}}\left(\chi_k\right)$

... and built iteratively by

$$\begin{cases} \upsilon_0 = 0 \\ \upsilon_k = \mathrm{Sup}\left[\mathrm{Inf}\left(\upsilon_{k-1}, 255 - m_k\right), \chi_k\right] \end{cases}$$

Similar procedure for the granulometric function

$$\begin{cases} \varsigma_0 = 0 \\ \varsigma_k = \mathrm{Sup}\left[\mathrm{Inf}\left(k, m_k\right), \varsigma_{k-1}\right] \end{cases}$$

Algorithm Enhancement

For each size k, from 1 to n+1, n being the maximal size of rocks:

- Definition of the mask M_k
- Modification of the mask $M'_k = \Psi(M_k)$
- Construction of a new granulometric function:

$$\varsigma'_k = \mathrm{Sup}\left[\mathrm{Inf}\left(k, m'_k\right), \varsigma'_{k-1}\right] = \underset{i \in [1,k]}{\mathrm{Sup}}\left[\mathrm{Inf}\left(i, m'_i\right)\right]$$

with $m'_k(x) = 255$, if $x \in M'_k$; $m'_k(x) = 0$, if not

- Construction of the new ultimate opening:

$$\upsilon'_k = \mathrm{Sup}\left[\mathrm{Inf}\left(\upsilon'_{k-1}, 255 - m'_k\right), \chi'_k\right]$$

with $\chi'_k = \mathrm{Inf}\left(\gamma_{k-1} - \gamma_k, m'_k\right)$

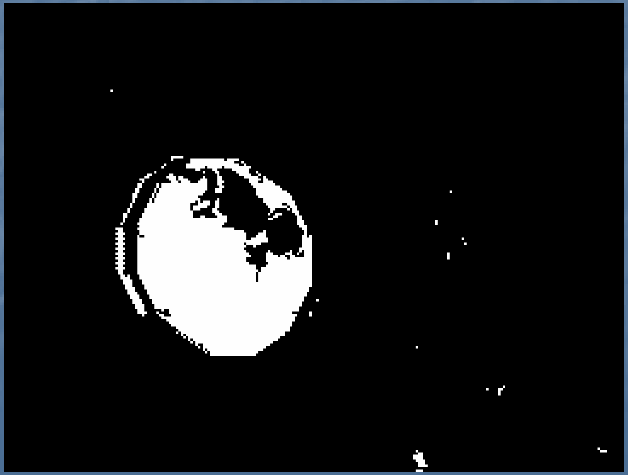
The Various Steps

3 modification steps are worked out:

$$\psi = \psi_3 \circ \psi_2 \circ \psi_1$$

The first step, ψ_1, is a simple hole filling procedure, denoted η

$$\psi_1(\mathrm{M}_k) = \eta(\mathrm{M}_k)$$

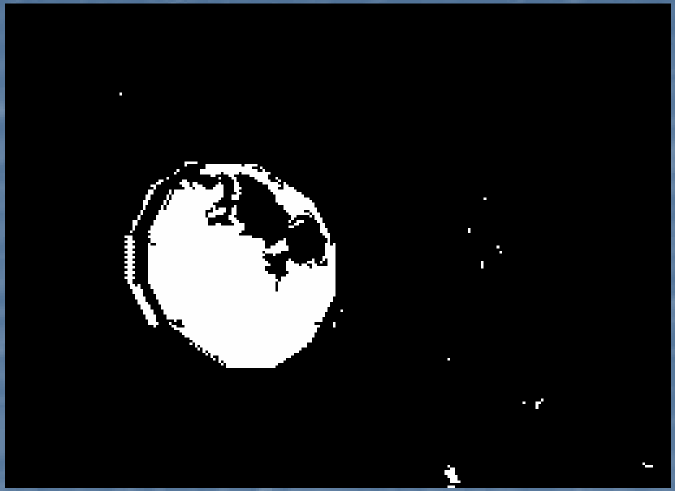


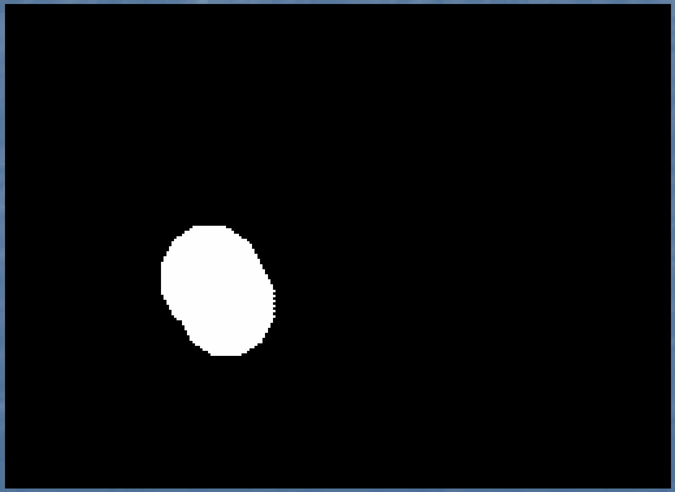
The Various Steps (2)

The second step consists in filtering connected components of the mask which are big enough to be fragments markers.

The filter is an opening of size k/a+b

$$\Psi_2 = \gamma_{(\frac{k}{a}+b)}$$

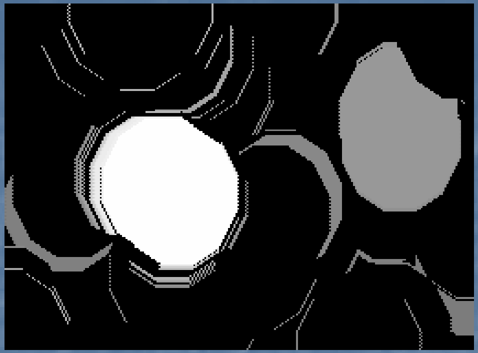




(a and b are the only parameters of the process. They depend only on the nature of the fragmented rocks)

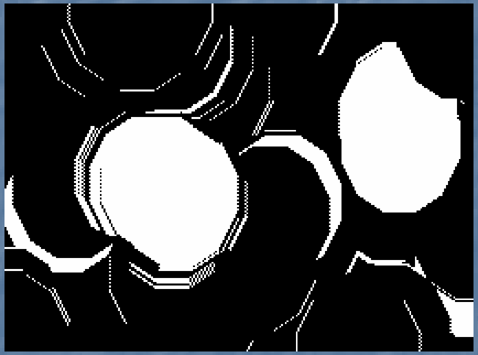
The Various Steps (3)

The third step is more complex. It uses the result of the two previous steps as a marker for a geodesic reconstruction of the size (k-1) maximal disks.



r_k

These disks are contained in the support R_k of the k order residue of the ultimate opening:

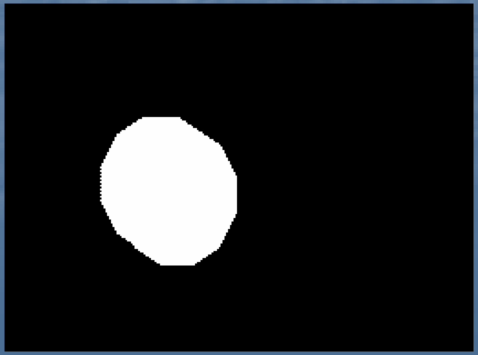


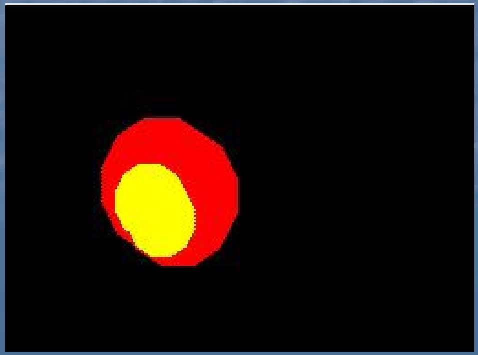
R_k

$$R_k = \left\{ x : \left(\gamma_{k-1}(f) - \gamma_k(f) \right) > 0 \right\}$$

They are preserved by an opening of size k-1

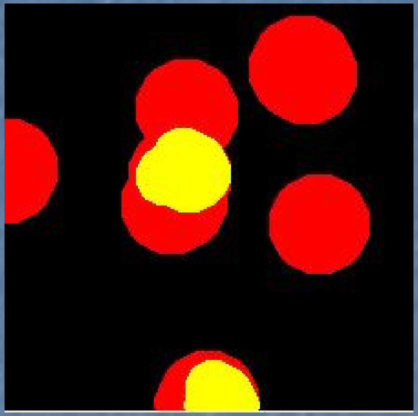
$\gamma_{k-1}(R_k)$

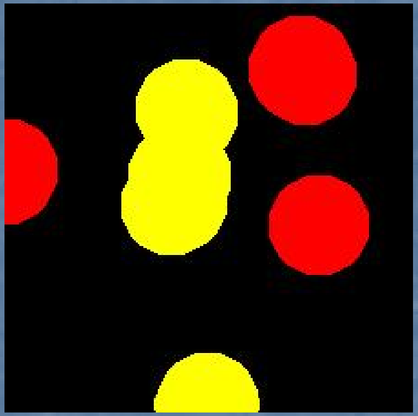




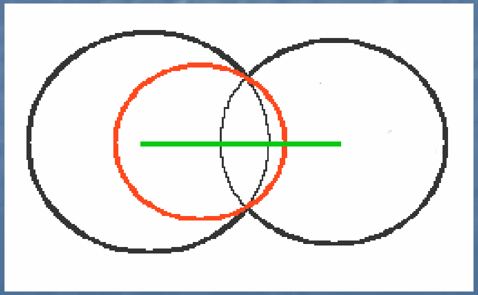
Step 3 (continued)

A simple geodesic reconstruction is not enough because, most of the time, too many maximal disks are rebuilt





This is why, instead of considering maximal balls, the reconstruction uses the critical disks of R_k

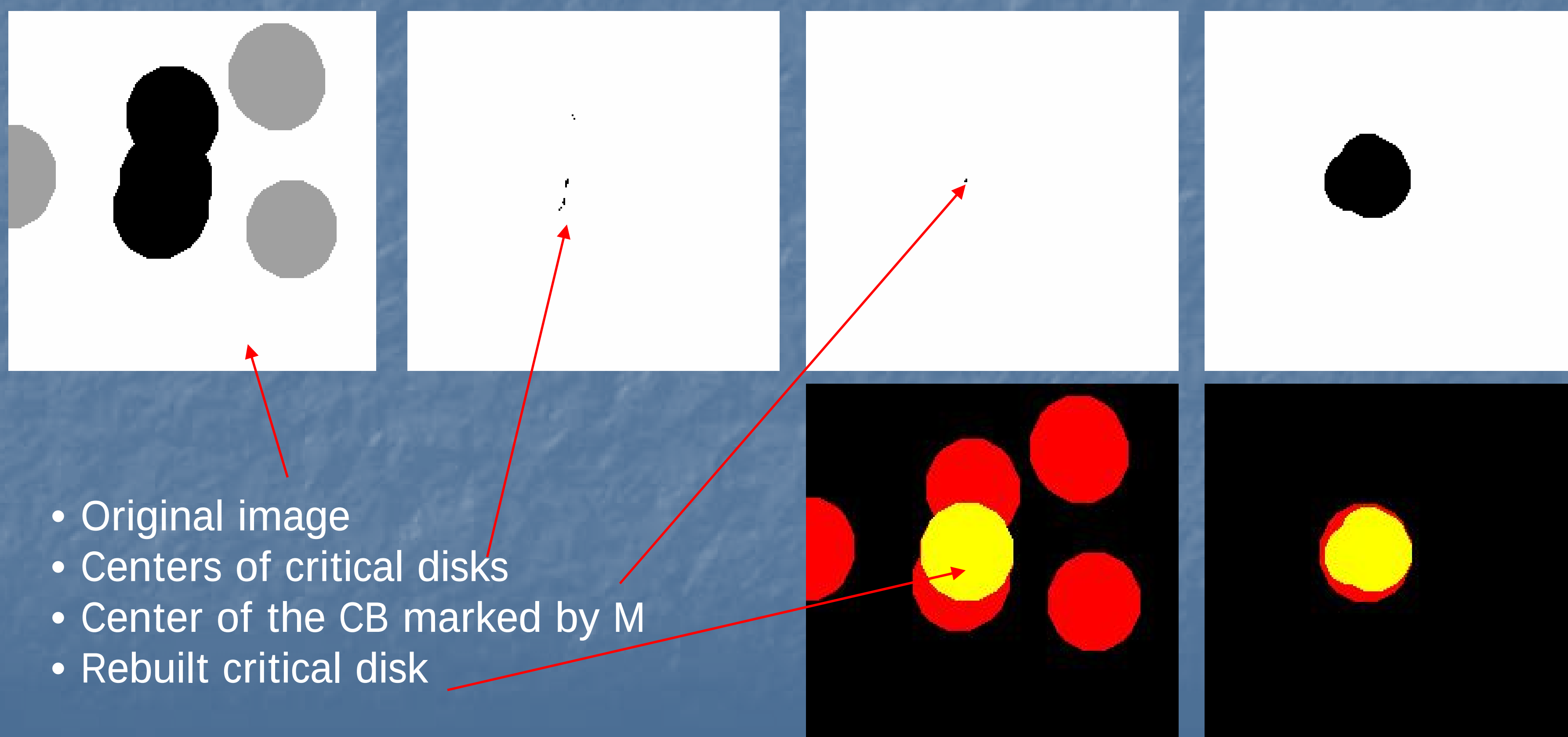


A maximal disk is critical when it is needful for the reconstruction (The red disk is maximal but not critical, only the two black ones are critical)

Step 3 (end)

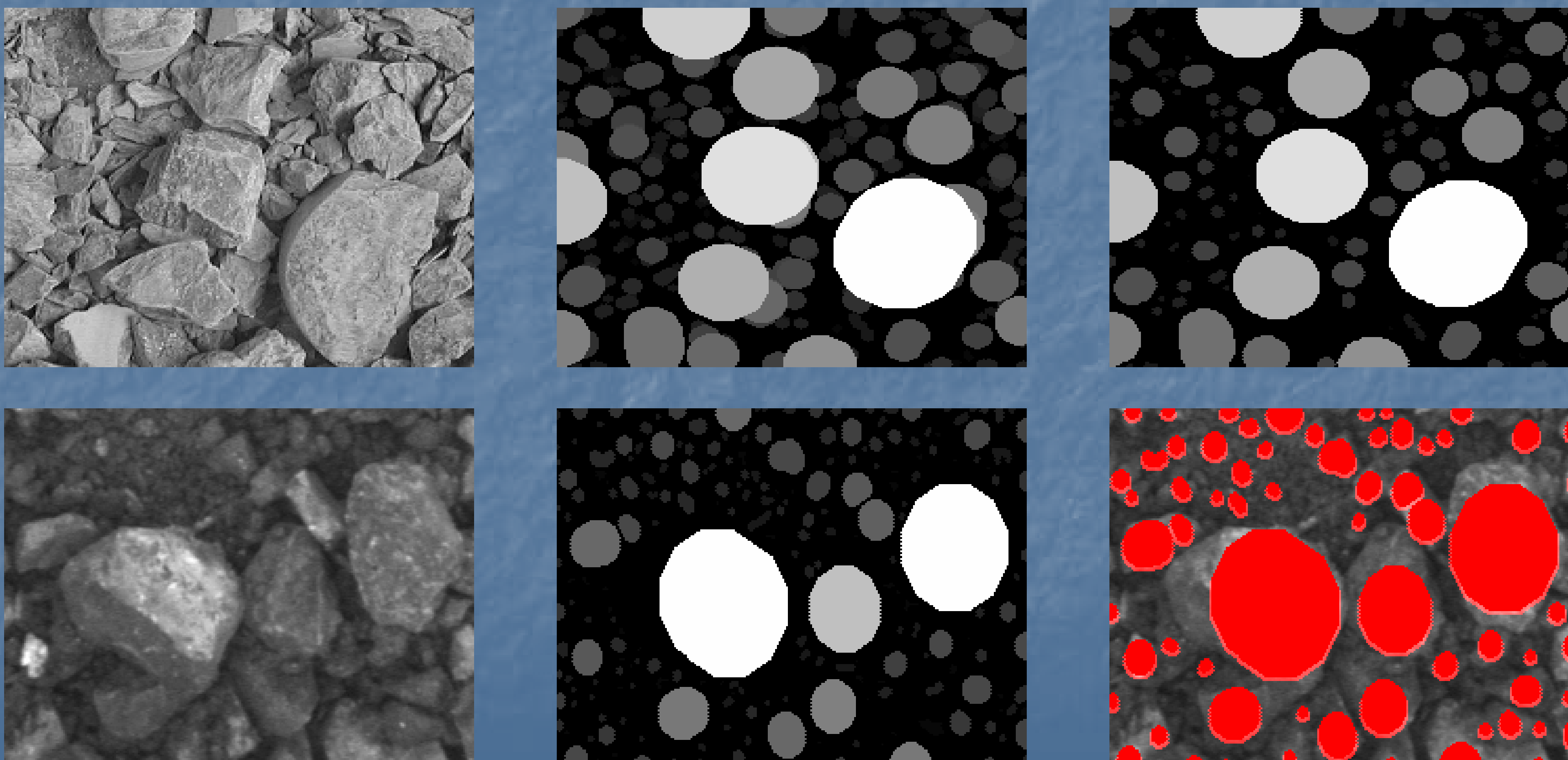
In the third step, the critical disk whose center is marked by the previous marker set M (result of the two preceding steps) is rebuilt.

$$\psi_3 = \mathrm{Rec\_cd}\left(\mathrm{M};\gamma_{k-1}\left(\mathrm{R}_k\right)\right)$$

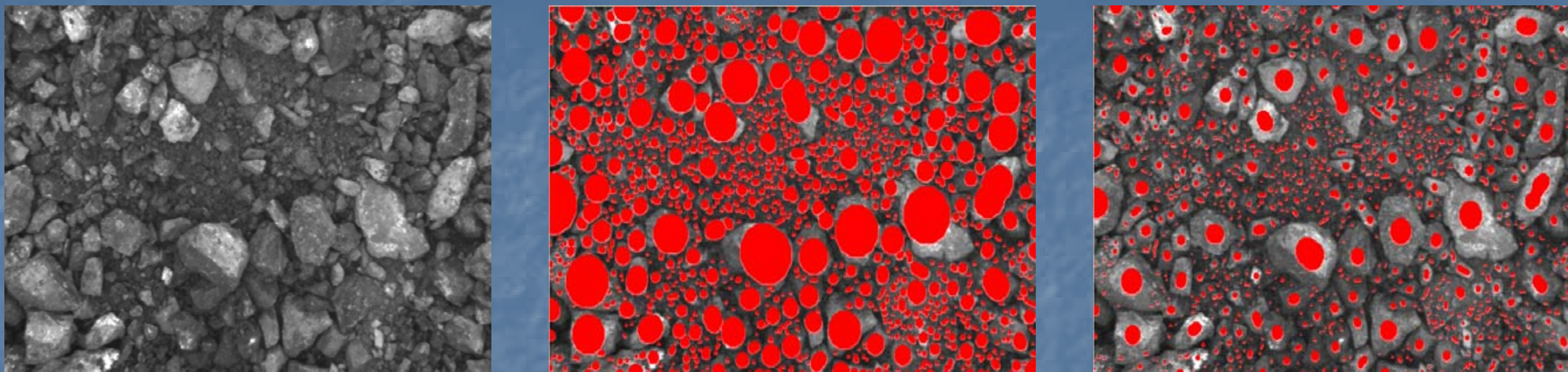


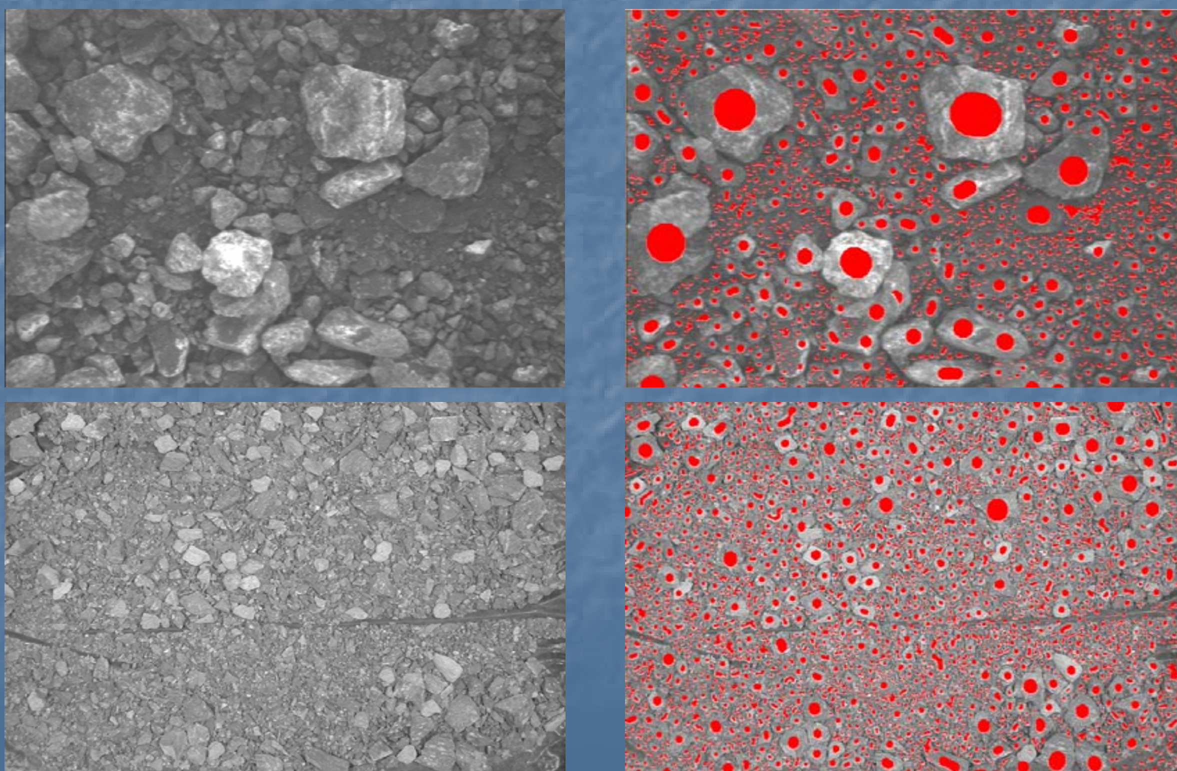
Construction of the Granulometric Function

$$M_k^{'} = \Psi\left(M_k\right) = \mathrm{Rec\_cd}\left[\gamma_{\left(\frac{k}{a}+b\right)}\left(\eta\left(M_k\right);\gamma_{k-1}\left(R_k\right)\right)\right]$$



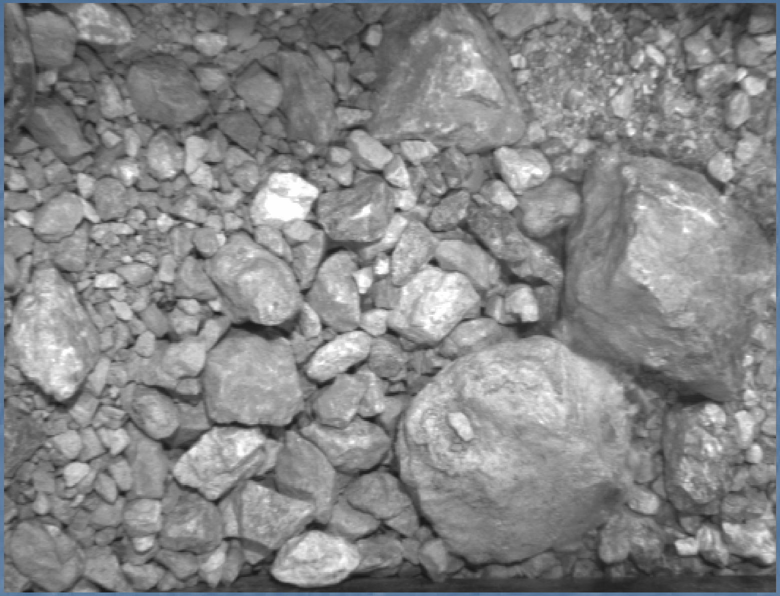
Some Results

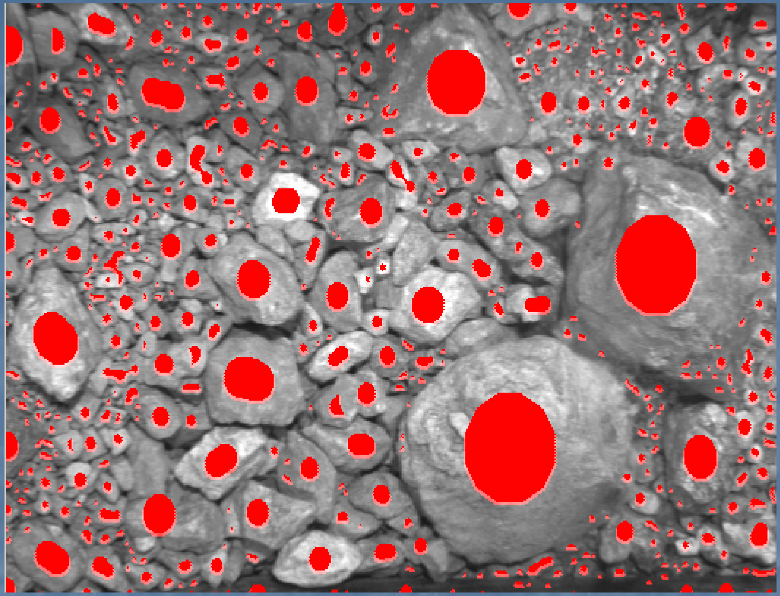


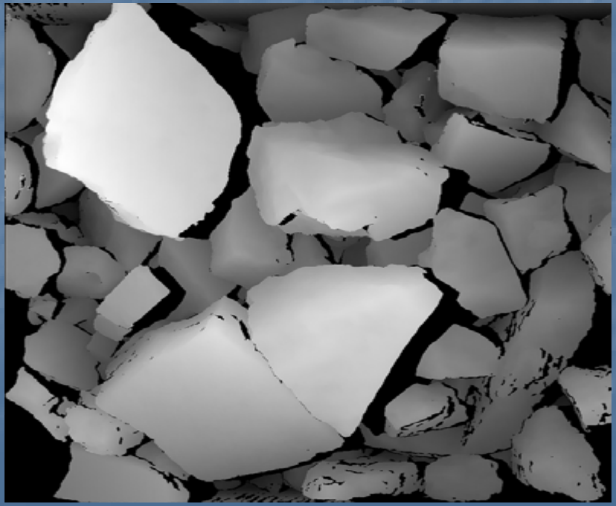


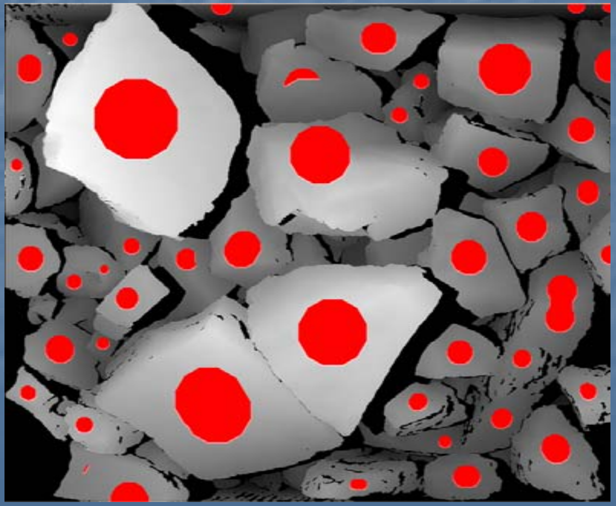
Heaps of rocks with a big amount of fine particles

Other Examples



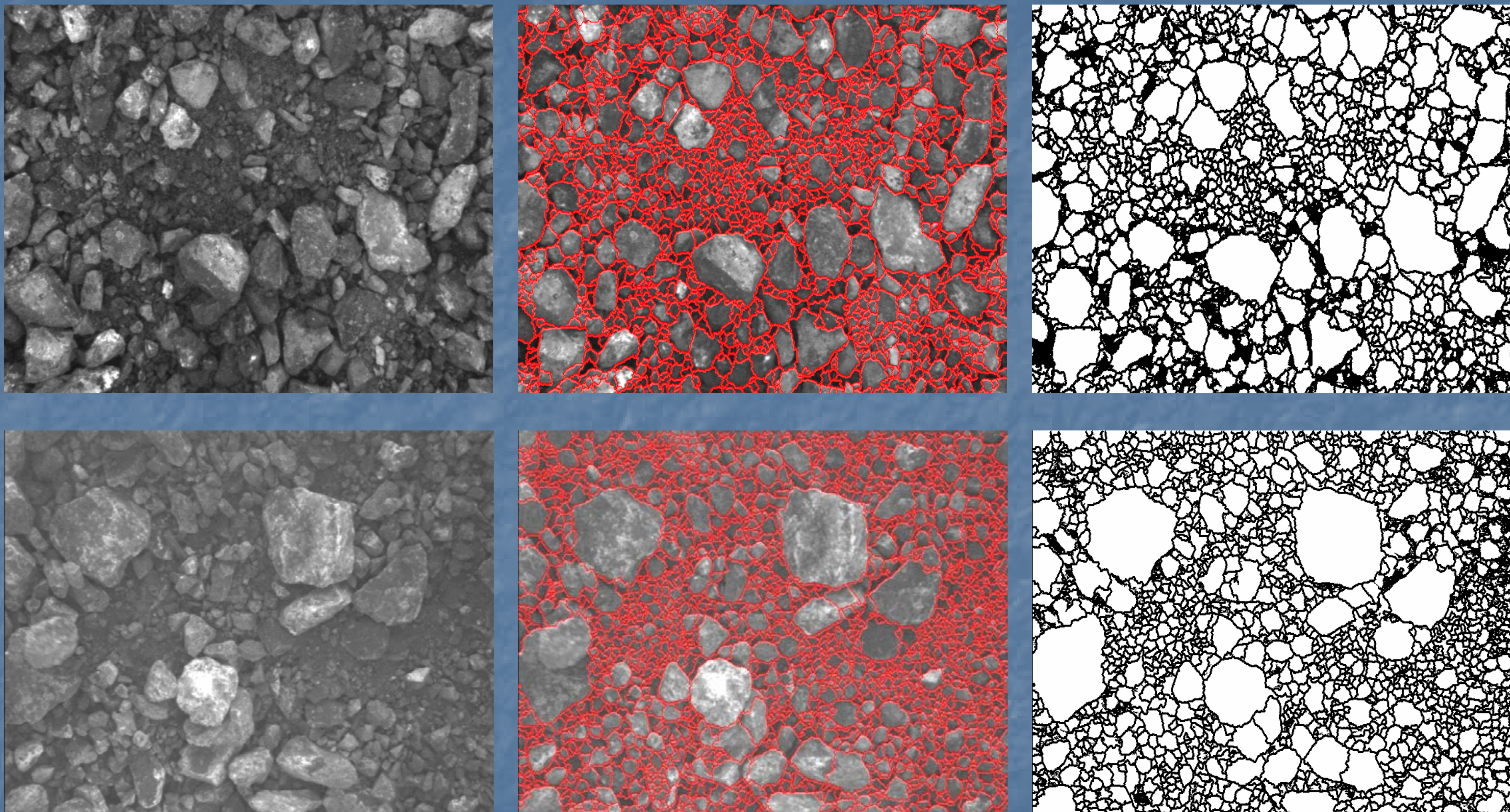




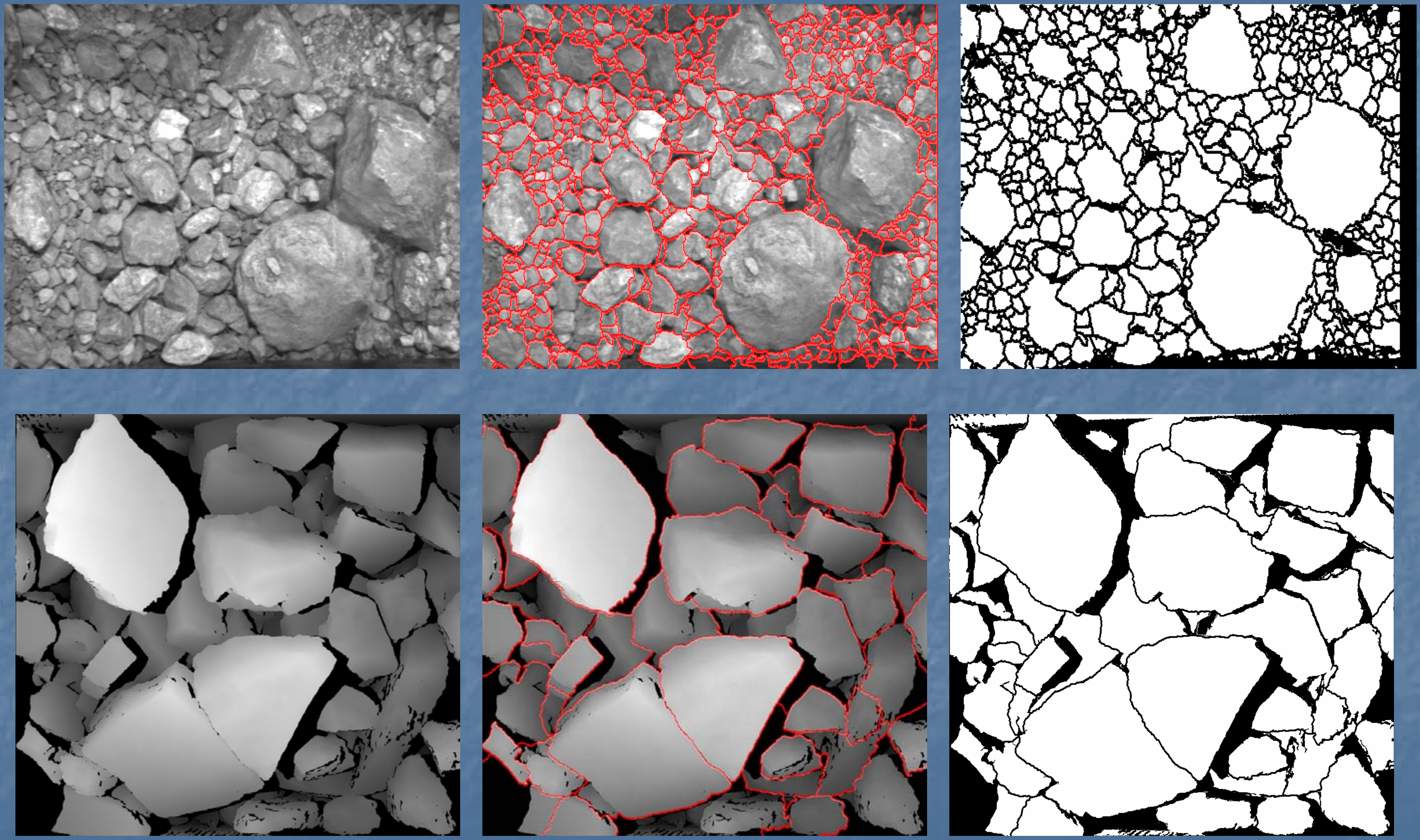


Laser triangulation image

Segmentation Results



Other Segmentations



Size Distributions