From Watersheds to Waterfalls and Beyond

A review of morphological tools for image segmentation

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- **Brief reminder of the watershed transform**

- **Importance of residual transforms in morphological segmentation**
  - **New operators**
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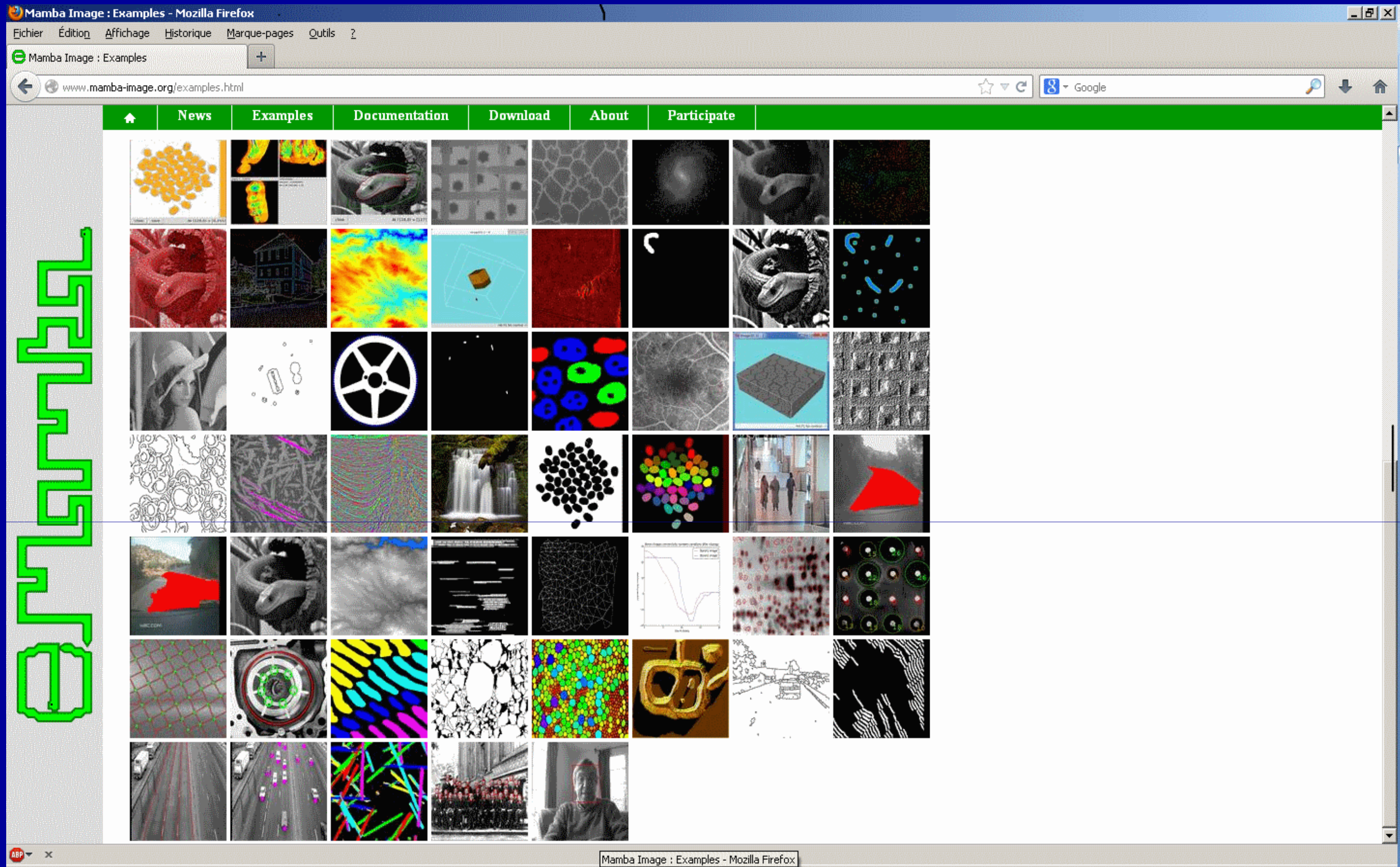
  - **Waterfalls transformation**
  - **Enhanced waterfalls**
  - **P algorithm**

How to Practice?

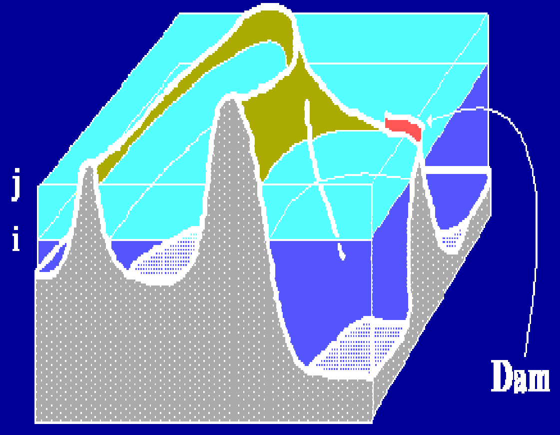


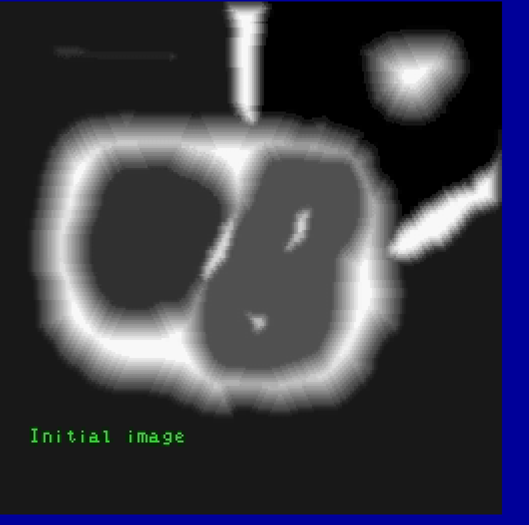
The tools presented here are available in the MAMBA library

http://www.mamba-image.org

Watershed Transformation, Quick Reminder

- It's a flooding process.
- Flooding sources are the minima of the function.
- The result is a partition of the image into catchment basins and watershed lines (dams).

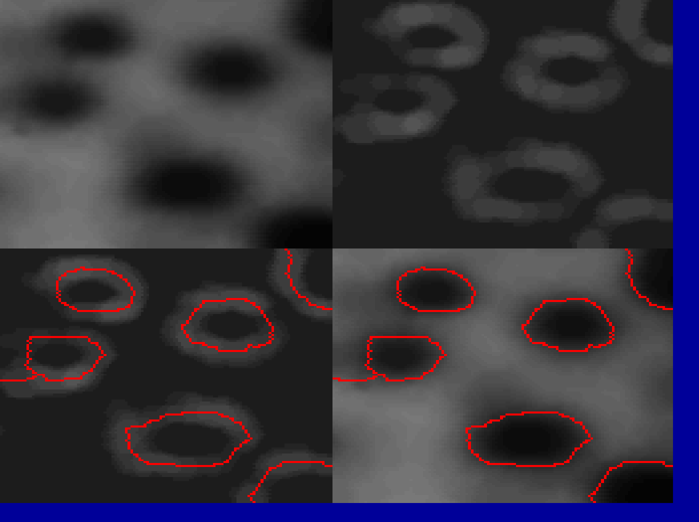




- Efficient implementations (real time) exist
- Properties and biases are known.

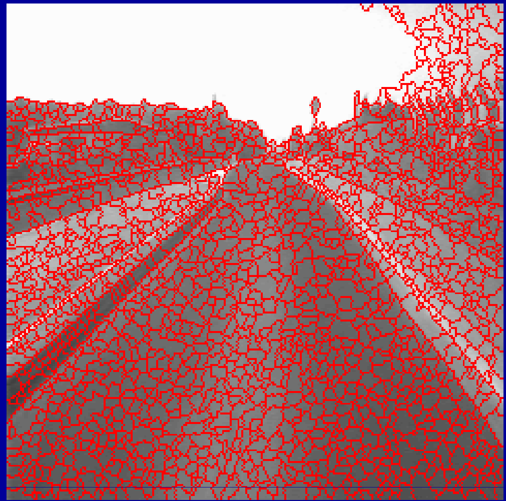
Use of the watershed transform for greyscale image segmentation (watershed of gradient)

Catchment basins correspond to homogeneous grey regions in the image.



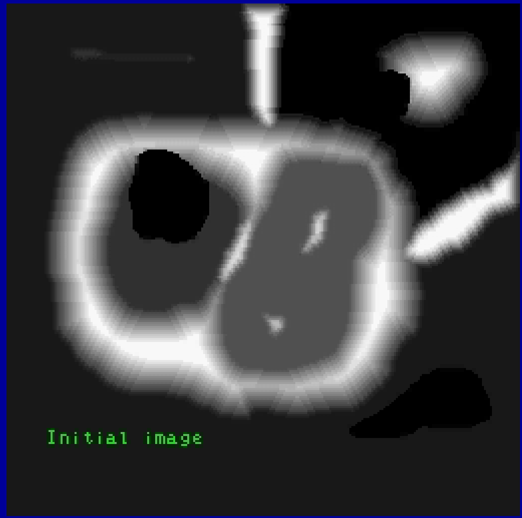
Coping with Over-Segmentation

The gradient watershed is over-segmented.



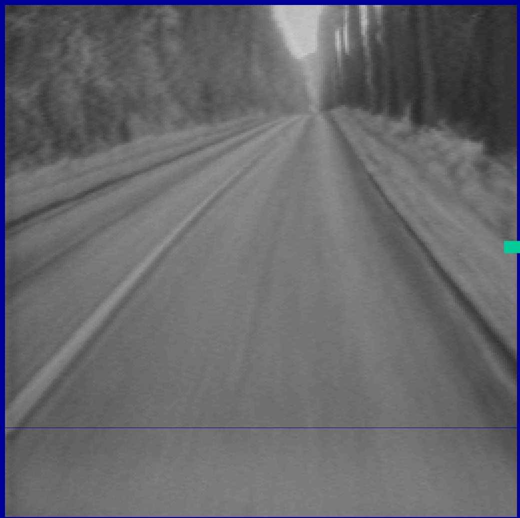
Gradient images are noisy and contain many minima. Each minimum generates a catchment basin in the WTS.

To avoid this over-segmentation due to numerous sources of flooding, one can select some of them (the markers) and perform *a marker-controlled watershed transform*.

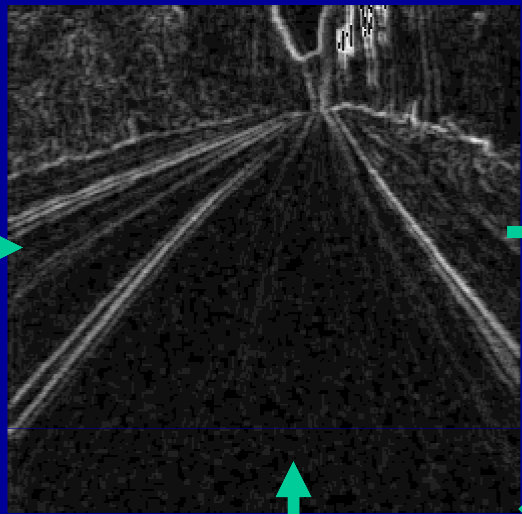


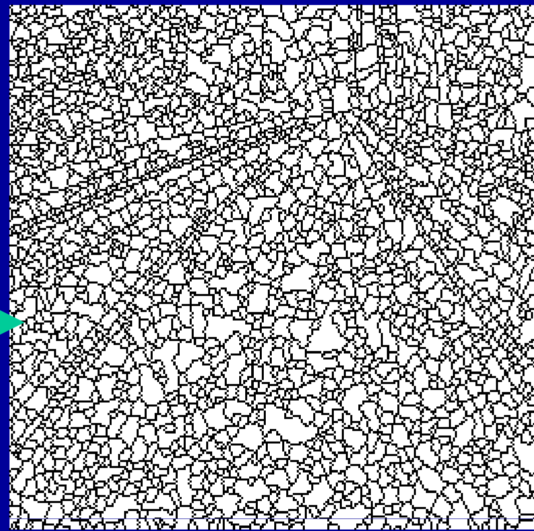
Advantages of the Marker-Controlled Watershed

original image



gradient





gradient watersheds

Markers corresponding to the two regions to be segmented (drawn by hand actually)

Supervised and Unsupervised Segmentation

- Using markers with the watershed transform is the « standard » approach,

→ Supervised segmentation

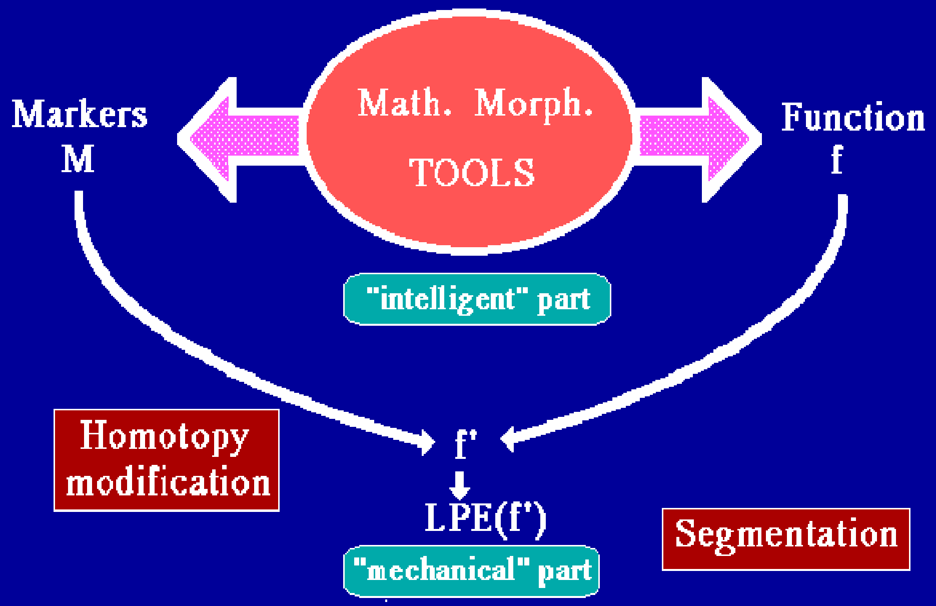
→ Class of operators providing these markers

- When markers are not available easily or when it is not possible to define relevant features to be segmented,

→ Unsupervised segmentation

→ Hierarchical segmentation

A Simple User's Guide



A morphological segmentation process is performed in two steps

- **The function f quantifies the criterion which is used by the segmentation**
- **The markers indicate the regions/objects to be extracted**

This scheme (segmentation paradigm) has the advantage to be generic and to be applicable to many segmentation problems (2D, 3D, greytone, color, multi-spectral, interactive segmentation, etc.).

It's, however, a simplistic user's guide…

Which Criteria? Which Markers?

- **Regarding greyscale (or color) images, contrast criteria are used and therefore functions quantifying differences between adjacent pixels and/or regions:**

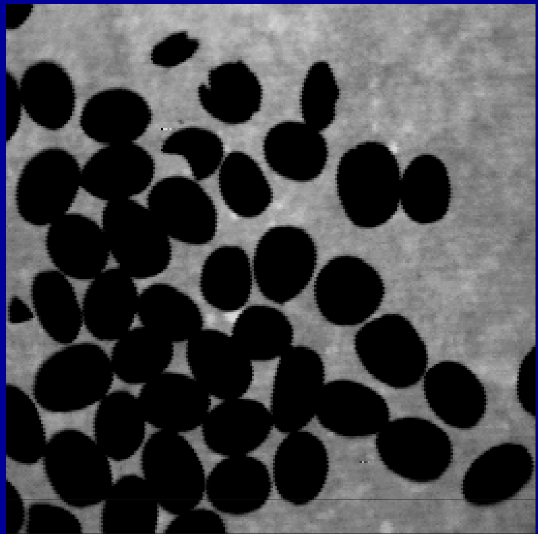
  Gradient
  Top-hat transform
  Various combinations

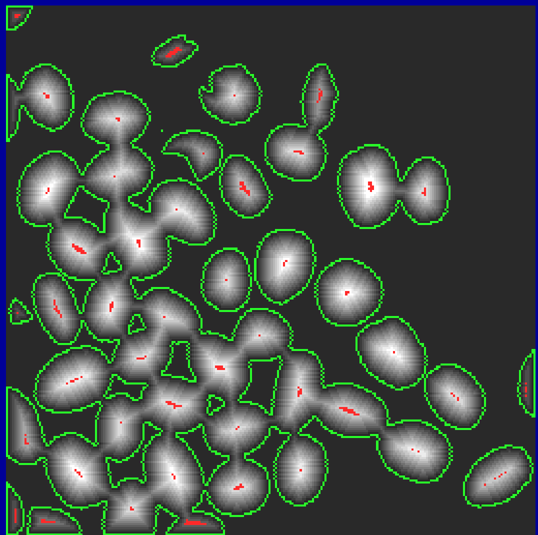
- **The watershed approach can also be used to segment sets according to their shapes and sizes. In this case, the *Distance function* is widely used.**

- **Markers are built by various means. They are often obtained from extrema (minima or maxima) of the criteria functions or by more sophifisticated approaches using a wide range of morphological tools (filters, geodesic operators).**

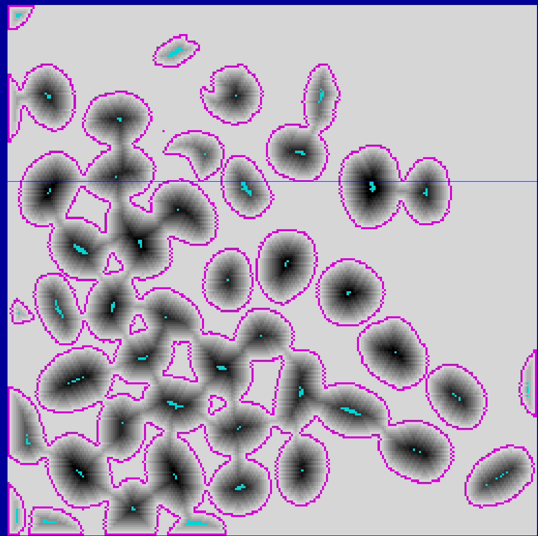
Set Segmentation, an Example

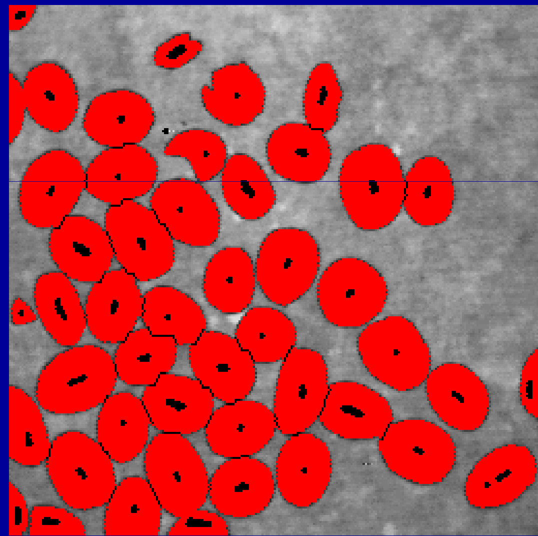
Coffee grains





The *distance function* of the set is computed. This distance function is inverted and its watershed is performed. The marker set is made of the maxima of the distance function.

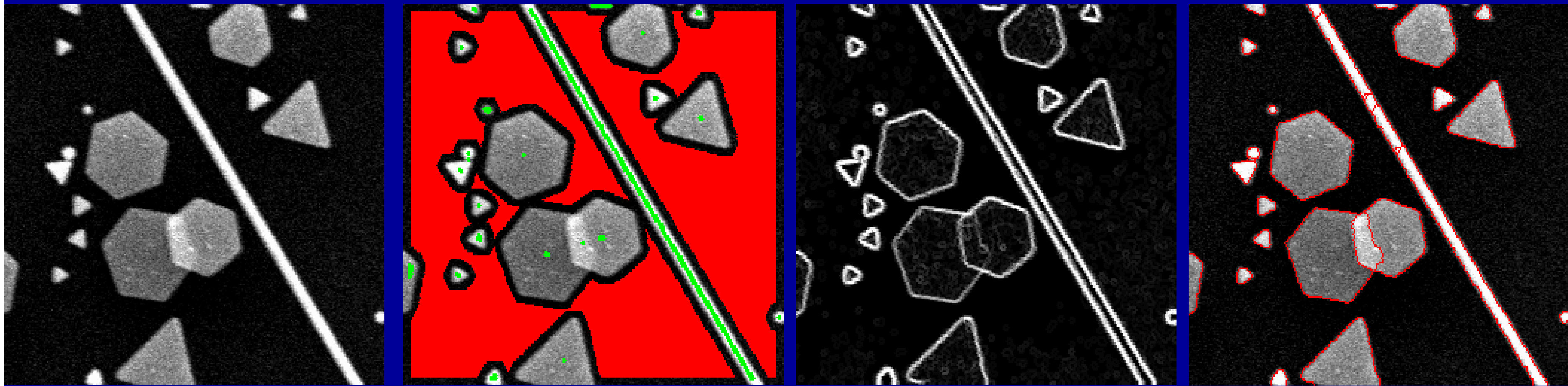




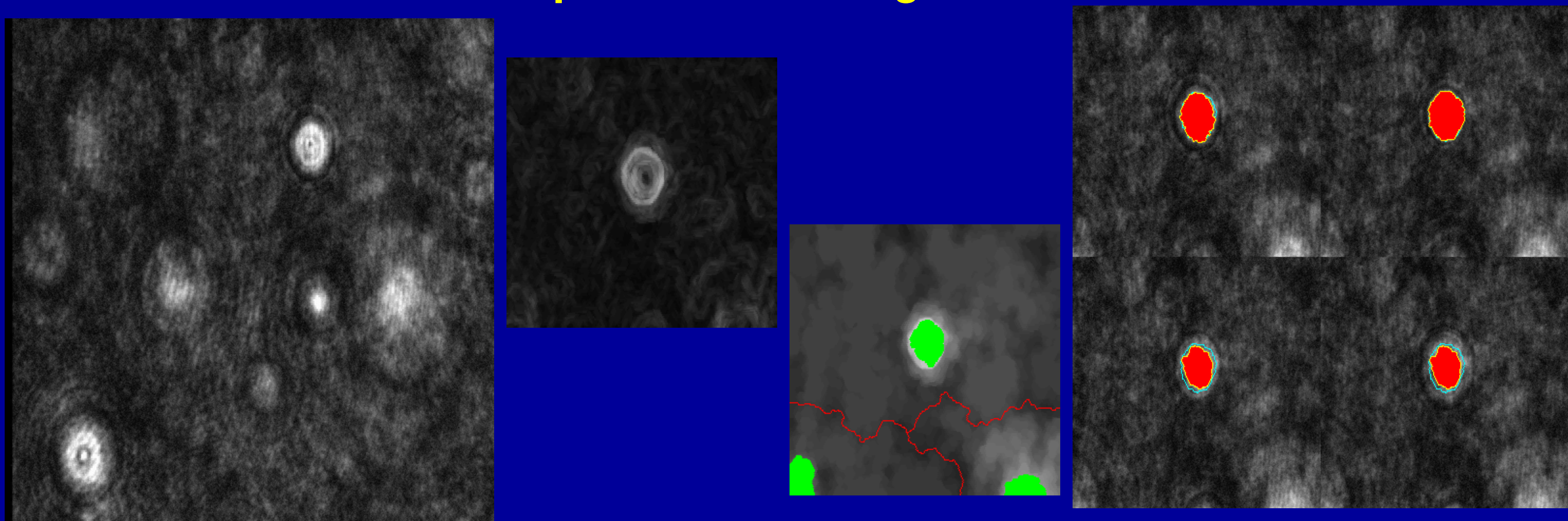
(Example available in MAMBA)

Some Examples of Applications…

Silver nitrate grains on a film

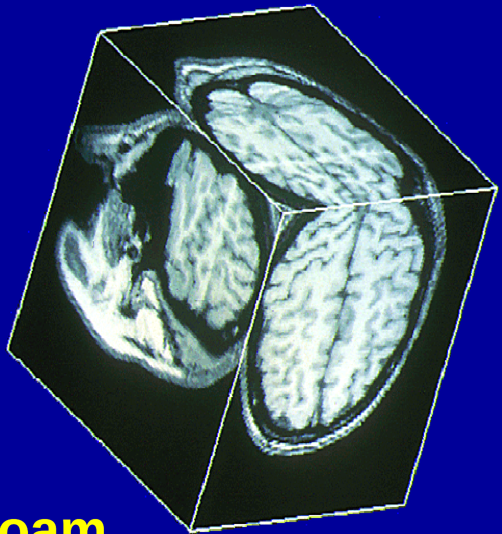


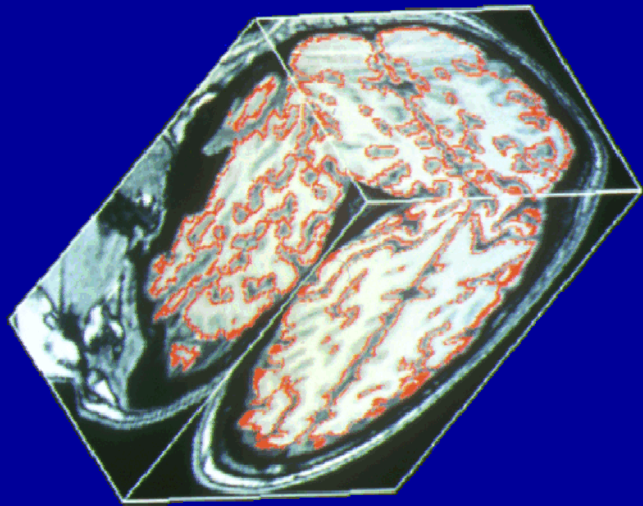
3D restitution of water drops from an hologram



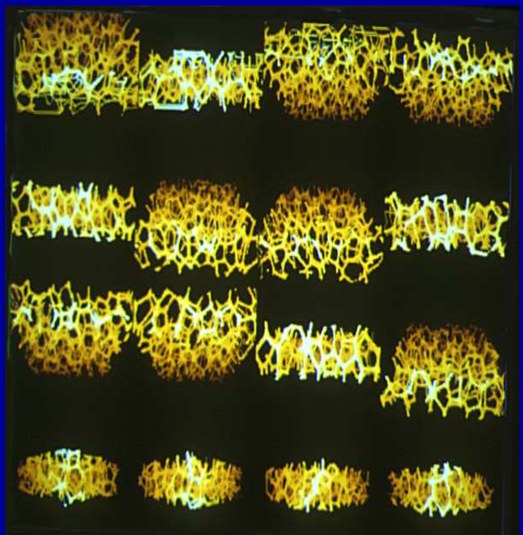
… With 3D Images

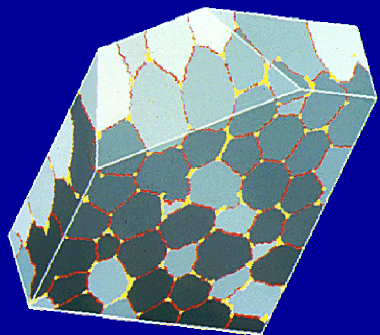
3D brain NMR image

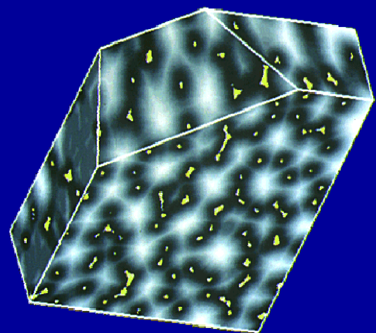


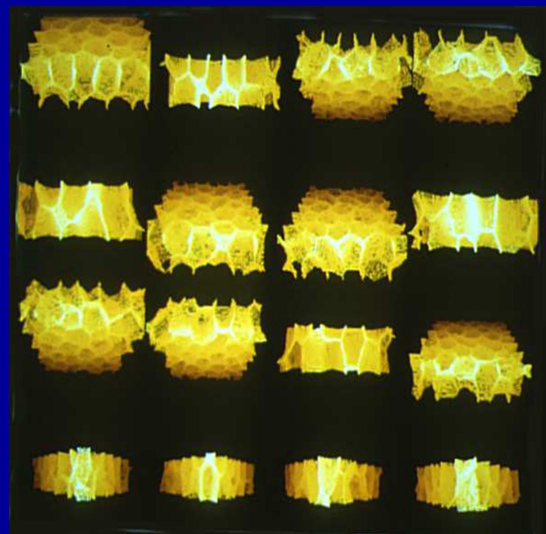


Polyester foam





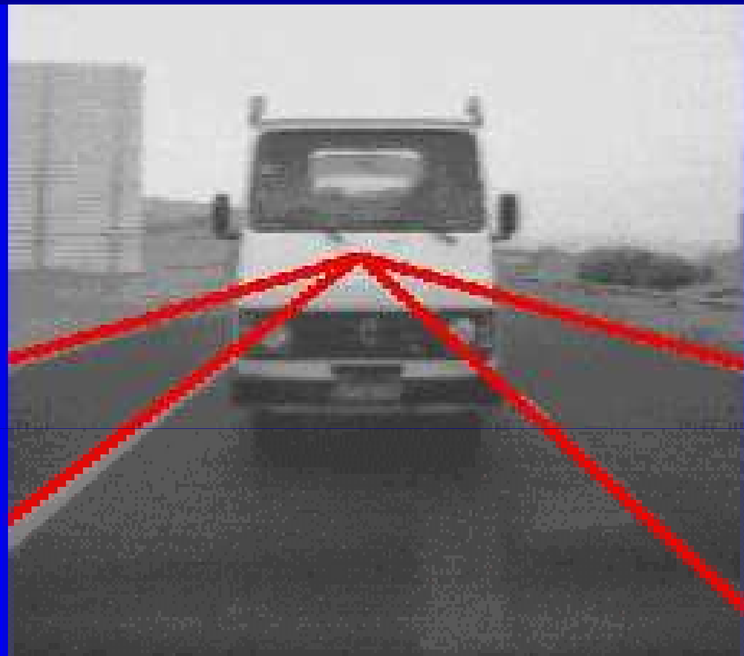




(Example available in MAMBA)

… And With Animated Images

The PROMETHEUS project: road segmentation and obstacle detection



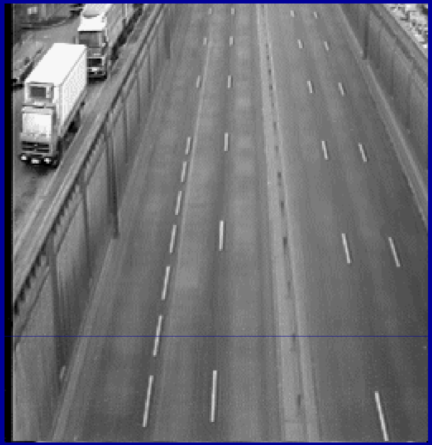
Lane detection based on a watershed segmentation applied on each image of the sequence

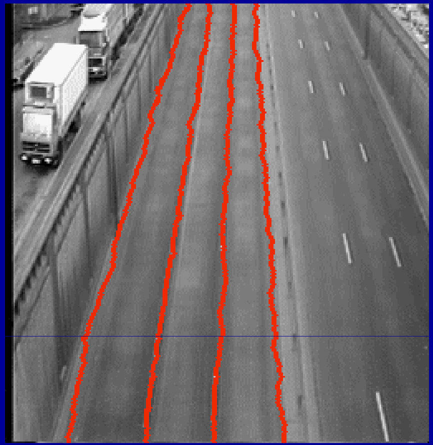
Lanes detection with re-use of the previous result as markers in the current image

(Example available in MAMBA)

Extending Shape Criteria to Greyscale Images, Is It Possible?

Until recently, it was difficult to apply shape and size criteria to greyscale image segmentation.





Example of traffic lanes segmentation: the lanes are not separated by a significant difference in grey levels. Therefore, using contrast criteria is irrelevant. Conversely, to use shape or size criteria, we need to work on the set corresponding to the road, which must be obtained by a… segmentation. ***(Example available in MAMBA)***

New operators allow to bridge the gap between these two kinds of criteria.

A Deeper Insight into the Tools

- **Limited toolbox**
- **Which class of tools?**
- **Is it possible to expand it?**

All the operators used with the watershed transformation are residues.
A residual operator is the difference of two operators called primitive functions:

- **Morphological gradient** $\longrightarrow \delta - \varepsilon$ **(dilation, erosion)**
- **Top-Hat transform** $\longrightarrow I - \gamma$ **(identity, opening)**
- **Distance function/ultimate erosion** $\longrightarrow \varepsilon_i \setminus \gamma(\varepsilon_i)$ **(erosion, opening of erosion)**
- **Maxima, minima of a function f** $\longrightarrow f - R_{f-1}(f)$ **(R, geodesic reconstruction)**

New residues can be defined and used

Numerical Residues, Short Introduction

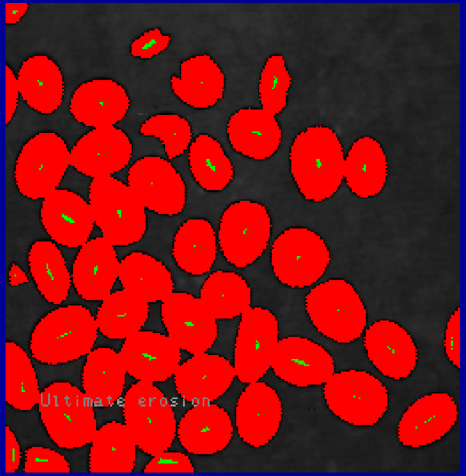
An elementary residual operator is defined by the difference of two operators.

<u>General definition</u>

Starting from two sequences of transformations ψ_i **and** ζ_i **with** $\psi_i \geq \zeta_i$ **, we define a doublet of operators:**

- **The residual transformation** $\theta = \underset{i \in I}{\mathrm{Sup}}\left(\psi_i - \zeta_i\right)$
- **Its associated function** $q = \arg\max\left(\psi_i - \zeta_i\right) + 1$

A residual transform is made of a couple of operators: the first one provides locally (at every point of the image) the maximum value of the residue, the second one indicates the value of the index i (it often corresponds to a size) which produces this maximum.
Both operators are important!



Ultimate erosion

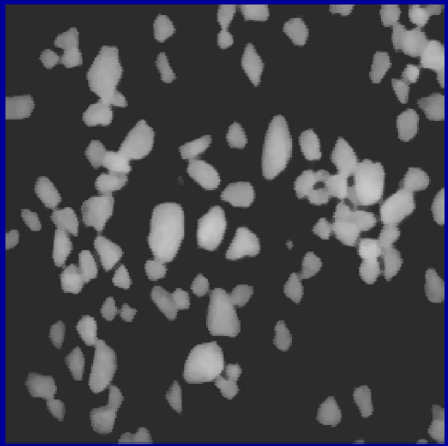
New Residues

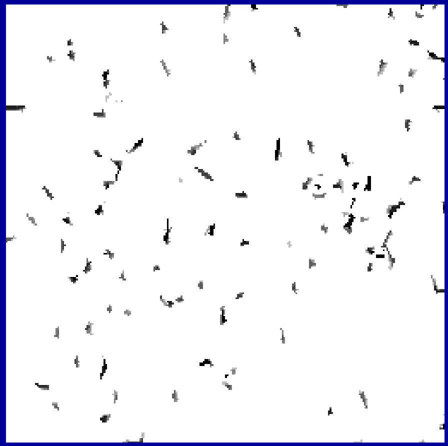
Thanks to this general definition of a residual transform, it is possible to extend to functions residues defined for sets:

$$\psi_i = \varepsilon_i$$

$$\zeta_i = \gamma_{rec}(\varepsilon_i)$$

Ultimate Erosion





It is also possible to define new transformations:

$$\left.\begin{aligned} \psi_i &= \gamma_i \\ \zeta_i &= \gamma_{i+1} \end{aligned}\right\} \rightarrow$$

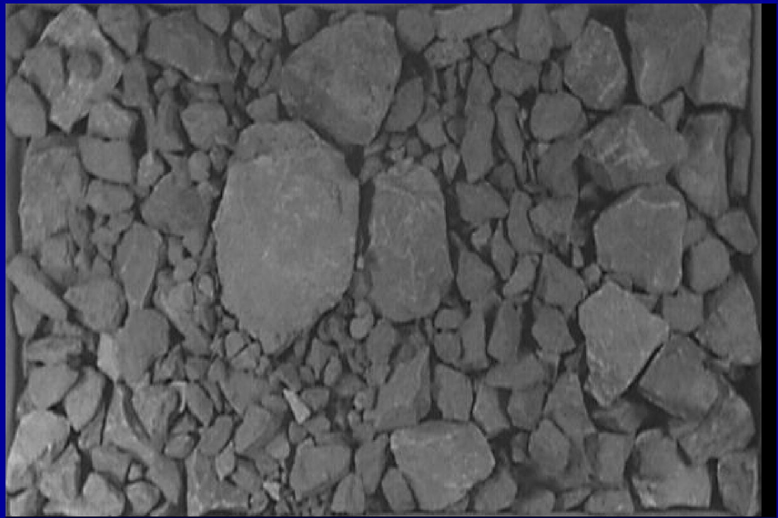
θ is named ***Ultimate Opening***

q is the ***Granulometric function***

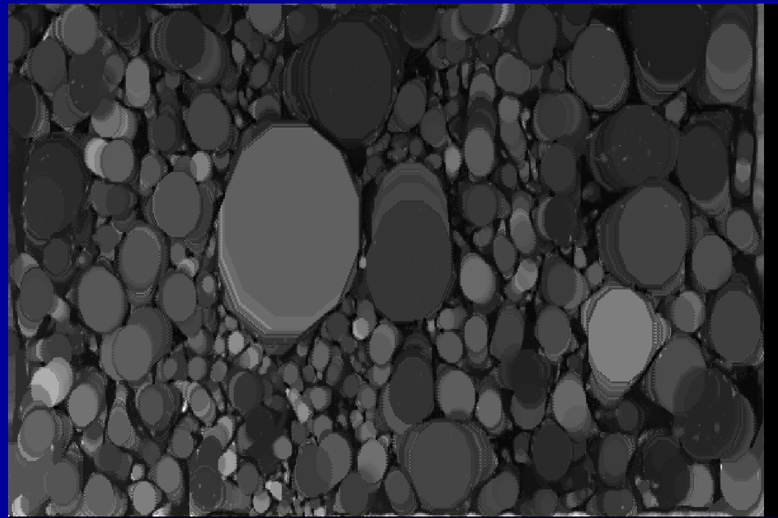
$$\left.\begin{aligned} \psi_i &= \varepsilon_i \\ \zeta_i &= \varepsilon_{i+1} \end{aligned}\right\} \rightarrow$$

q is called ***Quasi- Distance***.

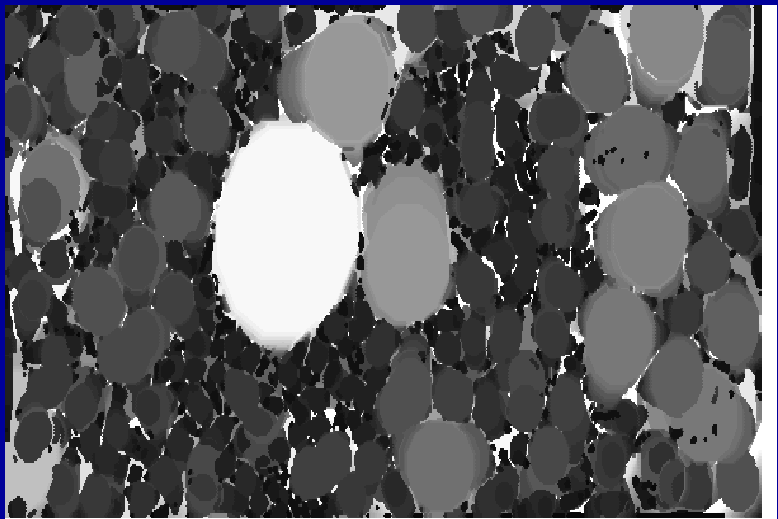
Ultimate Opening
Granulometic Function



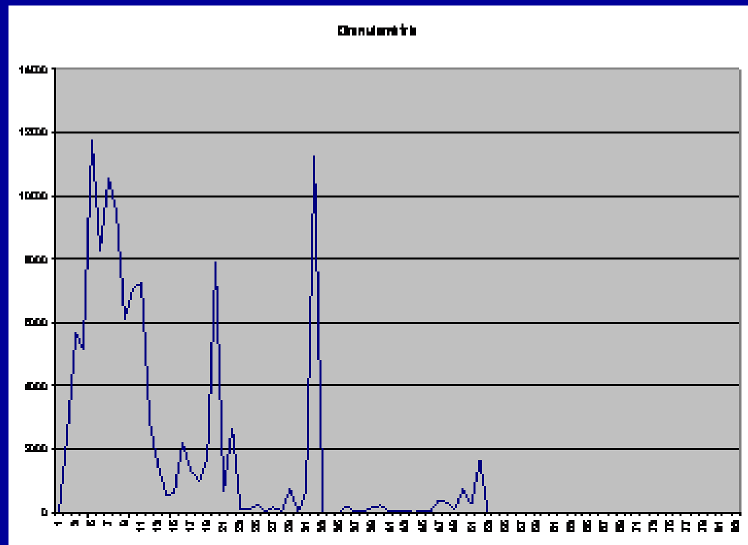
Heap of rocks



Ultimate Opening



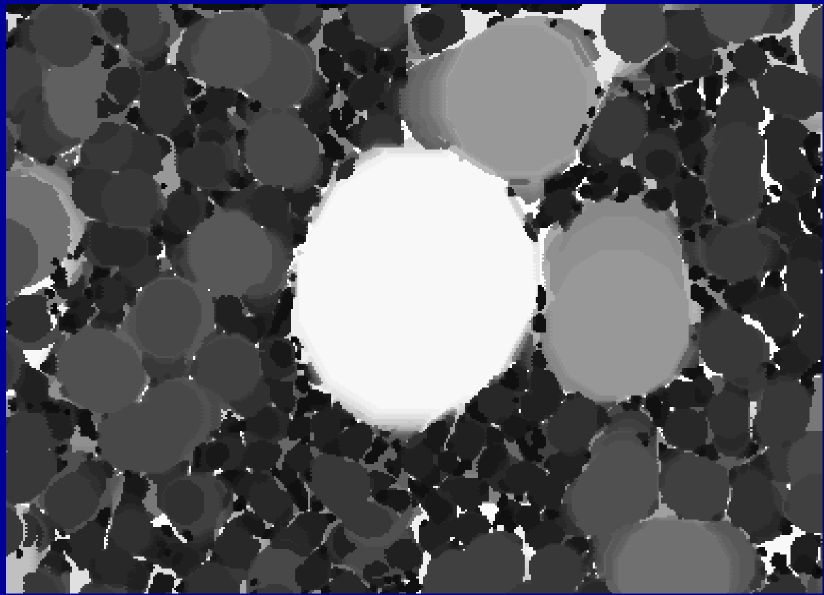
Granulometric function



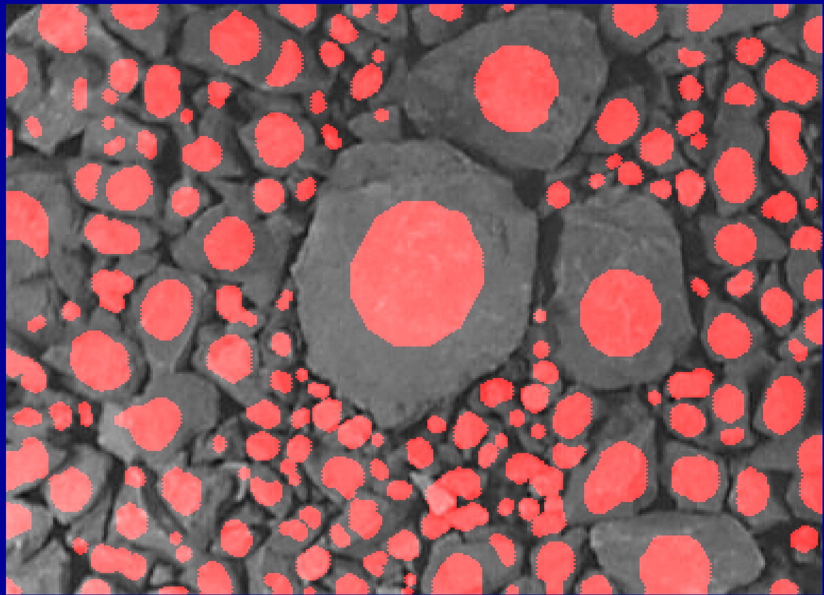
Markers Generation

The granulometric function can be used to produce efficient markers for the watershed segmentation.

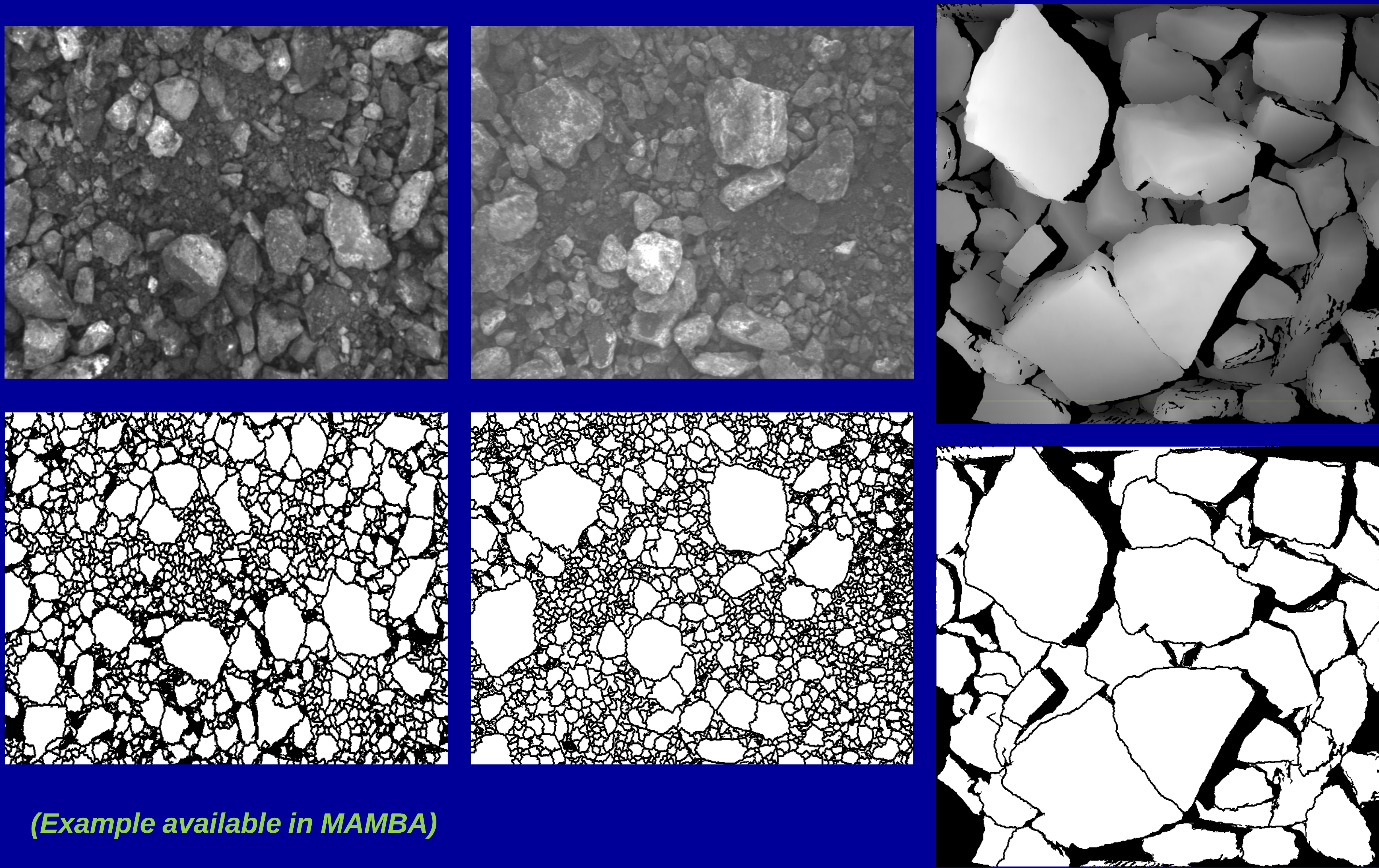
Each threshold λ of the granulometric function q is eroded by a disk of size $k\lambda$ ($k<1$)



This operation produces markers of blocks whose size is proportional to the size of the block. As a result, markers are better centered, even small particles are well marked and the watershed segmentation is of better quality.



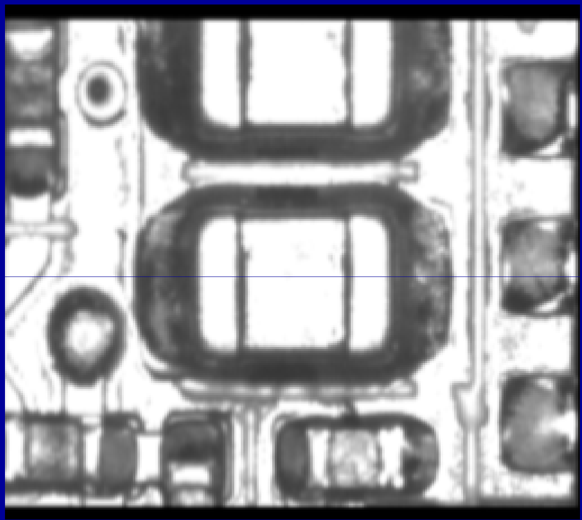
Some Results of Segmentation



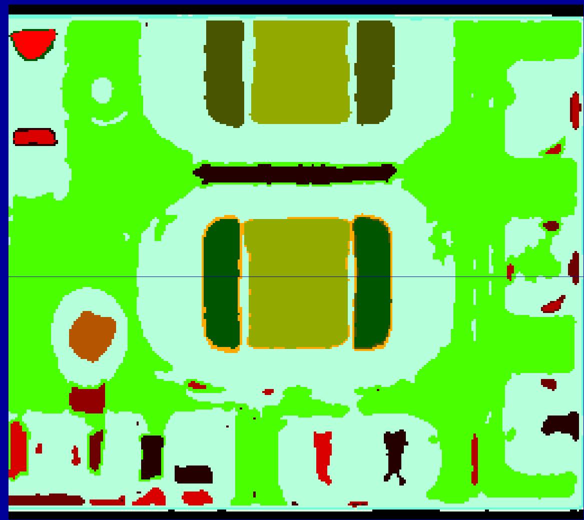
(Example available in MAMBA)

Ultimate Opening Can Be Defined From Any Opening Operator

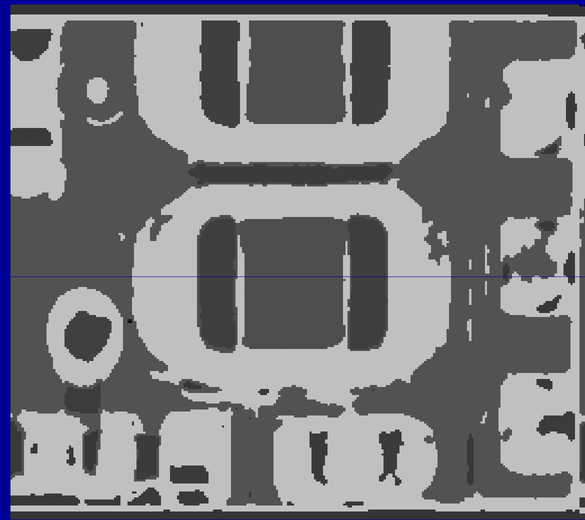
For instance, with opening by geodesic reconstruction…
This operator emphasizes the size criterion.



Initial image



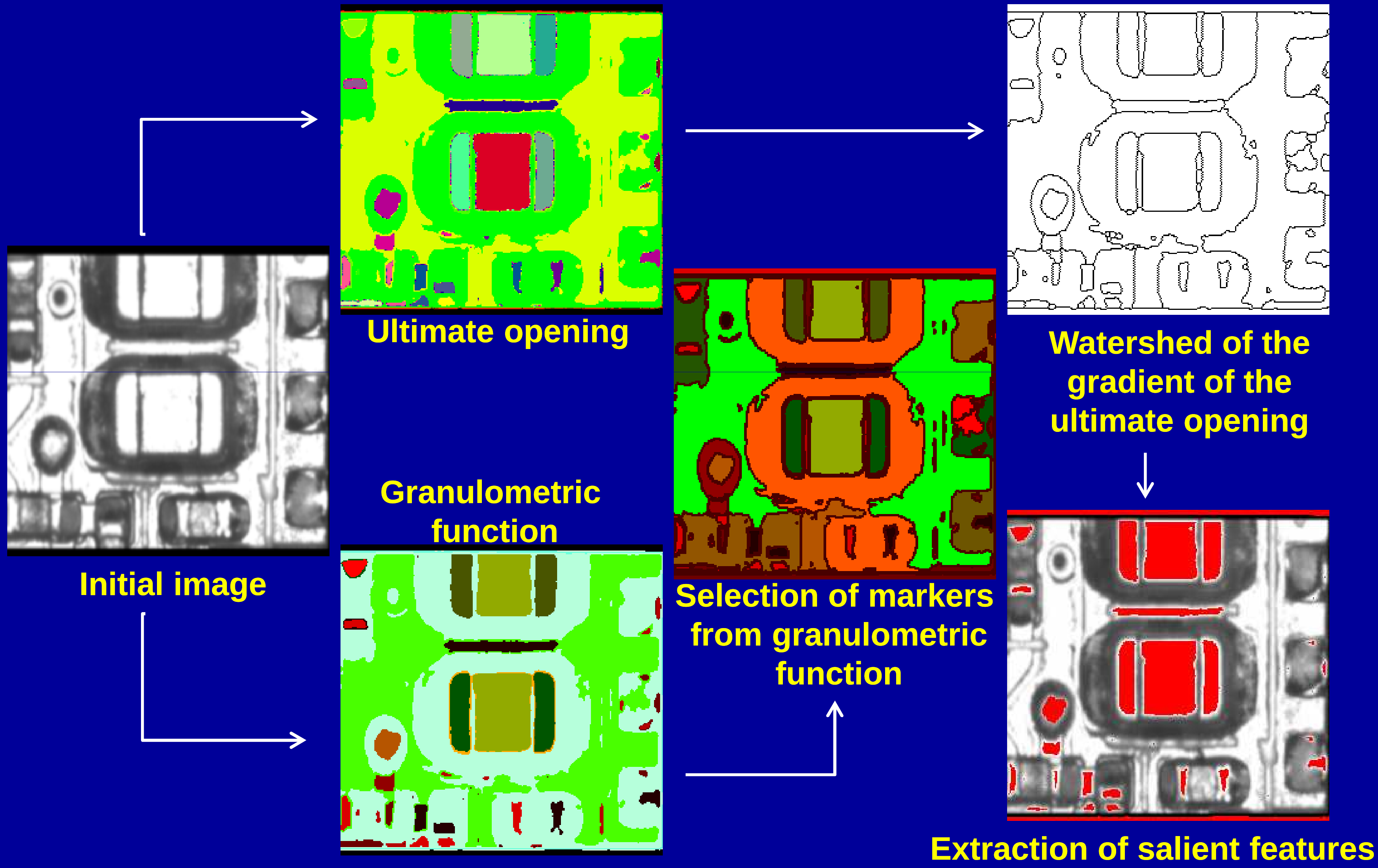
Ultimate Opening by reconstruction



Granulometric function

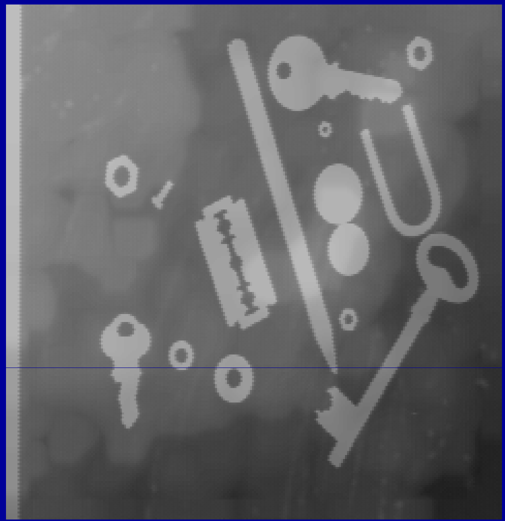
Residual operators provide efficient non parametric filters. The Ultimate Opening is a remarkable tool for marking and extracting salient features from an image.

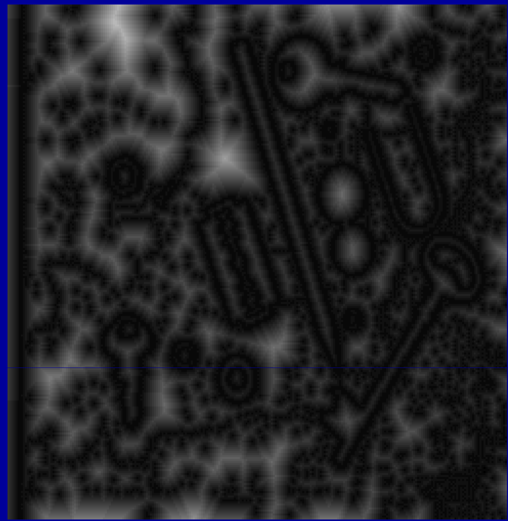
Segmentation with the Ultimate Opening by Reconstruction



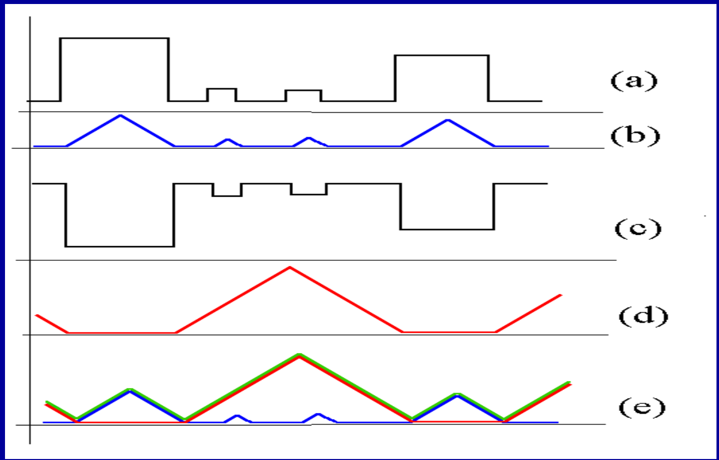
Quasi-Distance and Segmentation

A quasi-distance computed on a greyscale image provides the sizes of the flat (homogeneous) regions → Markers for a segmentation based on size and geometry (convexity).

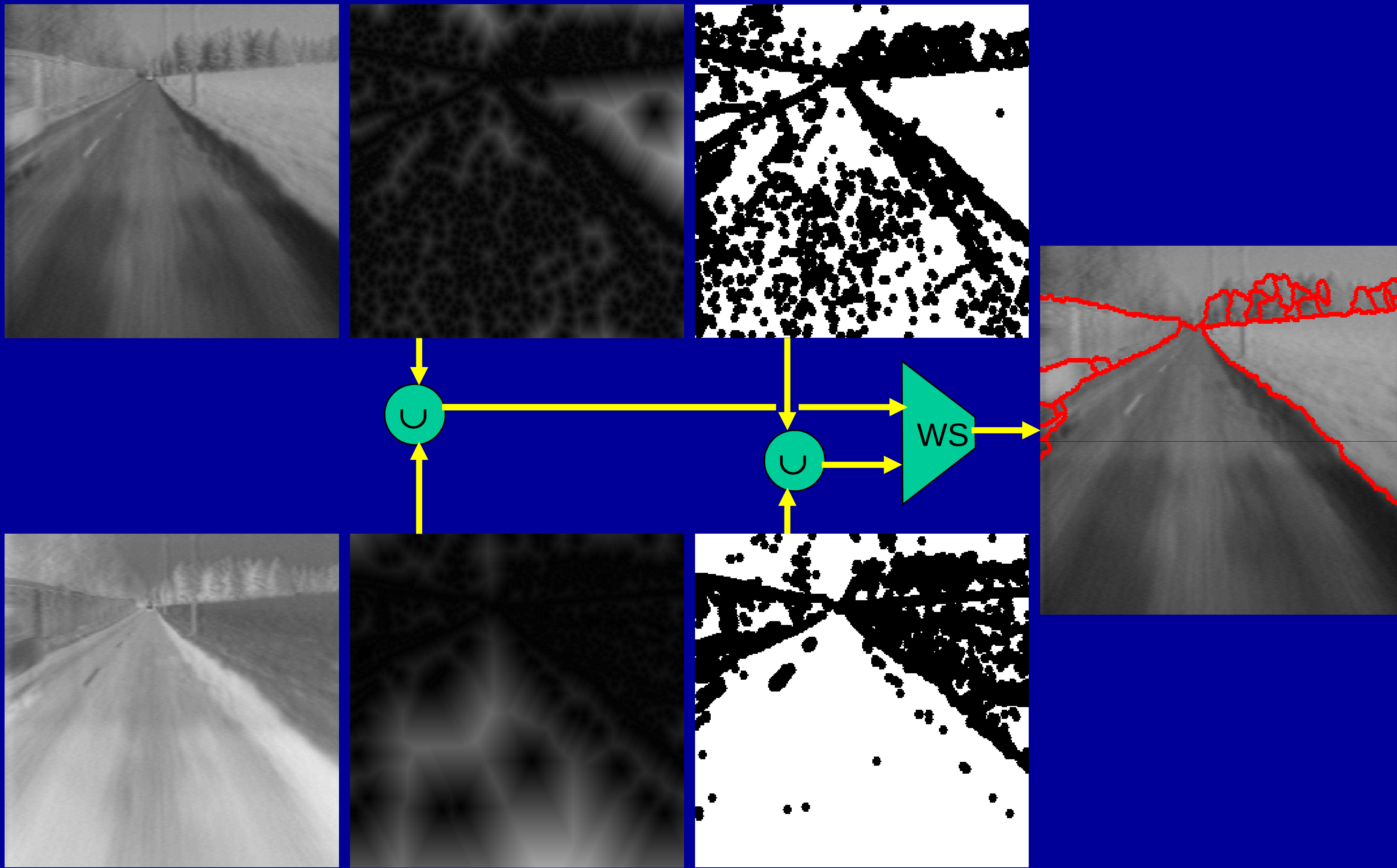




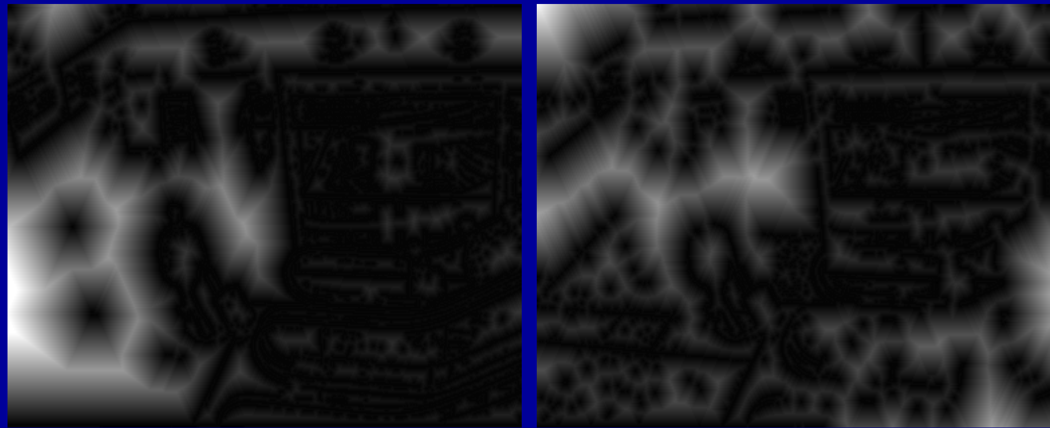
- **Quasi-distances performed both on the image and the complementary one → d, d'**
- **Sup of the results → h=sup(d,d')**
- **Markers extraction (maxima or threshold)**
- **Watershed of h**



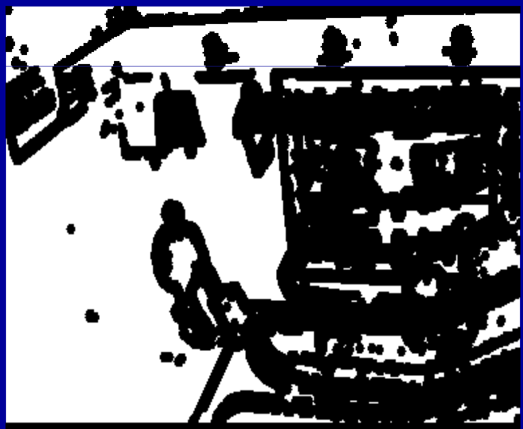
Segmentation With Quasi-Distances



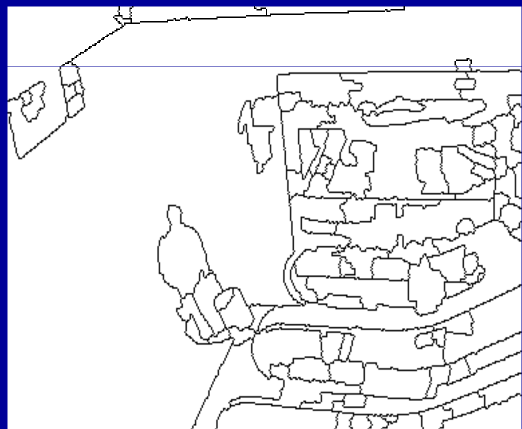
Another Example



Quasi-Distances



Markers

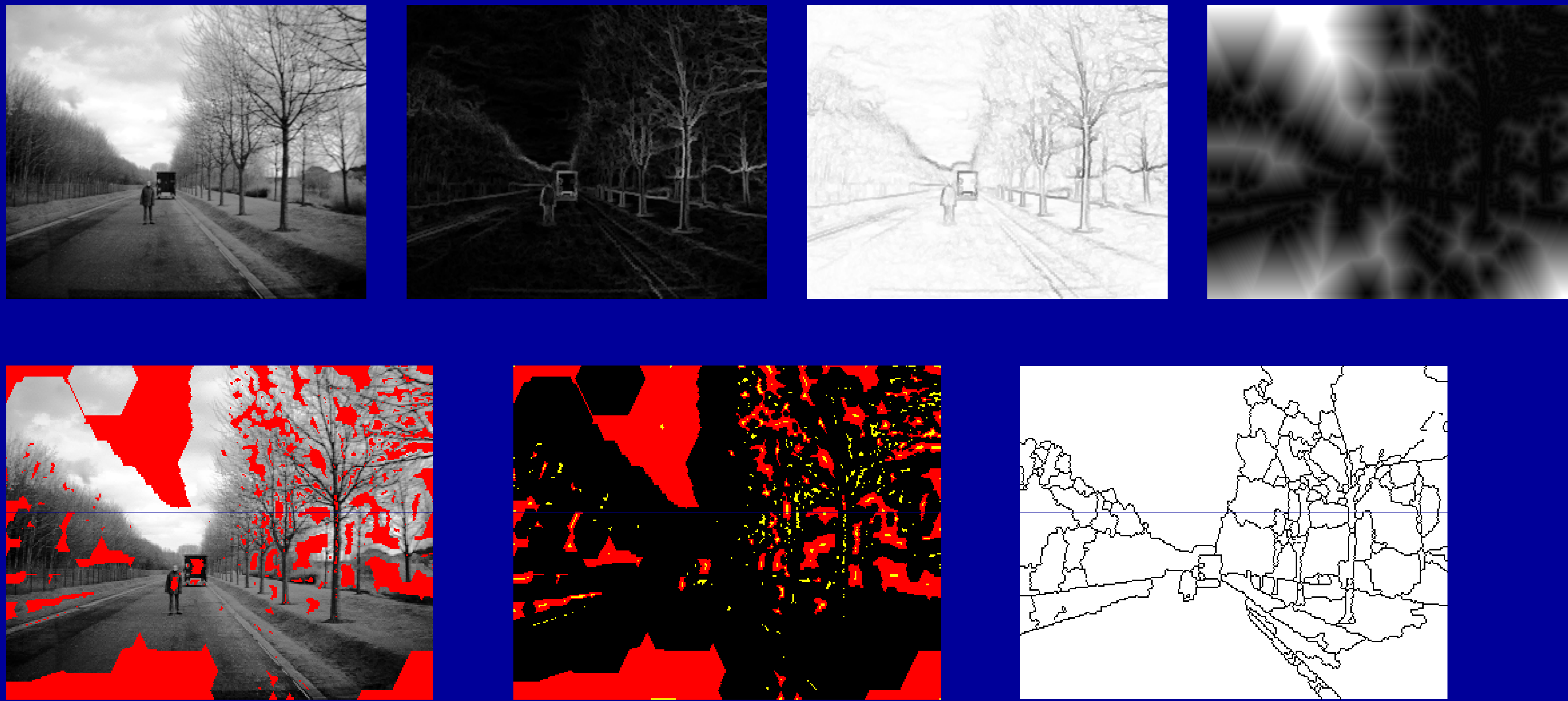


Segmentation

Video surveillance scene

(Example available in MAMBA)

Gradient and Quasi-Distance

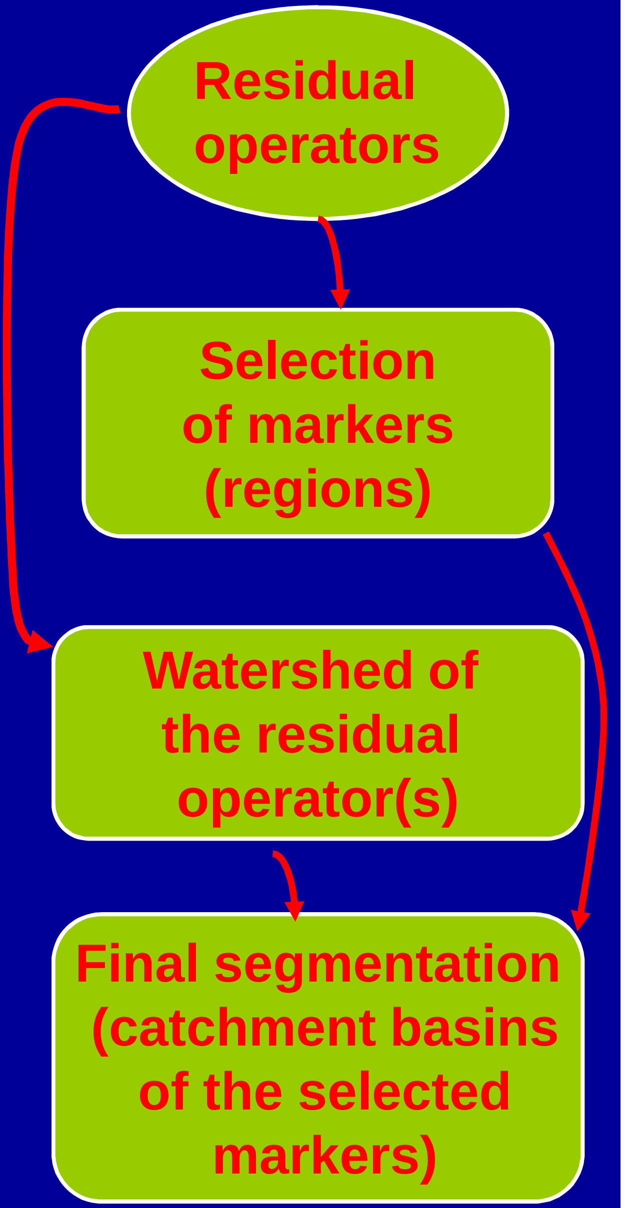


Quasi-distance can be computed on the inverted gradient function

- **Only one quasi-distance is calculated**
- **Hierarchy of regions based on their relative contrast**
- **The shape of regions is taken into account (closure of imperfectly closed regions)**

Towards a New User's Guide

- In the initial user's guide, no clue was given about the operators used for segmentation.
- Criteria functions belong to the residual transforms class and markers are linked to extrema of these functions.
- Residual transformations, not only, emphasize variations of some features (contrast, size, shape, etc.) but also, indicate where the greatest variations occur, what are their amplitudes and which index value produces them.
- New residual transforms have been introduced. They bridge the gap between the numerical and binary images regarding shape and size criteria.
- Other residues are also very efficient: regularised gradients, spatio-temporal gradients, critical balls, pilings (in hierarchical segmentations), etc.



Hierarchical Segmentation, Waterfalls

It is not always possible to prevent over-segmentation by marker-controlled watershed because it is not always possible to find good markers and/or segmentation criteria.

Therefore, another approaches of the segmentation which are not based on the a priori selection of markers exist. They aim at defining a hierarchy of segmentations.

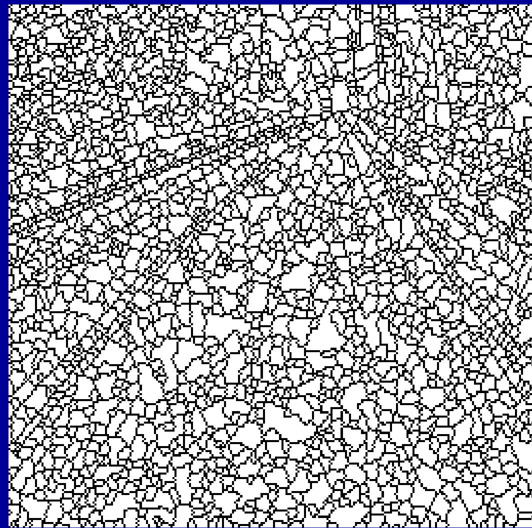
The Waterfalls transformation is the other way to solve the over-segmentation problem.
This approach is not new (1990). New developments (2006-2009) bring new perspectives and allow to define a unified approach of segmentation.

Some Definitions

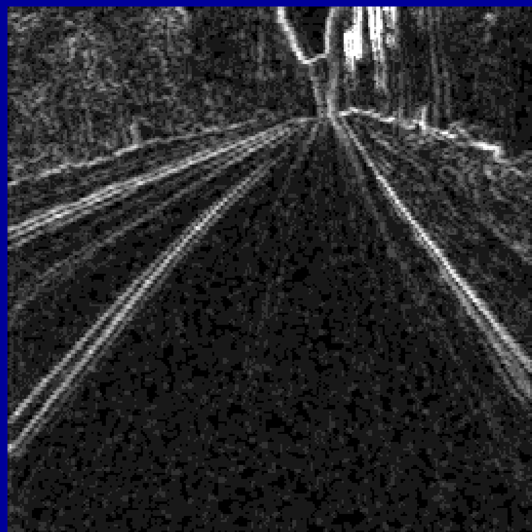
Valued watershed

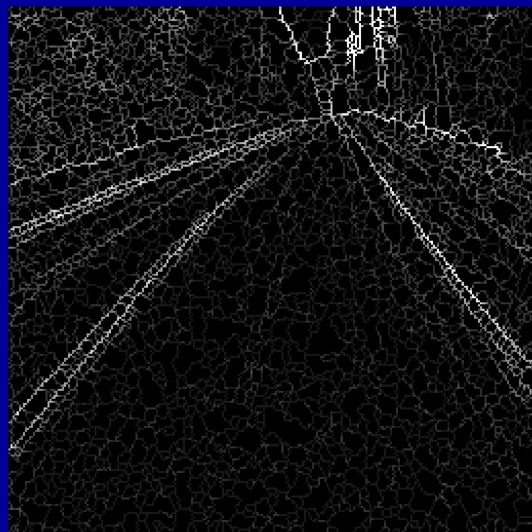
• The watershed of a function g is a set W(g)

• The valued watershed is a function w(g) with support W taking at each point of W the value of g



W





w(g)

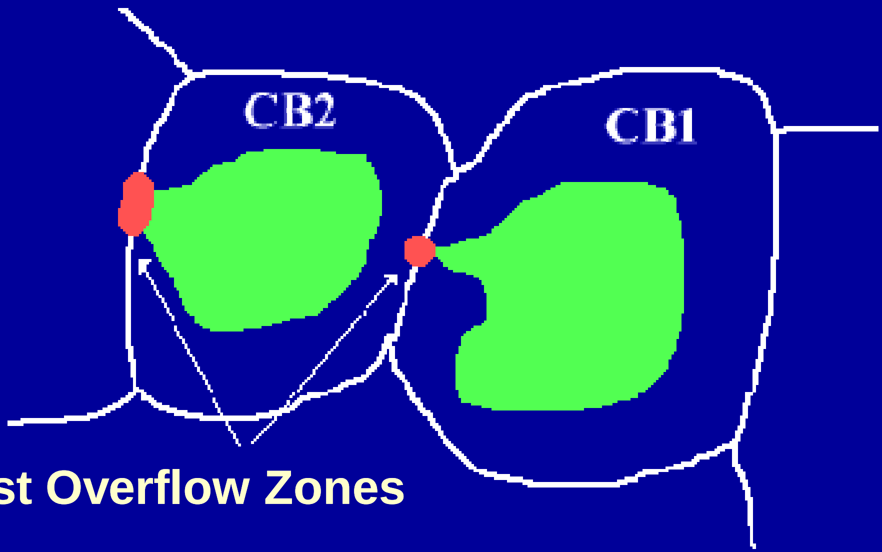
Image initiale

Gradient g

Definitions (continued)

First Overflow Zone (FOZ)

The FOZ of a catchment basin corresponds to the points of the watershed line surrounding this catchment basin where the first overflow occurs.



First Overflow Zones

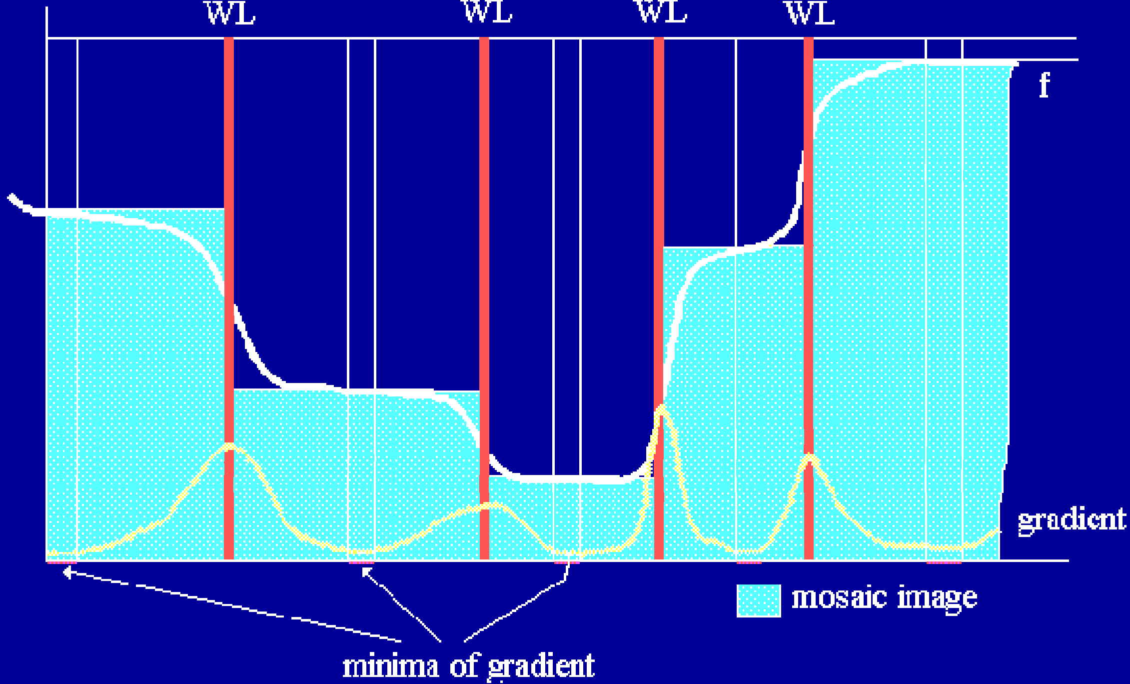
Lower Catchment Basin

It's the part of the catchment basin flooded before the occurrence of the first overflow.

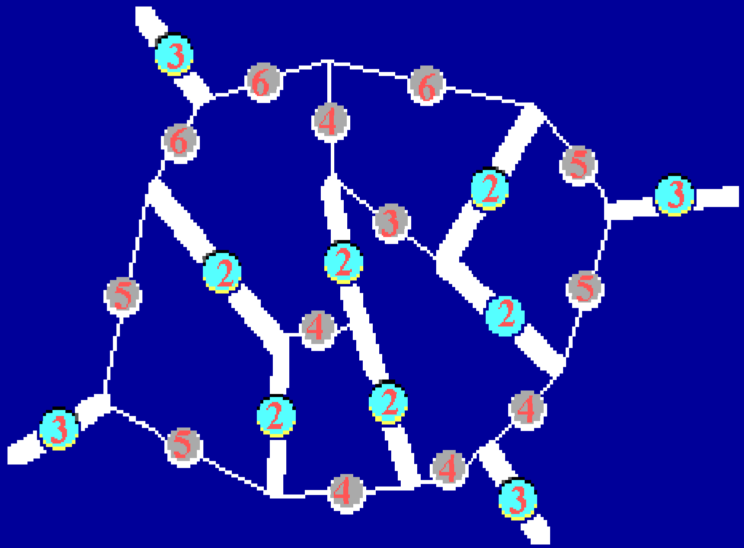
Mosaic Image and its Gradient

Building the mosaic image:

- **Watershed of gradient**
- **For each minimum of gradient, compute the corresponding grey value**
- **Fill in the catchment basin with this grey value**



Arcs Valuations in Gradient-Mosaic Image



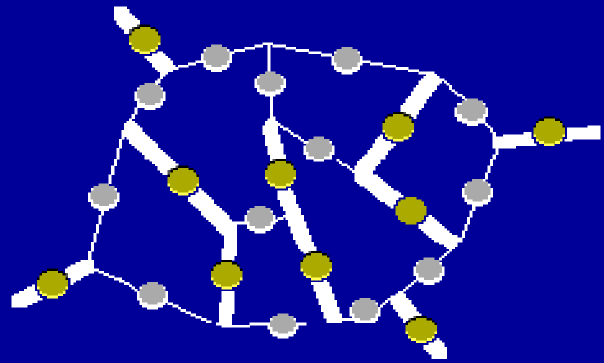
In the mosaic image, each arc c_{ij} separates two catchment basins CB_i and CB_j. The valuation v_{ij} of the arc is given by:

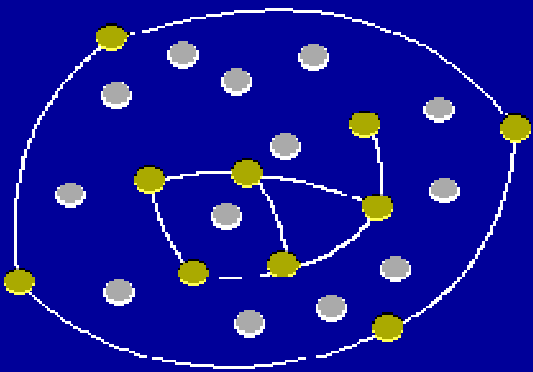
$$v_{ij} = | g_i - g_j |$$

where g_i and g_j are the grey values in the catchment basins.

In order to define properly minimal arcs, we need to define a neighborhood relationship between these arcs. This is achieved by means of the introduction of a new graph.

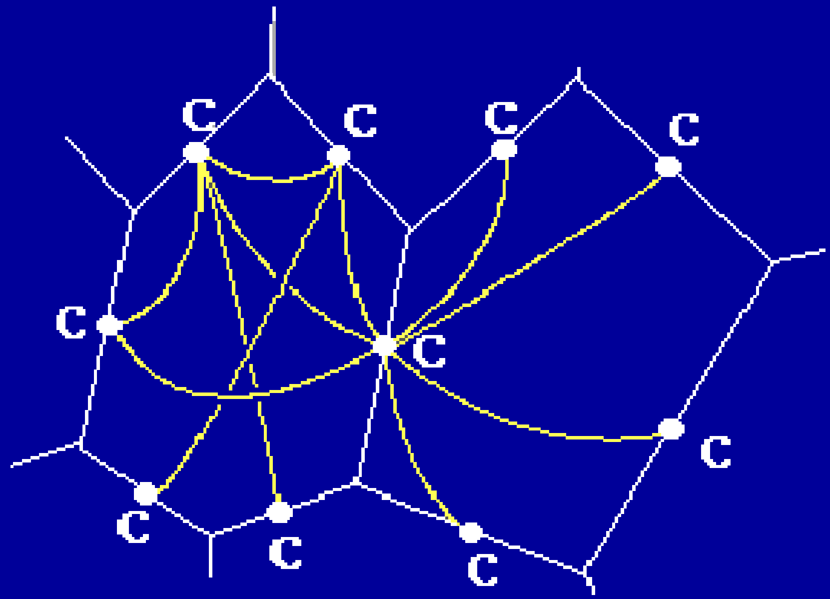
Graph Representation and Associated Watershed





Definition of a new graph

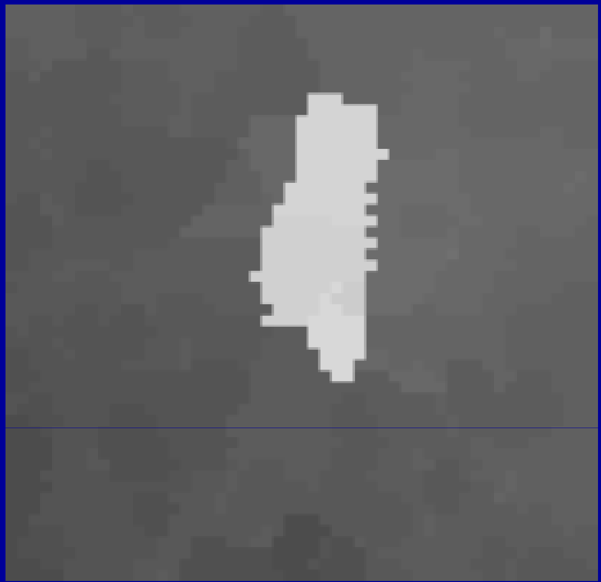
- its vertices correspond to the arcs of the mosaic gradient
- its edges link all arcs surrounding the same catchment basin
- each vertex is valued by the arc valuation as defined in the gradient mosaic

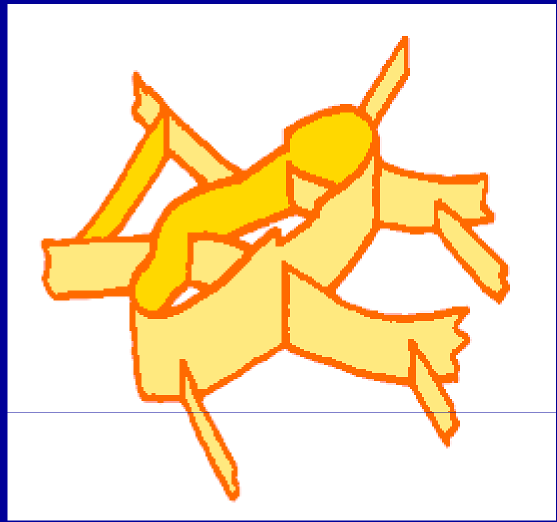


In this representation, the arcs surrounding the same catchment basin are adjacent. Therefore, minimal arcs can be connected although it is not the case in the mosaic gradient, as illustrated above (yellow summits correspond to minimal arcs).

Over-Segmentation and Perception of Images

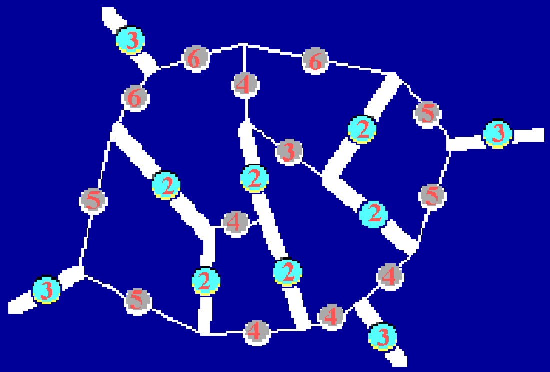
A simple illustration using a mosaic image





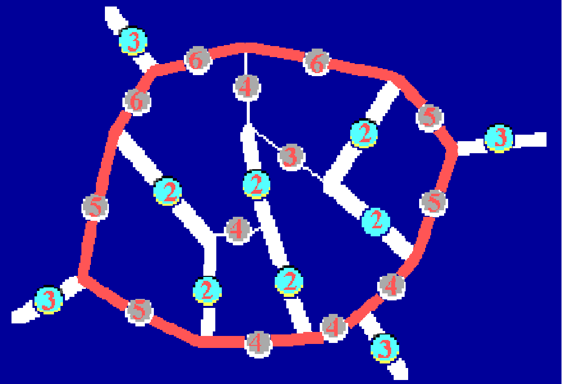
Despite the fact that the image is over-segmented, the white blob can be easily distinguished from the background because, at the same time, the boundaries between the regions inside the blobs and the boundaries inside the background are less contrasted than the boundaries which separate the blob and the background. Both the blob and the background are marked by boundaries with a minimal contrast.

Waterfalls transform: a watershed applied on graphs

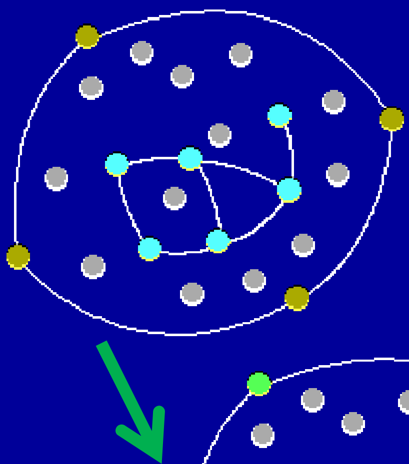


Initial watershed

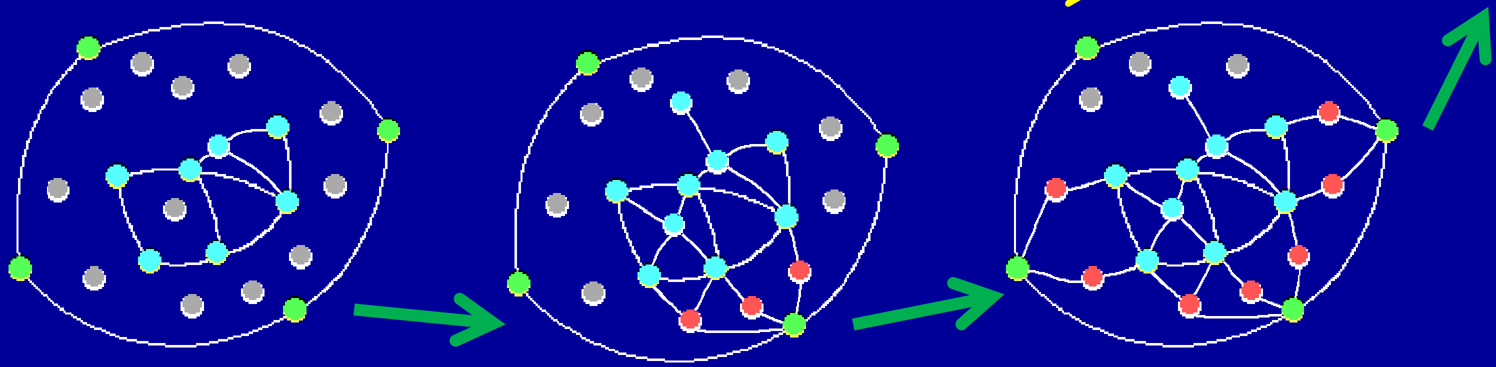
The waterfalls transform is a watershed transform (propagation) performed on a graph defined from the initial watershed image. The markers of this watershed are the minimal arcs of the initial watershed image.

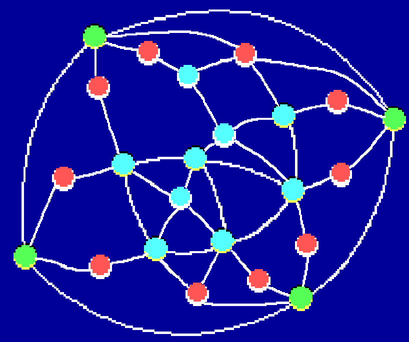


First level of hierarchy

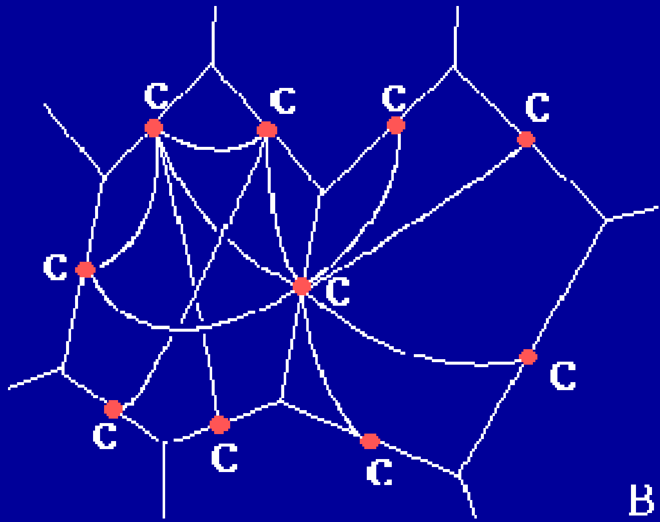


Successive steps of flooding

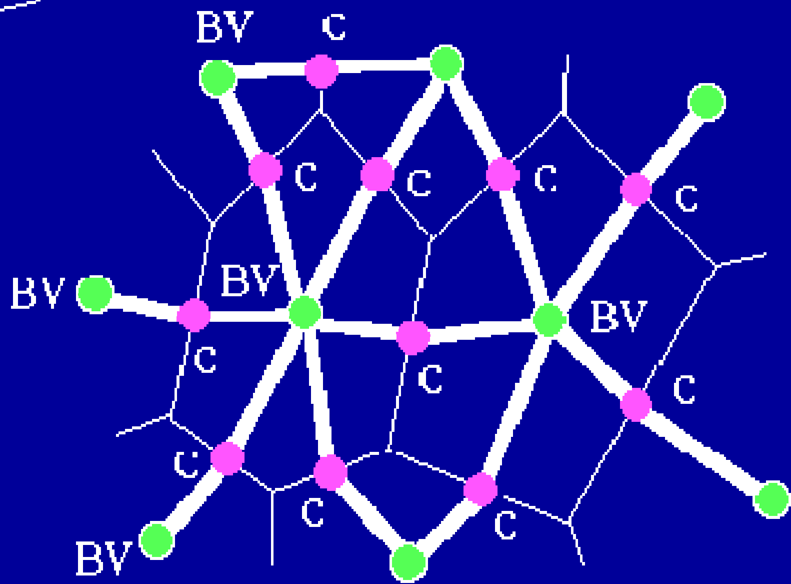




From a 3D to a Planar Graph



The previously defined graph is a 3D valued graph, which is not very handy.



This graph can be transformed into a planar one:

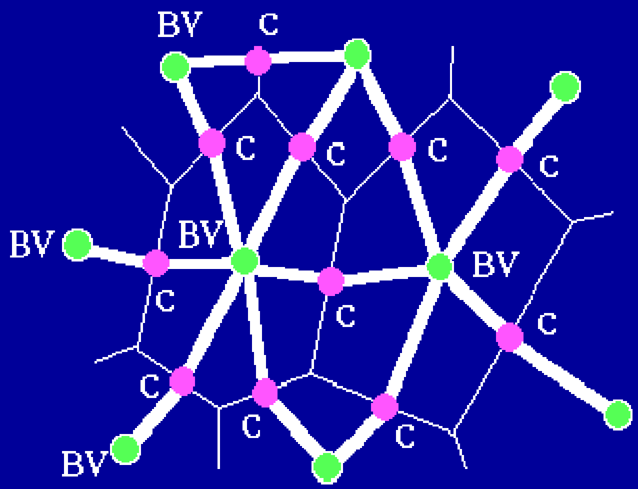
• A new vertex is added in each catchment basin (purple dot).

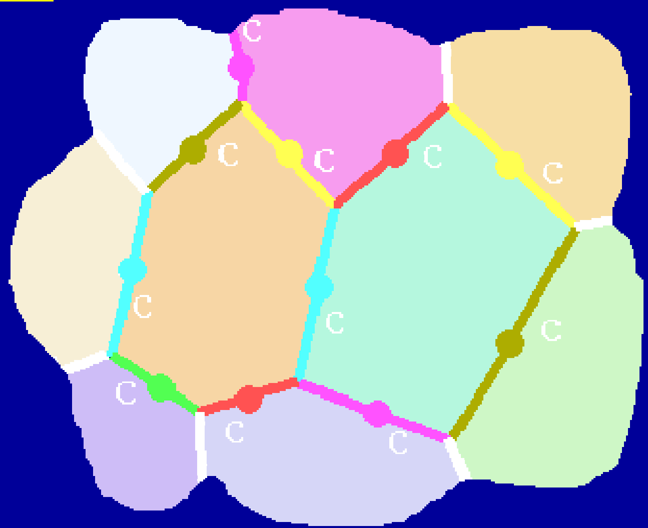
• The previous edges are replaced by two successive edges linking the original vertices through the new one.

• The valuation of the new vertex is equal to min (v_{ij}) where v_{ij} are the valuations of the arcs surrounding the catchment basin.

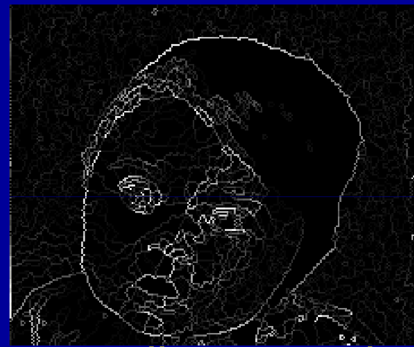
Image Representation

The hierarchical image






mosaic


gradient mosaic

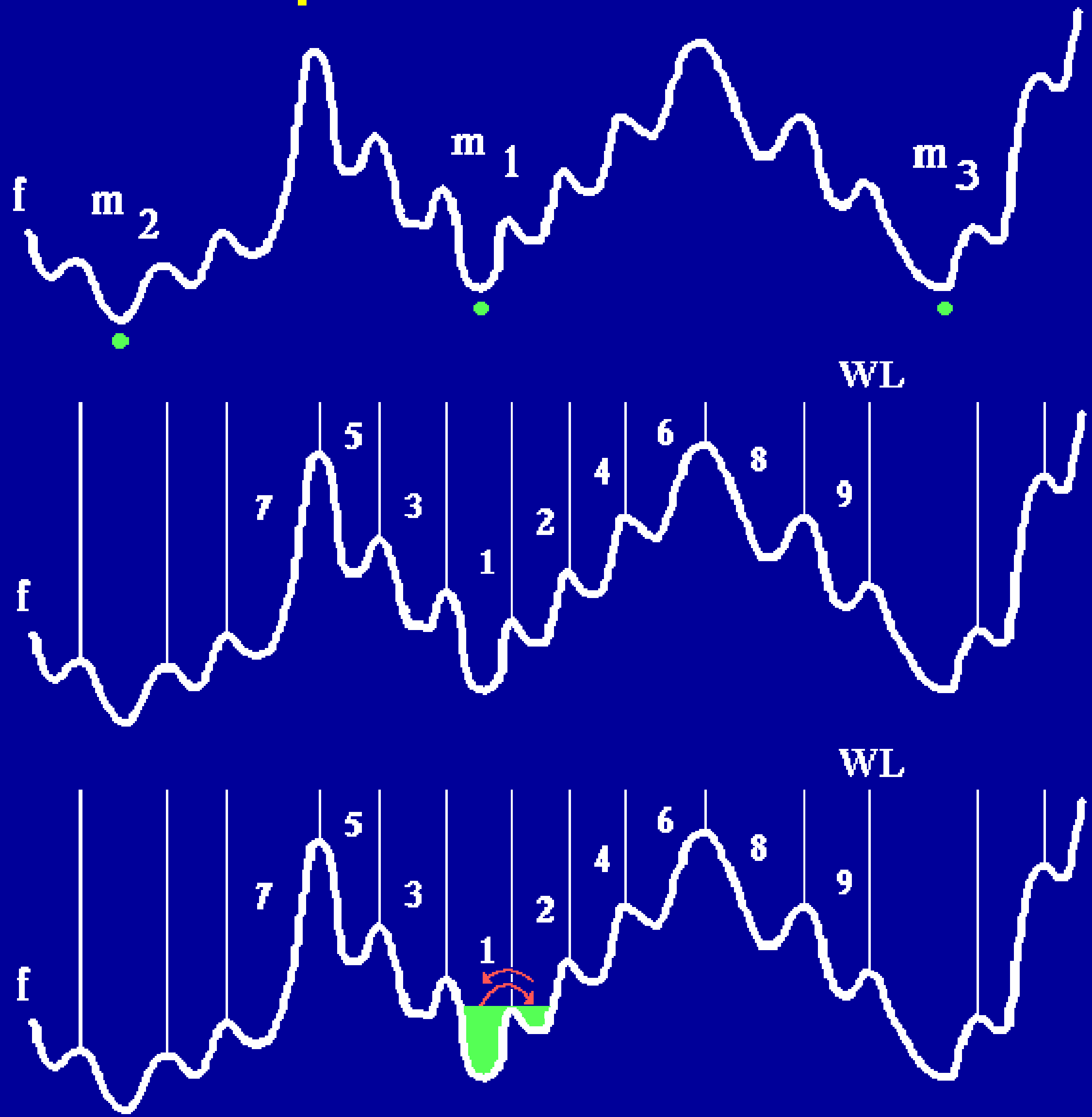

hierarchical image

An image, named hierarchical image can be build from the planar graph. The catchment basins of the gradient mosaic are filled with grey values corresponding to the valuation of the new added vertices.

The watersheds of this hierarchical image give the higher level of hierarchy (with some restrictions).

Where Are the Waterfalls?

A little explanation…



Consider the function f and its watershed. Various catchment basins are numbered from 1 to 9. Consider the flooding from the minimum m_1.

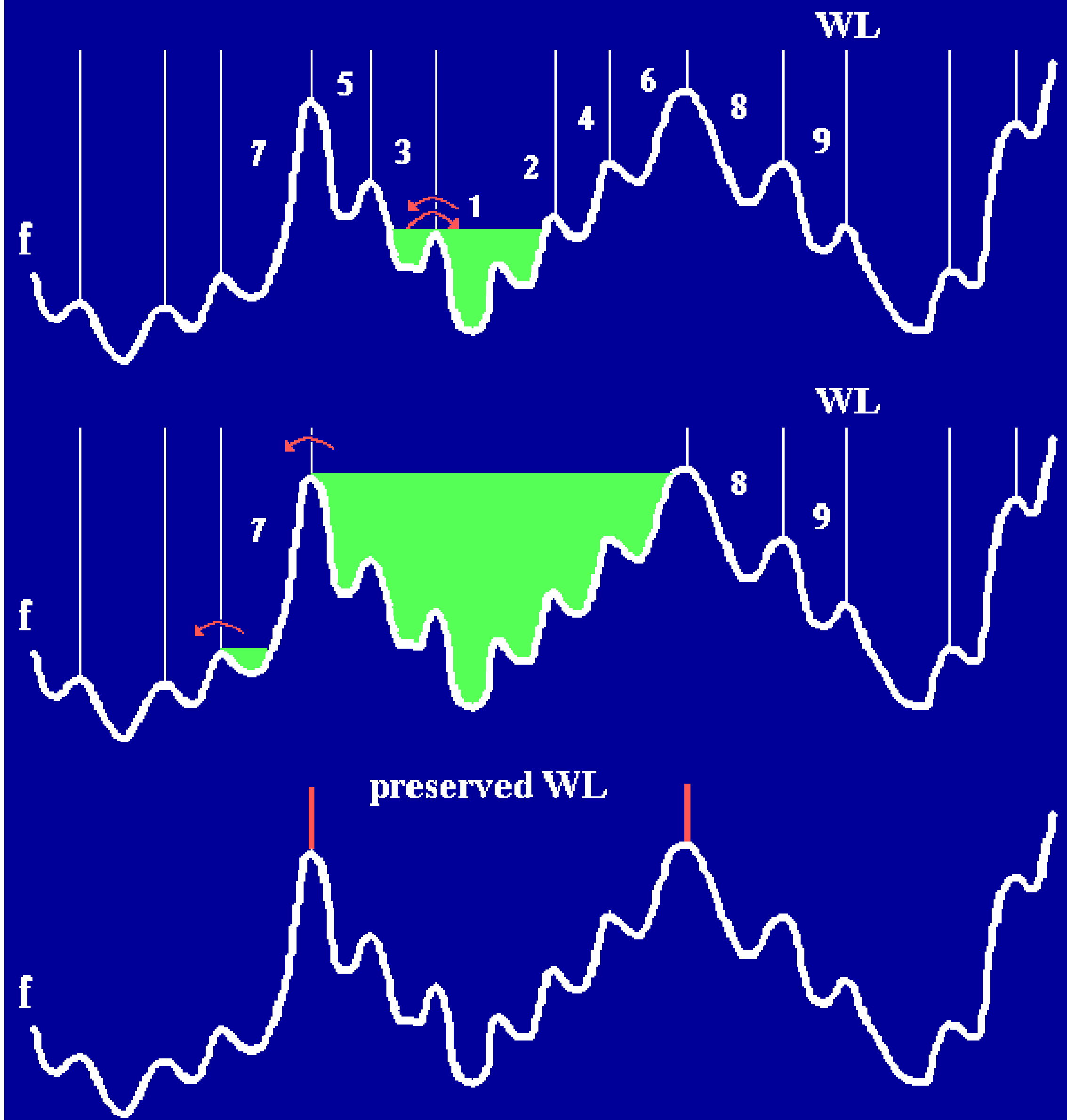
When filling CB1, an overflow occurs towards CB2.

Now, if we fill in CB2, the first overflow occurs towards CB1.

In this case, overflows (waterfalls) are symmetrical.

Therefore, the part of the watershed line separating CB1 from CB2 can be removed and the floods in CB1 and CB2 can be merged.

Waterfalls Are Here…

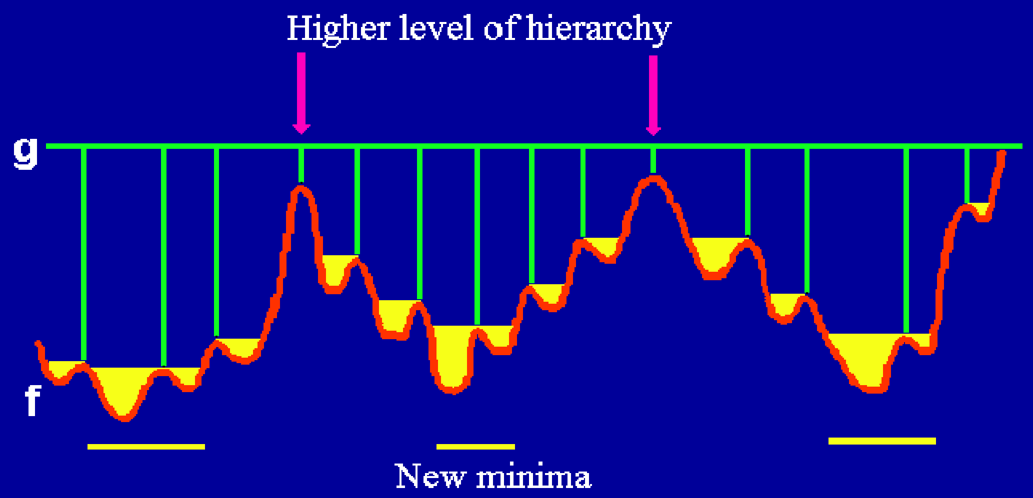


If this flooding process is iterated, the flood invades CB3 which in return, when flooded, pours into the merged basins CB1 and CB2. Here again the waterfalls being symmetrical, CB3 is merged to the flood.

Step by step, because, in each case, waterfalls are symmetrical, all the catchment basins from 1 to 6 are merged.

But, when the flood pours into CB7, the situation changes. Now, if we flood CB7, the waterfall is no longer symmetrical. Therefore, the watershed line between CB7 and the merged basins must be preserved.

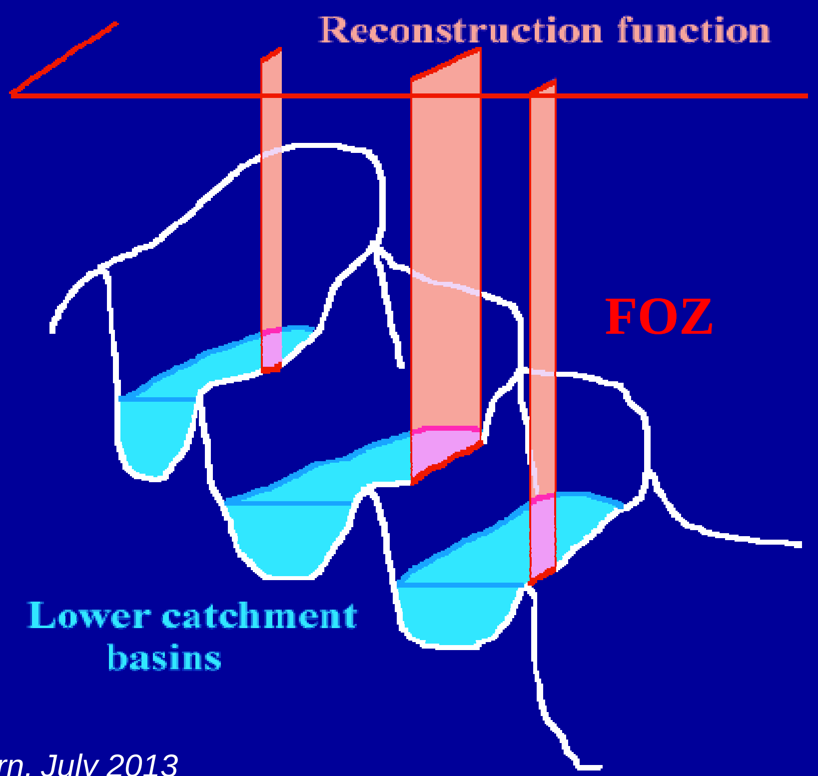
Filling the Lower Catchment Basins



The same result can be obtained by filling the lower catchment basins of the initial function and by performing the watershed transform of the new function.

The successive floods generate the lower catchment basins associated with each CB (flood just before the overflow through the FOZ).

This can be achieved directly by a geodesic dual reconstruction of the initial function by the FOZ.



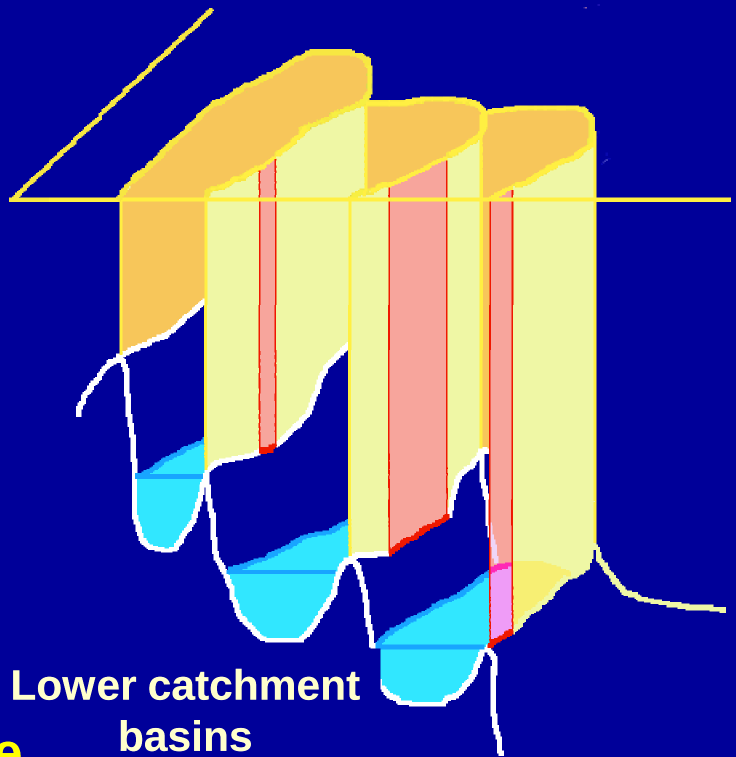
Reconstruction and Hierarchical Image

Instead of using FOZ (not easy to detect them), the whole set of watershed lines may be used. The result will be the same because the FOZ is the region at the lowest altitude bordering the catchment basin.

- f, initial function
- let us define g:

  g (x) = f (x) if and only if x belongs to the watersheds of f

  g (x) = max if not
- h = R^*_f (g) , result of the dual reconstruction of f by g
- W(h), watershed of h, produces the hierarchical segmentation of higher level

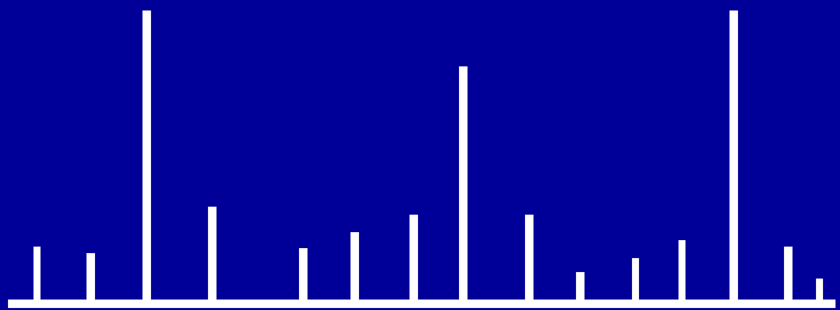


Lower catchment basins

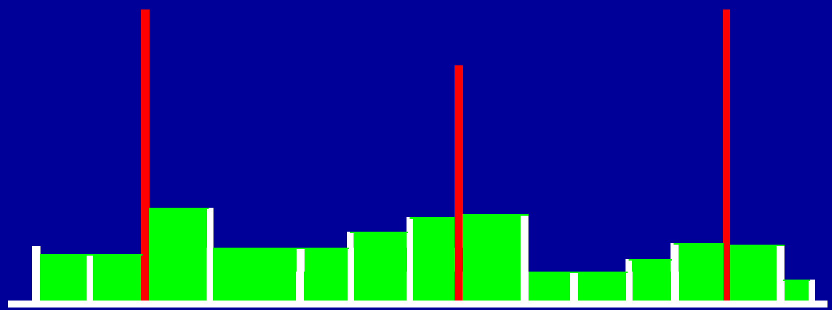
When f is a valued WTS, h is identical to the previously defined hierarchical image.

Waterfalls and Hierarchical Images

The waterfalls transformation can also be obtained by performing the watershed transform of a new image, the hierarchical image.



Initial watershed image

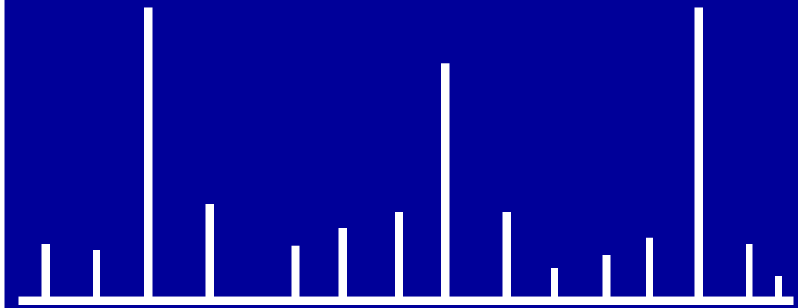


Hierarchical image and waterfalls transformation

- **Each catchment basin of the initial watershed image is flooded and filled (in green).**
- **This new image is called hierarchical image.**
- **The watershed transform of the hierarchical image is identical to the waterfalls transform (in red).**

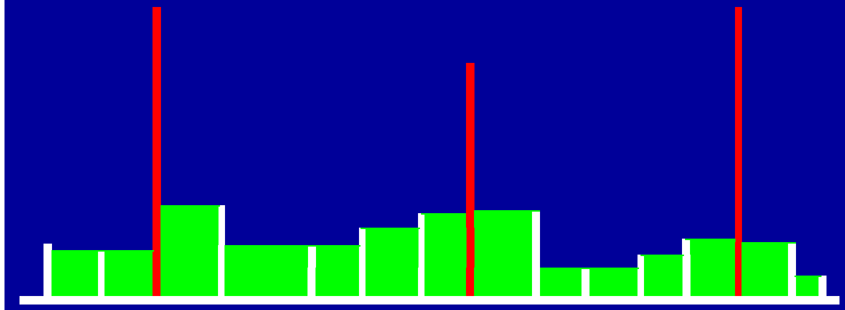
Waterfalls and Mosaic Images

In this case, the hierarchical approach and the waterfall approach are identical. The waterfall transformation is the generalisation for any function of the hierarchical approach.

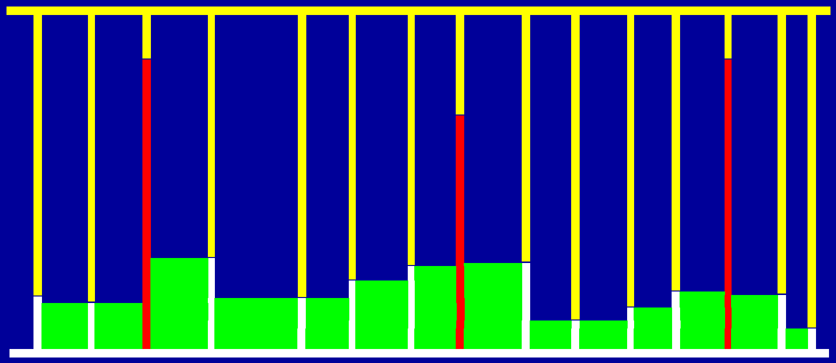


Gradient mosaic image

The minimal valuation of the catchment basin corresponds to the height of the lower FOZ. This valuation produces the same result as the reconstruction of the gradient mosaic function by the lower FOZ.



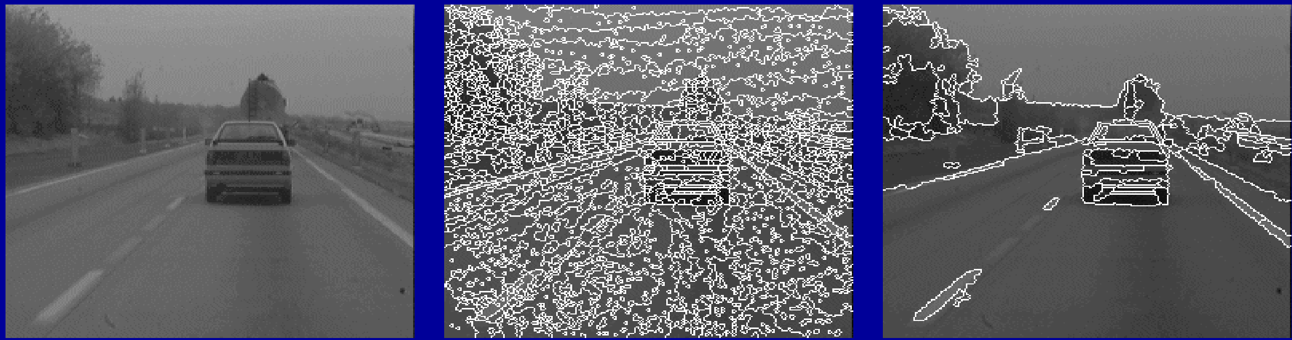
Hierarchical image



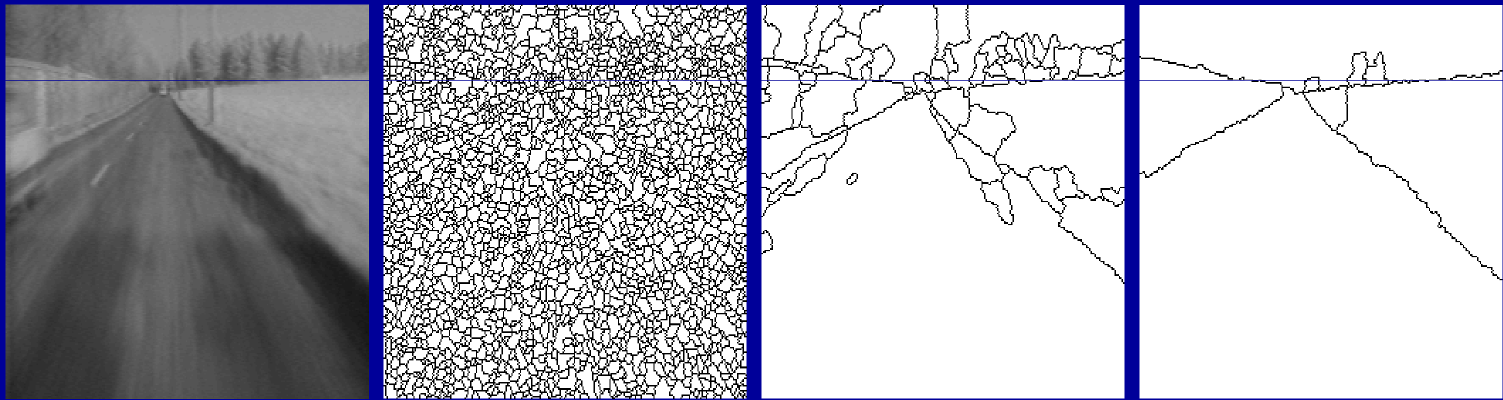
Waterfall image

Using the Waterfalls Transform

- **Efficient and non parametric approach to reduce over-segmentation.**



- **The process can be iterated.**



→ ?

S_0 S_1 S_2

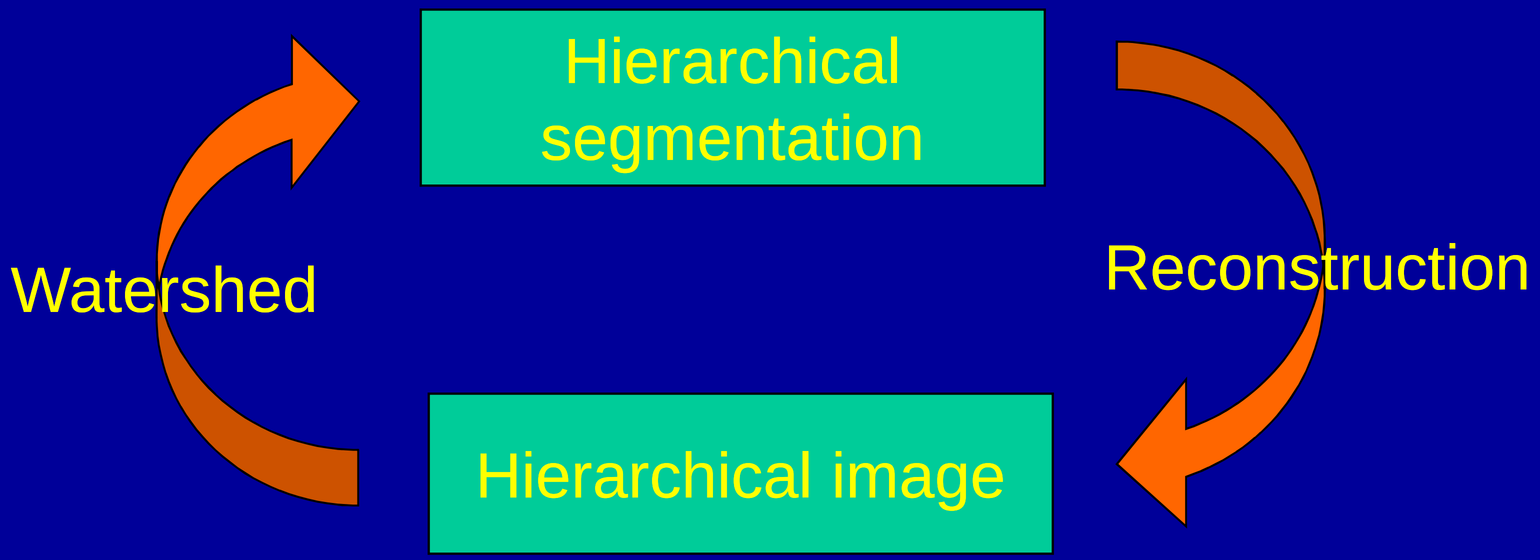
- **But the process ends up with the empty set (no stop criterion).**
- **It is difficult to select a « good » hierarchical level.**
- **Another annoying problem appears…**

Iterating the Waterfalls Transform

Protocol

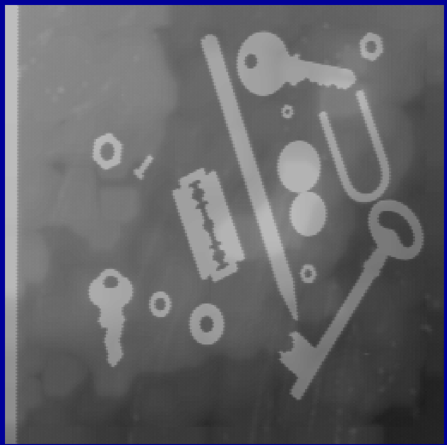
- **One starts from an initial valued watershed s_0**
- **An iterative process produces successive hierarchical segmentations s_i:**

$s_i = w(h_{i-1})$ where h_{i-1} is the hierarchical image associated to the segmentation s_{i-1}

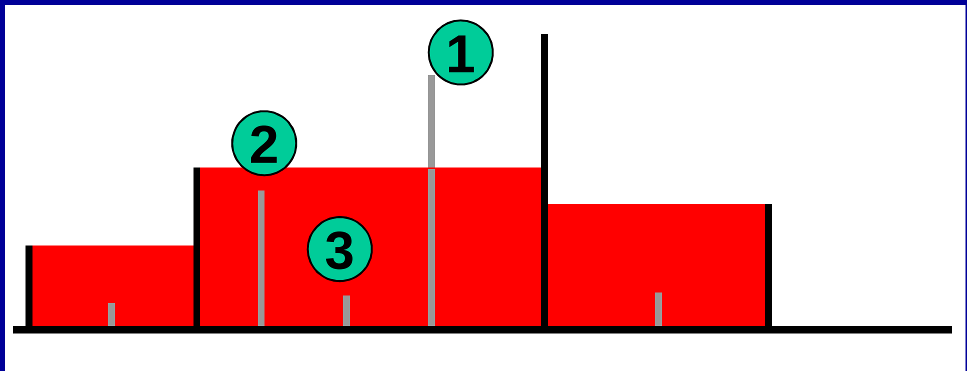


Waterfalls Short-sightedness

- The successive hierarchical levels are far from being relevant…



The waterfalls transforms removes too many contours!



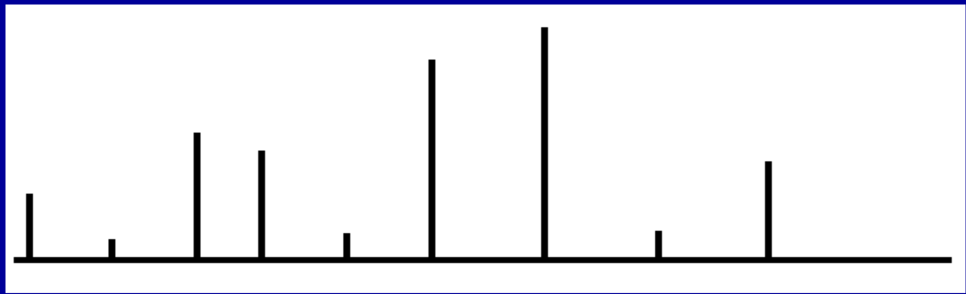
In red, hierarchical image h associated with the next level of hierarchy

Three different kinds of removed contours appear:

1. Contours whose altitude is higher or equal to h
2. Contours whose altitude is lower than h but closer to it than to 0
3. Contours whose altitude is close to 0

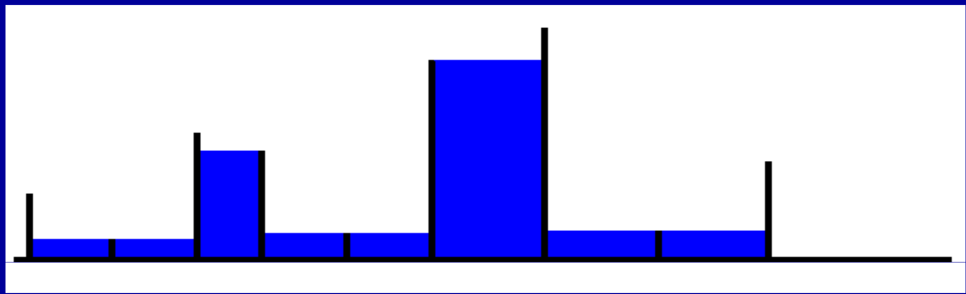
Only the removal of type (3) contours is legitimate!

Problem with the Waterfalls Operator

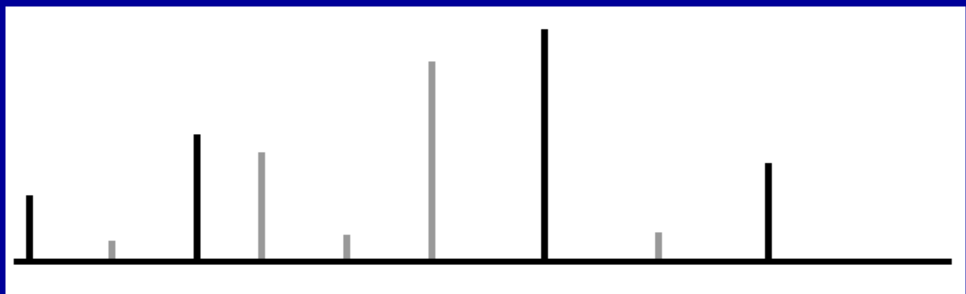


Initial Segmentation s_0

(valued watershed)



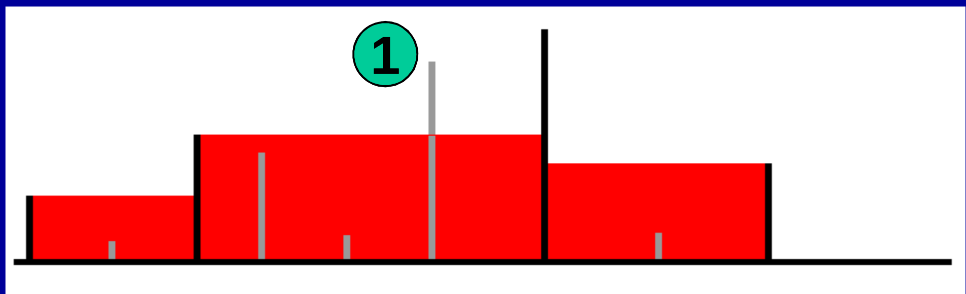
Hierarchical Image h_0



Hierarchical Segmentation s_1

$s_1 = w(h_0)$

(In grey, removed contours)

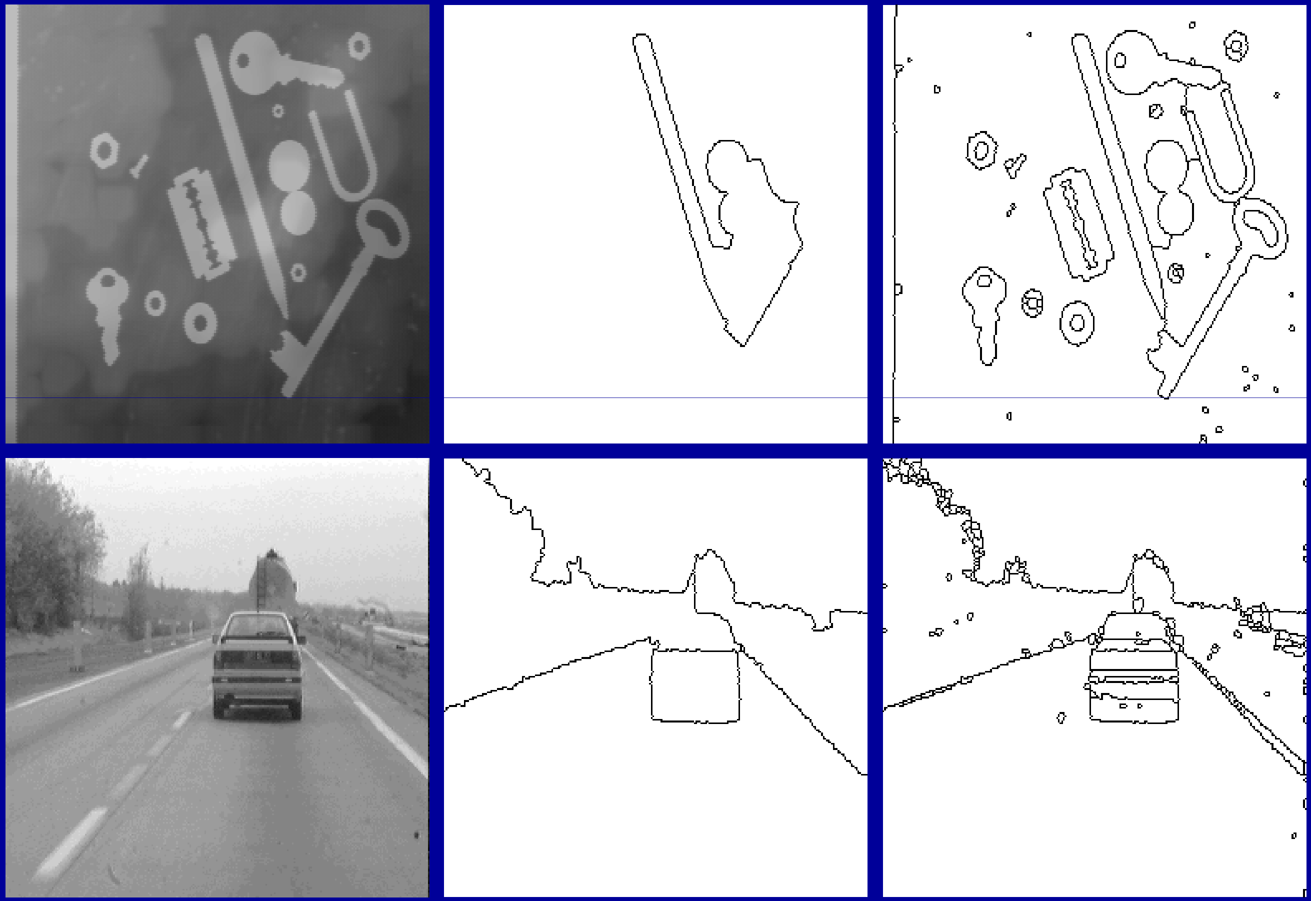


The labelled contour is higher than the hierarchical image h and therefore, it should be preserved!

Enhanced Waterfalls

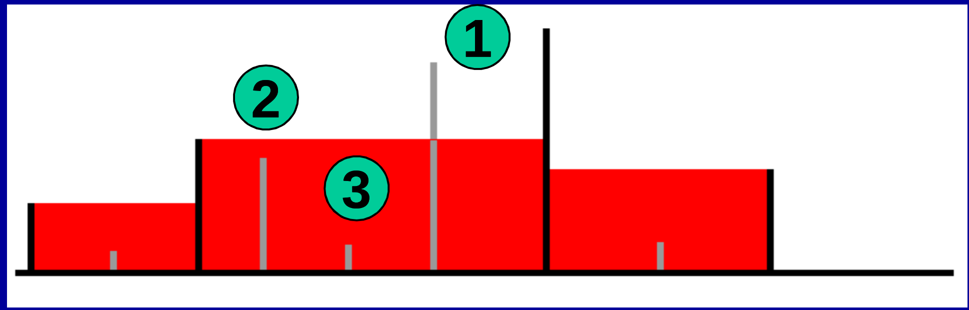
The Waterfalls transform can be improved by re-introducing type 1 contours at each level of hierarchy.

- **The number of intermediary levels of hierarchy remains the same**
- **This is a self-locking process (the last hierarchical image being equal to 0, the previous contours are re-introduced).**



Comparison between the last levels of hierarchy of the « classical » waterfalls transform and the enhanced one.

Standard Segmentation Algorithm

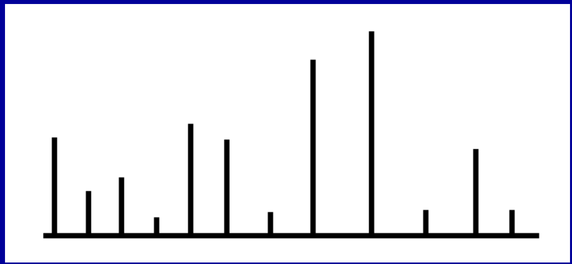


When type 2 contours are also re-introduced (contours whose altitude is lower than h but closer to it than to 0), the corresponding algorithm is named standard algorithm.

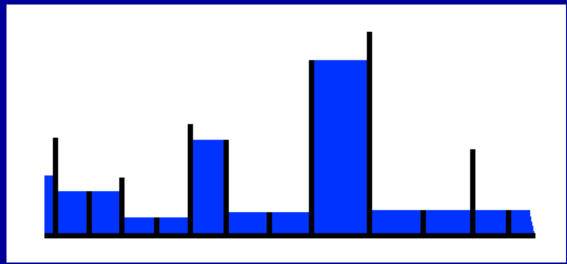
<u>At each step i</u>:

- **Segmentation s_{i+1} is obtained from a watershed of h_i**
- **Hierarchical image h_{i+1} is computed**
- **The height of contours of $inf(s_i, s_0)$ – those initial contours which still belong to s_i – is compared to h_{i+1}. If $s_0 > h_{i+1} - s_0$ (it is a type 1 or 2 contour), the contour is kept.**
- **But before adding it to s_{i+1}, its height is modified and is set to h_{i+1}.**

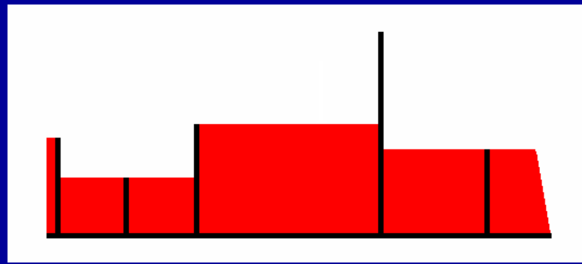
Iterating the standard algorithm



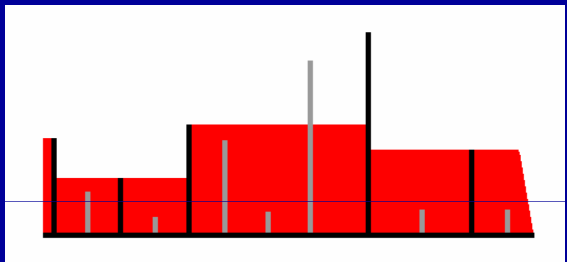
Initial segmentation s_0



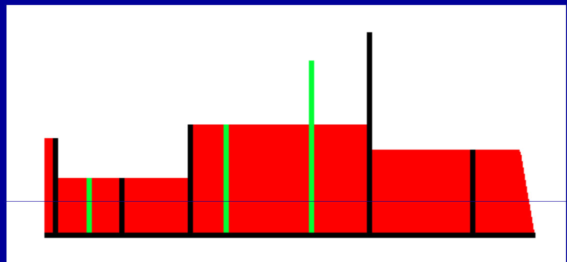
Initial hierarchical image h_0



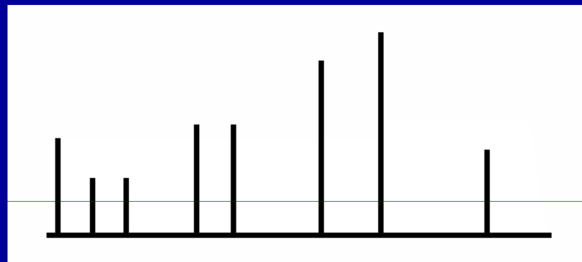
In red and black, h_1
In black, $s_1 = w(h_0)$



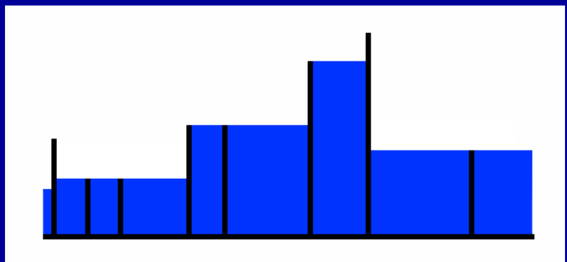
In grey, contours to be checked (removed by the initial WTS of h_0)



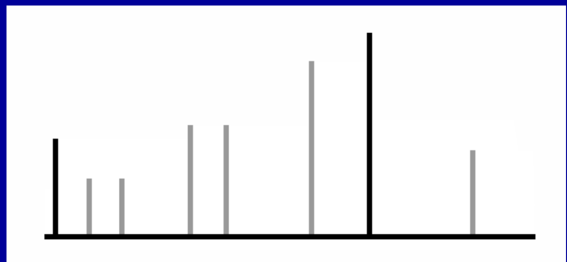
In green, preserved contours. Height of type 2 ones is modified



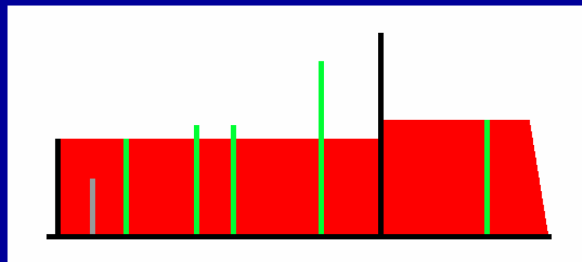
Final segmentation s'_1



Next step, segmentation s_1 and hierarchical image h_1 (blue and black)

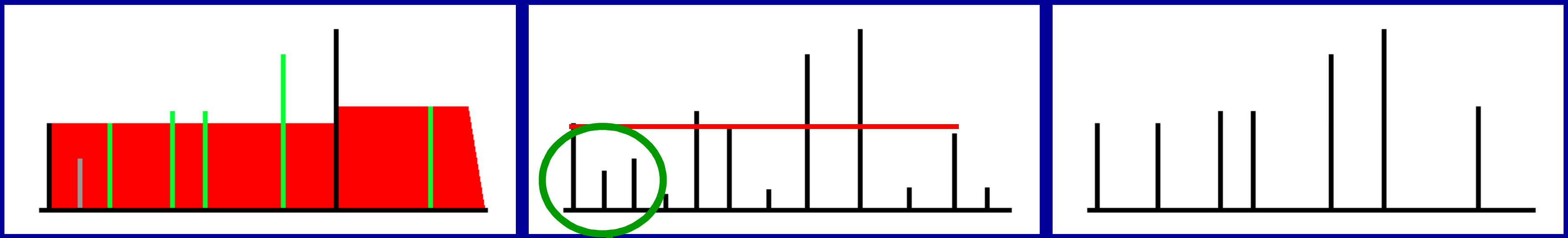


In black, initial segmentation s_2, $s_2 = w(h_1)$. In grey, contours to be checked



In red, hierarchy h_2 . In green, preserved and modified contours. In grey, removed ones

Iterating the standard algorithm (continued)



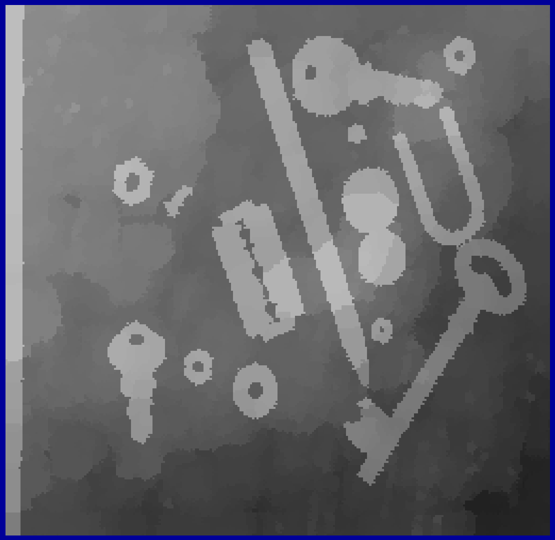
Grey contour is removed (its initial height in s_0 is not sufficient compared to h_2

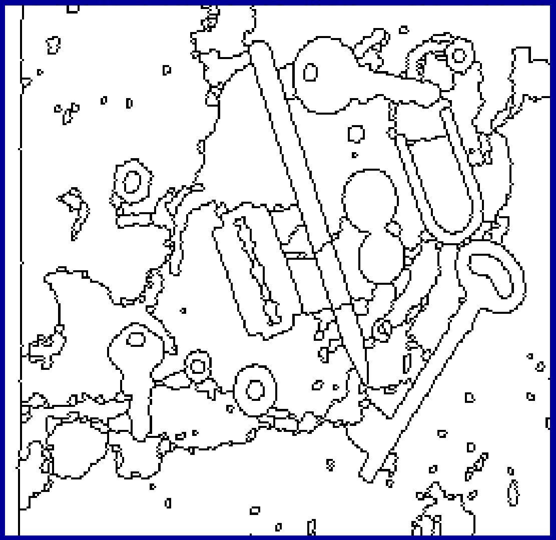
Initial s_0. In red, height of h_2

$s_0 < h_2 - s_0$

Final segmentation s'_2

The height of type 2 contours must be modified in order to be able to iterate the process.





The number of hierarchy levels is unchanged compared to the enhanced waterfalls

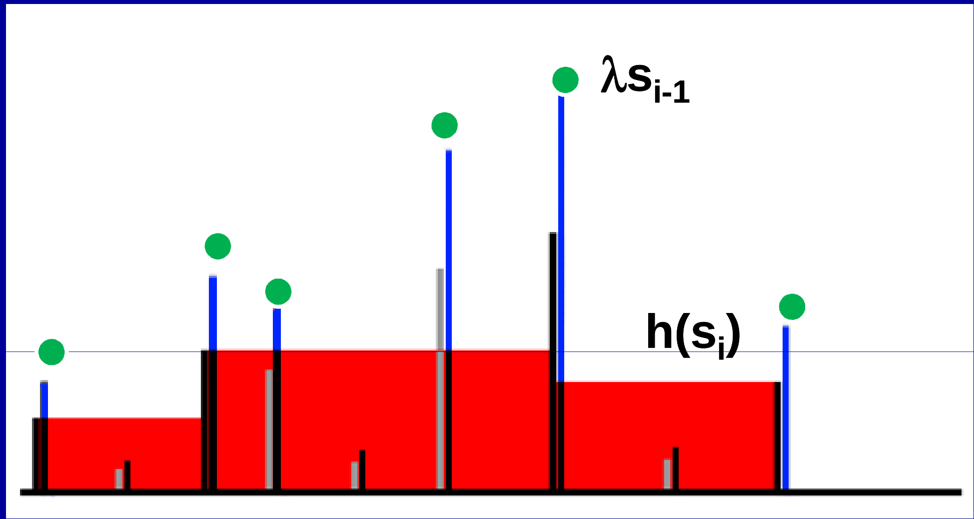
Final level with standard algorithm

A General Standard Segmentation Algorithm

Enhanced waterfalls and standard algorithm are two particular cases of a more general algorithm controlled by a parameter λ .

The heights of the contours belonging to s_{i-1} are multiplied by a factor λ and compared to the current hierarchy. Those which are higher define a mask m (green dots):

$$m = \{x : \lambda(s_0 \wedge s_{i-1}) \geq h(s_i)\}$$



The marked contours are reintroduced in the current segmentation s_i:

$$s'_i = m \wedge [h(s_i) \vee s_0]$$

When λ is equal to 1, we obtain the enhanced waterfalls.
When λ is equal to 2, we get the standard algorithm.

P algorithm

In the previous algorithms, the contours which are preserved or removed in the current hierarchical level i+1 always belong to the segmentation s_i.

However, if the height of the contours of s_0 [instead of $\inf(s_i,s_0)$] is compared to h_{i+1}, we define a new procedure named P algorithm.

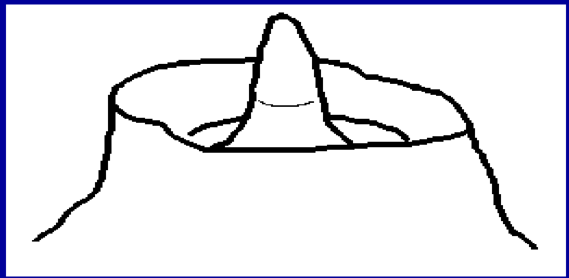
- **This new algorithm allows to re-introduce contours which have been eliminated in the previous levels of hierarchy.**

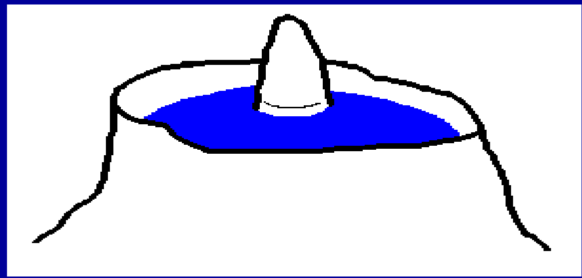
- **Some of these contours are inside particular structures of the hierarchical image called maxima-islands.**

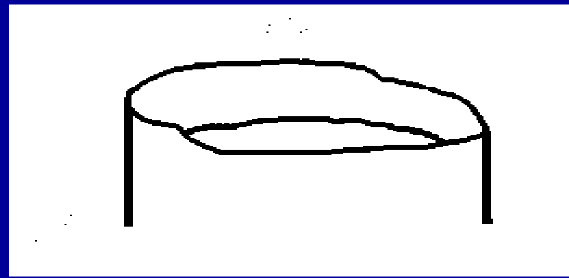
- **These islands are suppressed by the watershed transform because this operator is semi-homotopic.**

- **P algorithm, by re-introducing these maxima-islands, let them contribute to the hierachical process.**

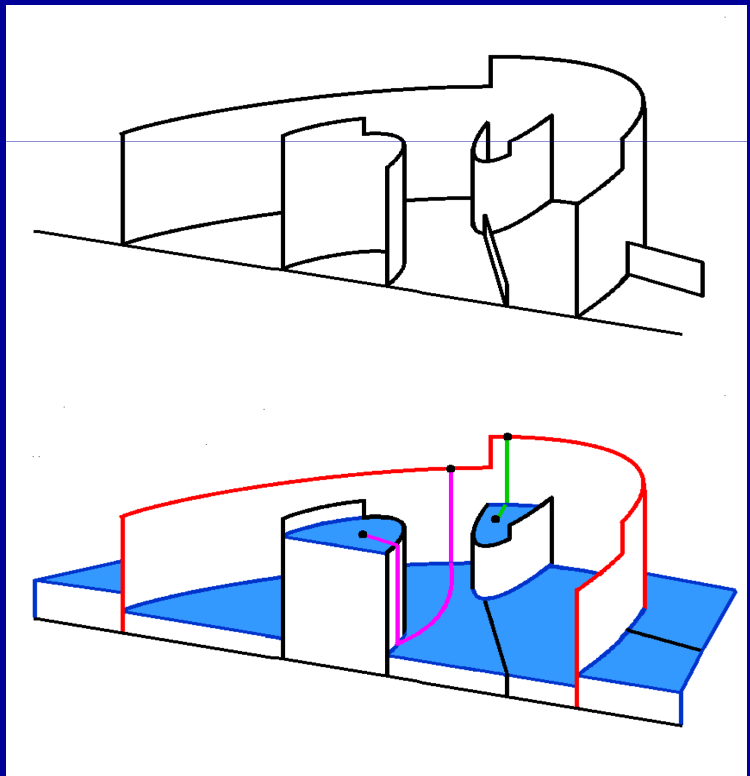
Semi-Homotopy of the Watershed Transform







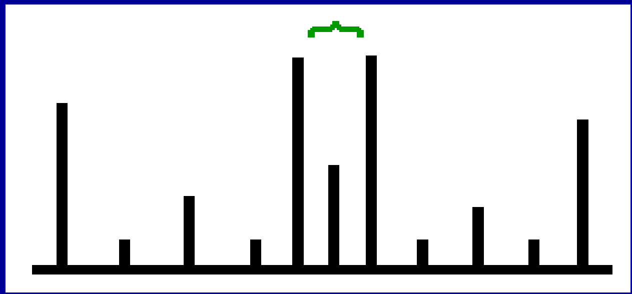
The watershed transform is semi-homotopic, it preserves minima but it removes maxima inside catchment basins.



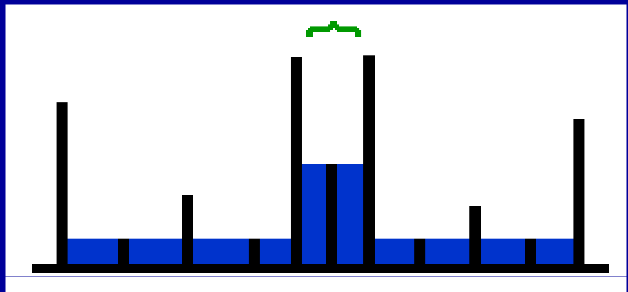
- **However, many relevant maxima-islands are likely to appear in the hierarchical images.**

- **P algorithm, by re-injecting some contours, gives the capability of these maxima-islands to be re-introduced when they have been removed in the previous steps.**

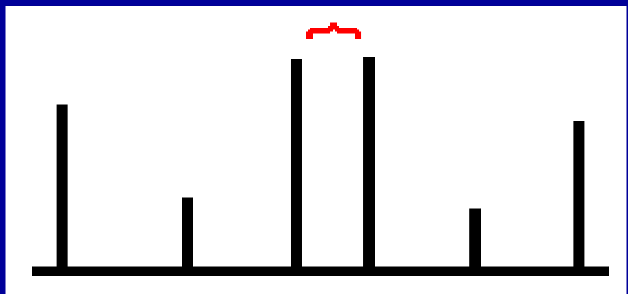
Steps of P Algorithm



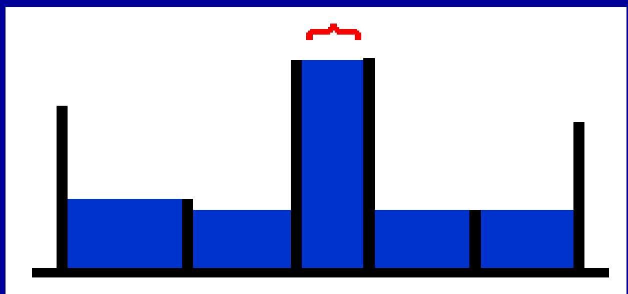
Hierarchical segmentation s_{i-1}
A maximum-island is likely to appear (green bracket)



Hierarchical image h_{i-1}

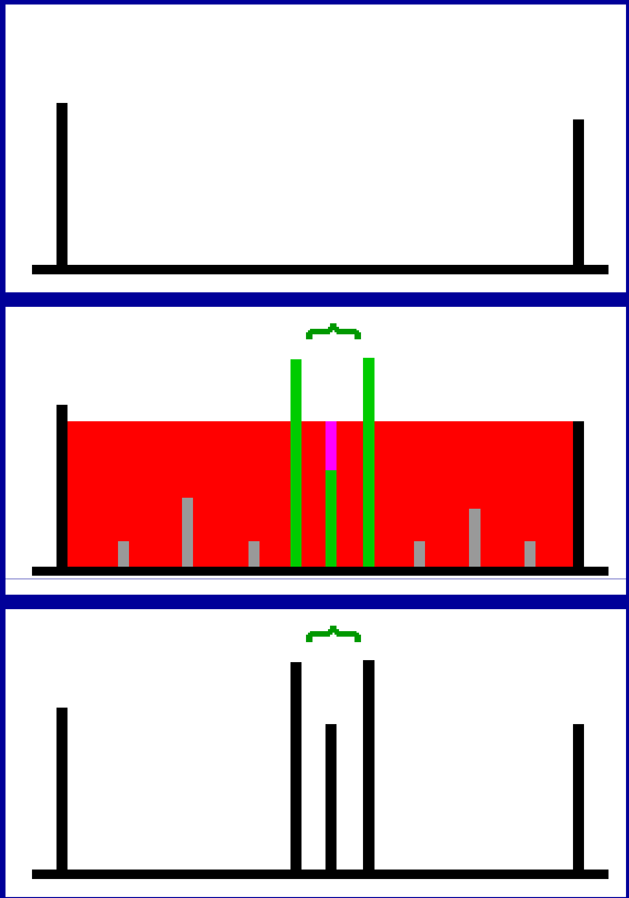


Initial hierarchical segmentation $s_i = w(h_{i-1})$
The maximum-island appears. Final segmentation $s'_i = s_i$ (no added contour)



Hierarchical image h_i

Steps of P Algorithm (continued)



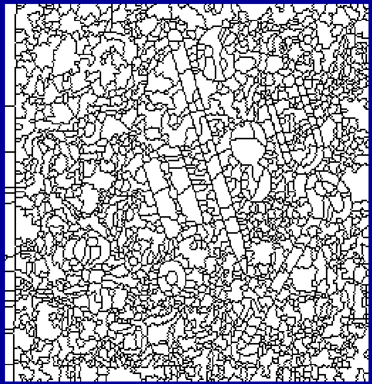
Initial hierarchical segmentation $s_{i+1} = w(h_i)$. Due to the semi-homotopy of the watershed transform, the maximum-island is removed.

All the contours of s_0 are compared to h_{i+1} (in red). Green contours are added. The height of the contour inside the maximum-island is modified.

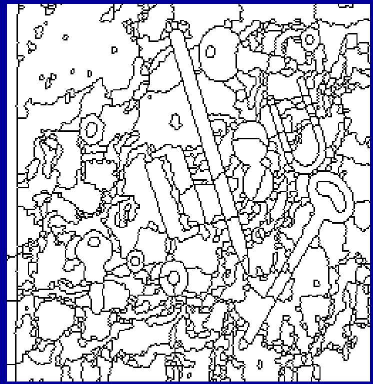
Final hierarchical segmentation s'_{i+1}.

Re-introducing the internal contour allows the reappearance of the maximum-island which will contribute to the genesis of the next hierarchical image.

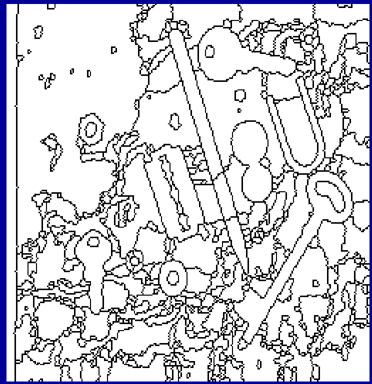
Properties of P Algorithm



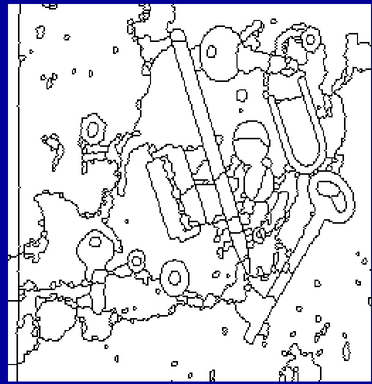
S_0



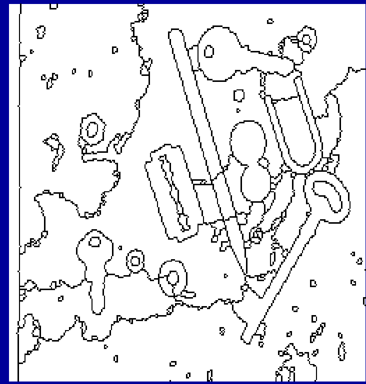
S_1



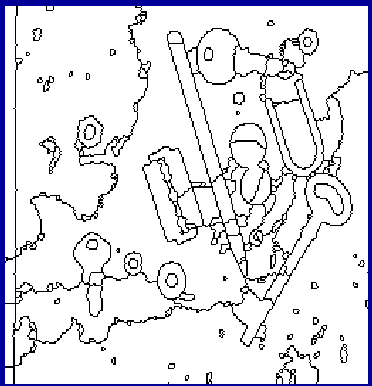
S_2



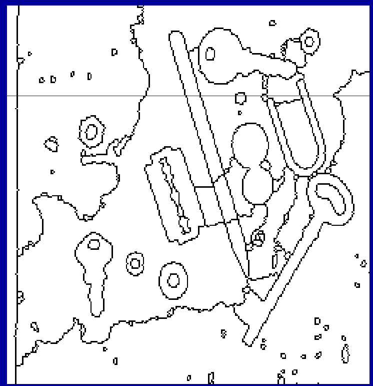
S_3



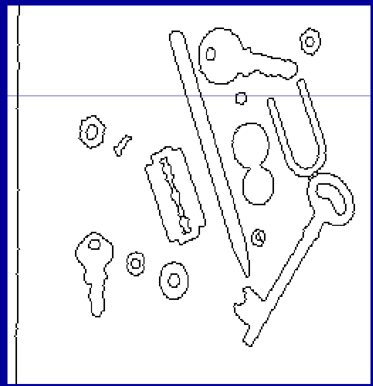
S_4



S_5



S_6

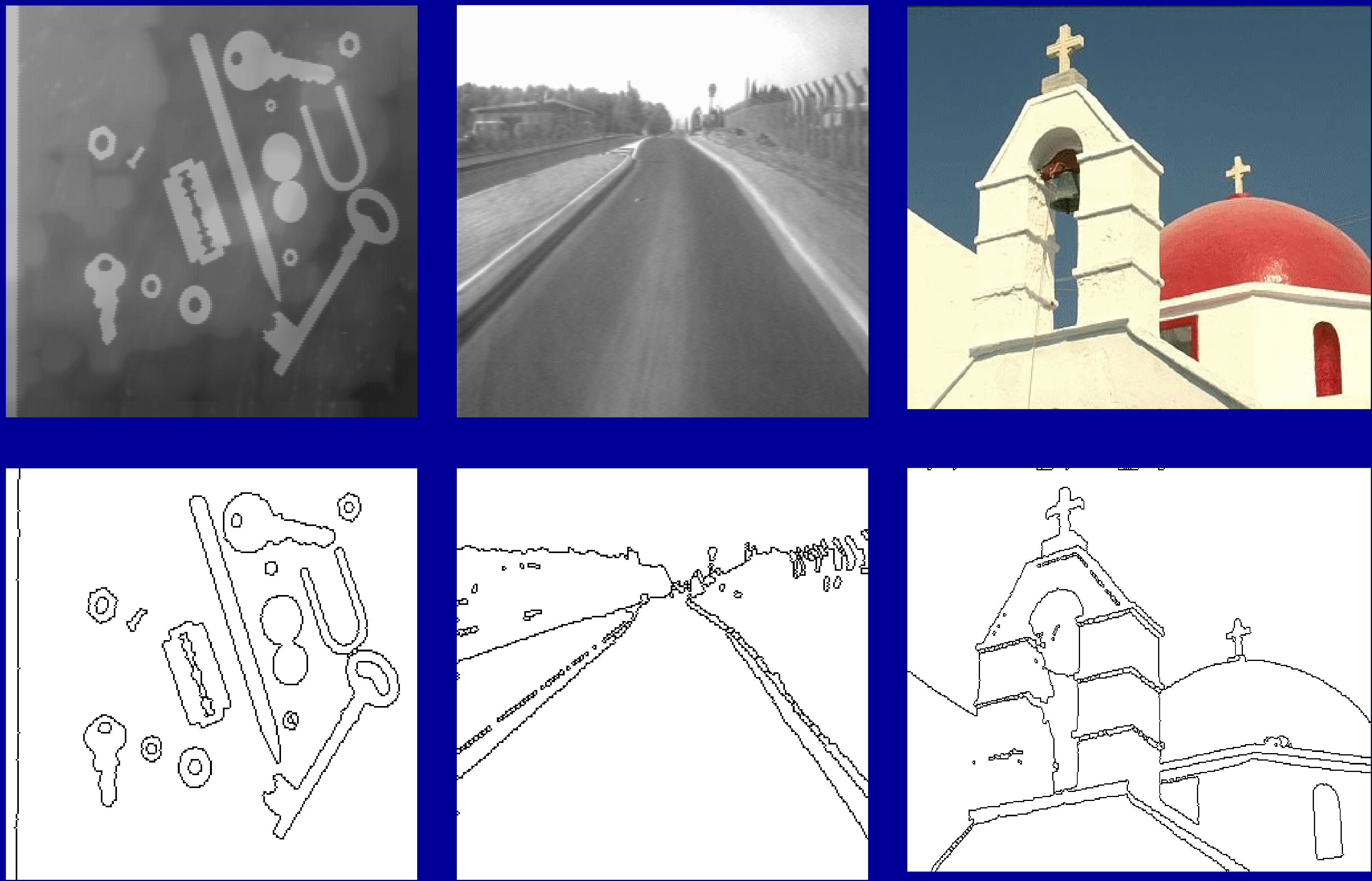


S_7

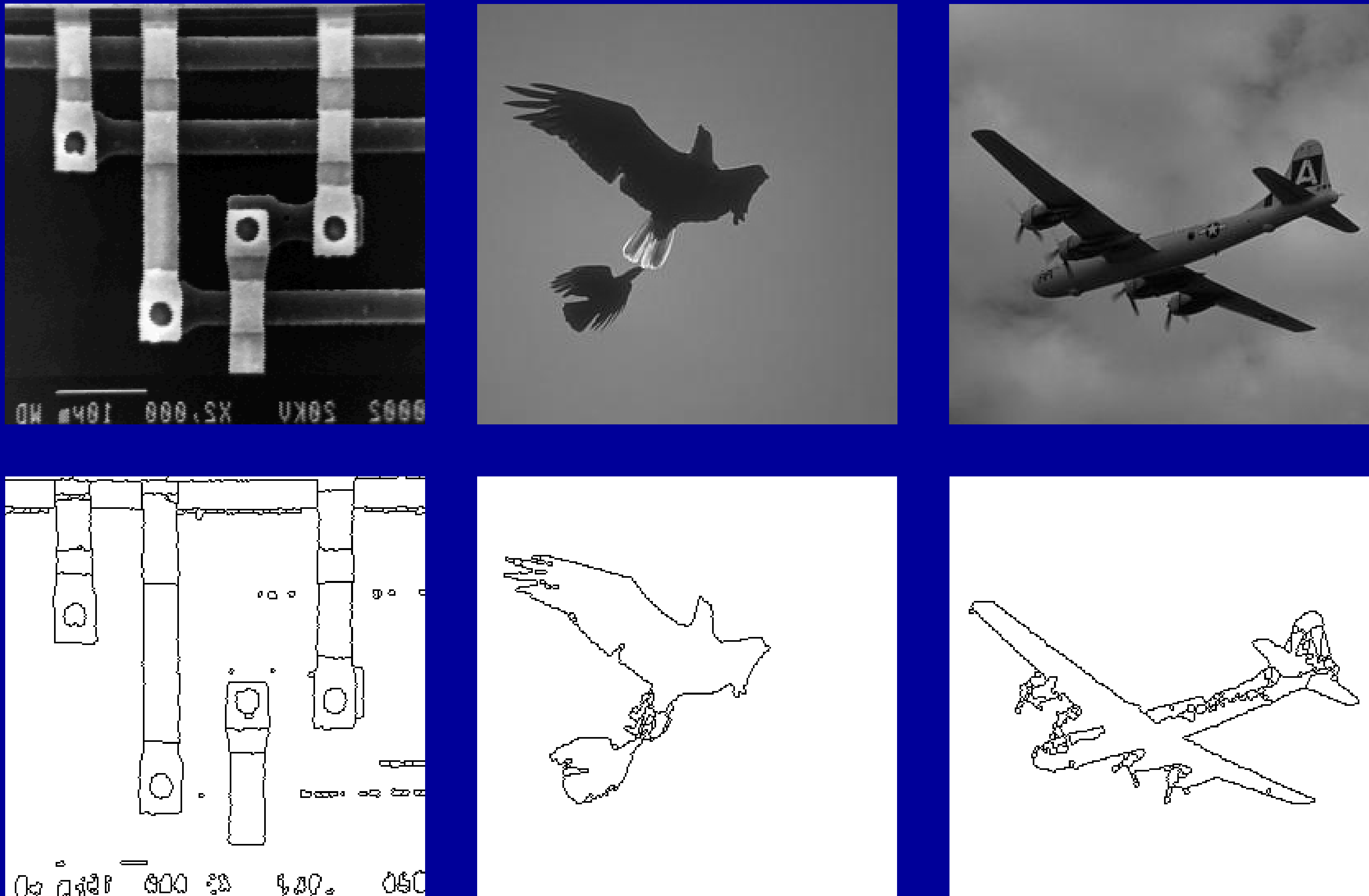
Occurrence of oscillating contours

- **Any periodicity may occur. Moreover, it can change during the process.**
- **However, the algorithm is self-locking (no infinite oscillation, at least for digital images).**

Examples of segmentations with P Algorithm

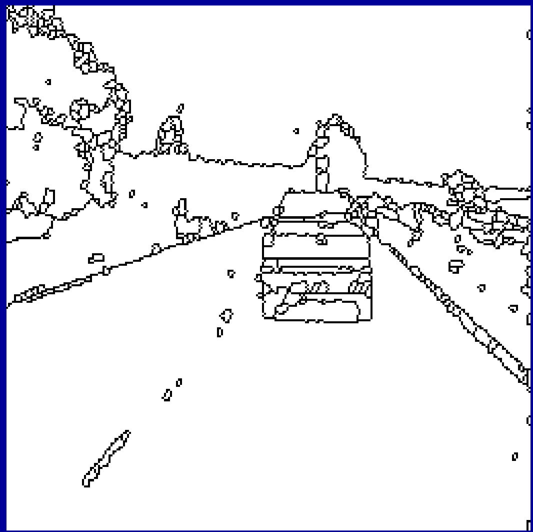
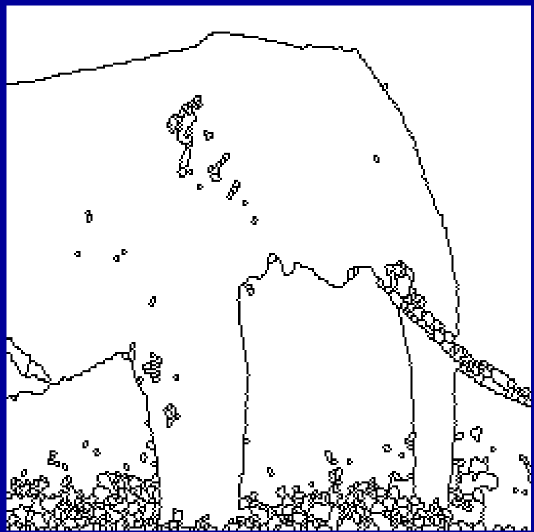


Examples of Segmentation (continued)



More Examples of Segmentation

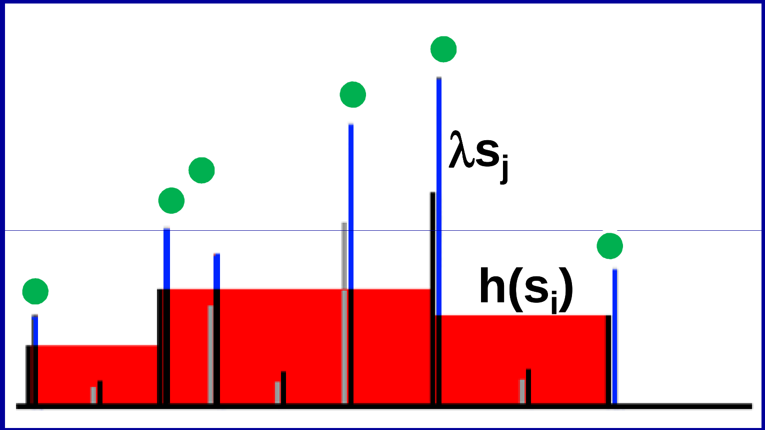
  

Towards a General Hierarchical Segmentation Algorithm

The previous generalisation can be extended by introducing a second parameter which defines the offset between the current segmentation s_i and an older one s_j (with I > j) which it is compared to.

- **Selection of the level of hierarchical segmentation s_j to be compared with the current hierarchical image $h(s_i)$. Sup(i – j, 0) is called the offset.**
- **The heights of the contours belonging to s_j are multiplied by a factor λ, called gain, and compared to the current hierarchy. Those which are higher define a mask m (green dots):**



$$m = \{x : \lambda(s_0 \wedge s_j) \geq h(s_i)\}$$

- **The marked contours are reintroduced in the current segmentation s_i:**

$$s'_i = m \wedge [h(s_i) \vee s_0]$$

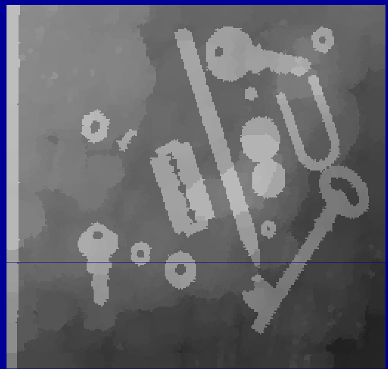
Offset maximum (the initial segmentation s_0 is always compared to the current hierarchy) and $\lambda = 2$ ⟹ P algorithm

Offset = 1 ⟹ Enhanced waterfalls and standard algorithm

(Operators and examples available in MAMBA)

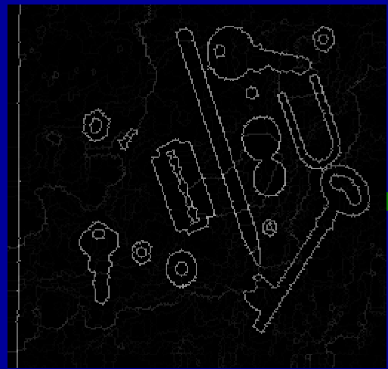
Hierarchical Segmentation and Thresholding

Regarding the last level of hierarchy, P algorithm, standard one and enhanced waterfalls boil down to a simple threshold of s_0 (followed by the removal of isolated contours).

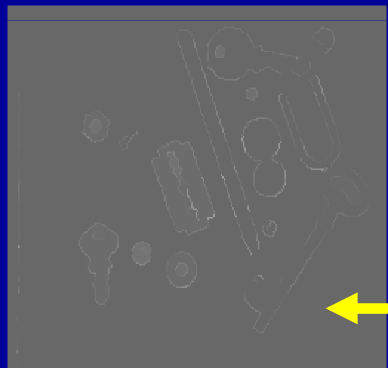


Initial image

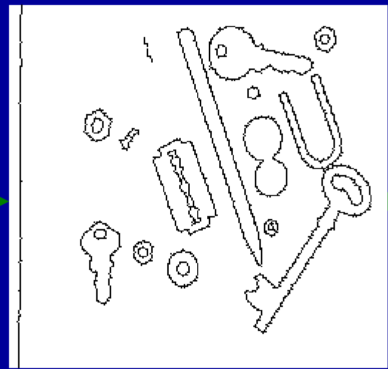
s_0



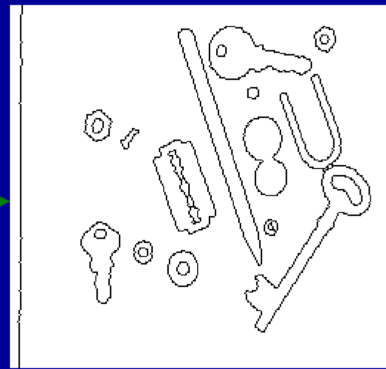
h_N



Height of minimum =52



Threshold of s_0 at value 26

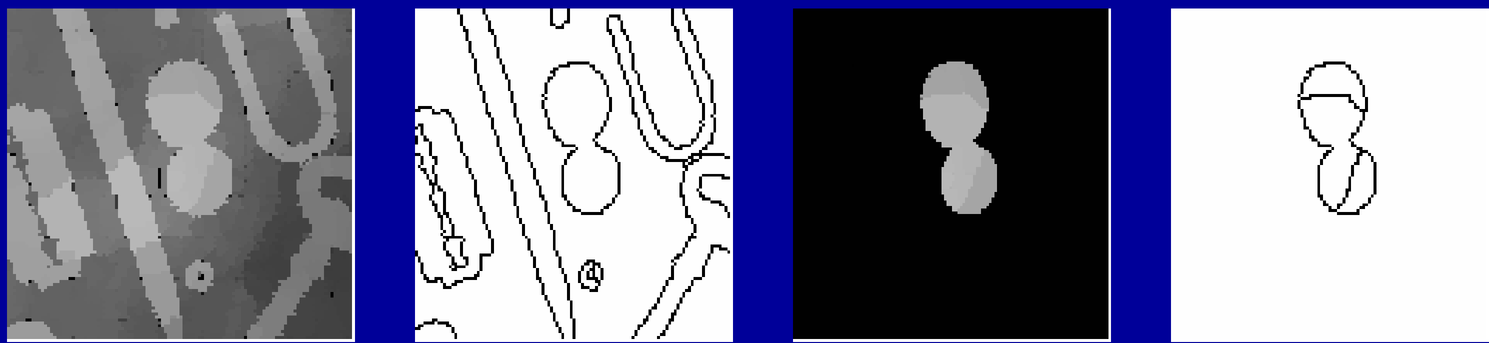


Clipping (Algo P)

The threshold value is determined automatically according to some saliency criteria (not entirely understood).
The same threshold is applied on the WHOLE image (and it can't be the other way….)

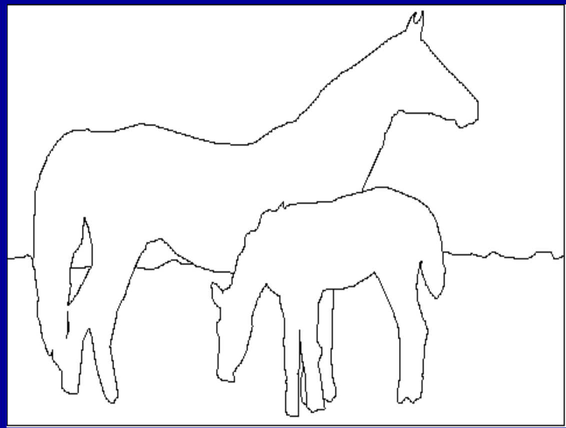
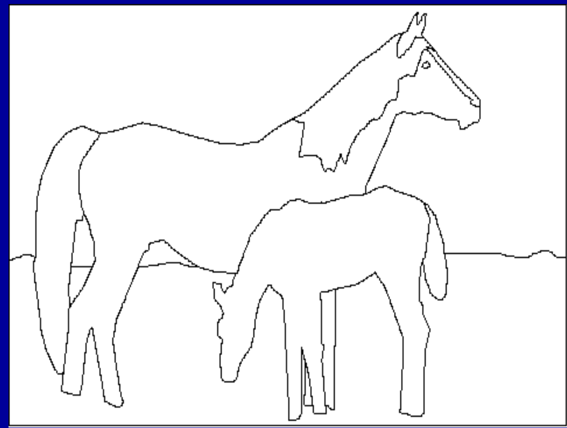
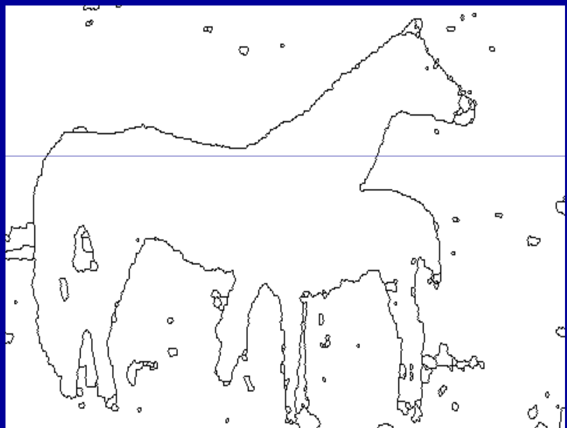
Contours Relevance

- Assume that the relevance of the contours can be (and must be) assessed locally by comparing them to their neighborhood.
- What are the size and shape of this neighborhood?
- It is likely that, the smaller the size of the neighborhood, the greater the number of relevant contours.
- At the end, if we assess the relevance of contours very locally, no doubt that all the contours of the initial watershed segmentation will be considered as relevant!



When restricting the field of analysis, contours inside the coins which where not relevant (2nd image) are finally preserved by P algorithm (4th image).

« Ground Truth » and Segmentation

On top, two human segmentations of the mare and foal image (Berkeley dataset) . Bottom image, result of P algorithm

- **P, standard algorithms are not, by any means, content-based segmentation tools.**

- **Comparing the results provided by these algorithms to human segmentations (Berkeley dataset for instance) is meaningless for at least two reasons:**

✓ a huge part of semantic knowledge is involved

✓ drawing needs to focus locally one's attention on the drawn contour.

Residues, a New Impulse for Morphological Image Segmentation

- Almost all the morphological tools used in supervised segmentation belong to the residual transformations class.
- These operators can also be used for unsupervised segmentation (pilings operators).
- Their main advantage lies on their capability to automatically adapt the parameters of the primitive transforms to the local characteristics of the image.
- The results provided by the residual operators are so efficient that the Watershed transform becomes… unnecessary.
- However, some operators are still slow. But new implementations are under study.
- The residual operators open new prospects in morphological image segmentation.

A Unified Hierarchical Segmentation Approach?

- All the hierarchical operators introduced here belong to a unique type of operators derived from the initial waterfalls transform.

- They are controlled by two parameters: the gain and the offset.
- Regarding the offset, results seem to be more efficient when it is equal to the infinite (P algorithm in particular).

- The gain brings also a way to cope with anamorphoses. P algorithm in particular is very sensitive to anamorphosis operators (Gamma correction for instance).

- These operators however are not very efficient with textured images (it is already the case for the watershed transform). Segmenting images where textured regions appear needs an a priori segmentation of these regions (by means of residual filters for example).

- Some of these operators are still slow (computation time greater than 40 ms).

P Algorithm and Visual Perception

- P algorithm and more generally segmentations based on an infinite offset have a remarkable capability: the foreground/background separation. This makes P algorithm a very powerful tool in perception. This principle is very important in Gestalt theory.

- Other principles of Gestalt are also fulfilled (closure principle for instance).

- Other phenomenons of visual perception may also be of primary importance (articulation, anchoring, color constancy, Weber-Fletchner law, etc.).

- It is a matter of fact that a lot of work still needs to be realised to better understand the underlying perception laws which are at stake in these operators.

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