MATHEMATICAL MORPHOLOGY AND GEOLOGY: WHEN IMAGE ANALYSIS USES THE VOCABULARY OF EARTH SCIENCE A REVIEW OF SOME APPLICATIONS

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ABSTRACT

Mathematical Morphology has proved to be a very powerful methodology in various applications of image analysis: industrial control, medical imagery, telecommunications, material sciences... This approach, however, has not yet fully entered the geological domain although many morphological tools can be described by referring to the vocabulary of earth science. A morphologist commonly uses concepts and tools named erosions, watersheds, catchment basins, flooding, levelings, crest lines, summits, ridges, valleys and so on.
The aim of this lecture is to present some of these morphological tools and, through some applications in petrography, geology and seismic data processing, to explain their use and their interest in various image processing tasks such as filtering, segmentation and feature extraction.
keywords: image analysis, image segmentation, geodesy, mathematical morphology, watersheds.

I. Introduction

Mathematical Morphology (MM) is a methodology of image analysis which provides a great variety of tools, some of them simple, others more or complex. However, their complexity seldom comes from the underlying concepts but from their algorithmic sophistication. It is a matter of fact that, although some morphological tools are very elaborated, most of them remain easy to handle because the morphologist can naturally describe their effects. This is far from being the case with many other image analysis transforms. This explains the success of this methodology: the end-user can see which kinds of modification are made on the images. Moreover, the appropriate tools can be chosen in the MM toolbox to emphasize and extract some relevant features from these images.
This paper aims at introducing this toolbox and, more specifically, at showing how the main morphological concepts can be easily visualized. Some examples of their application to the geological domain, especially to image segmentation, are also given.

II. Basic transforms in MM

Set theory is the mathematical background of Mathematical Morphology. All the morphological transforms are applied to sets and they produce new sets. MM was first used with 2D sets. But it can be used on 3D or any multi-dimensional sets as well.

II.1. Set transforms

The basic morphological set transformations use the concept of *structuring element.* A structuring element is a given set B which is used as a probe to collect information on the shape and/or size of the set X under study. Two basic operators can be defined: *erosion* and *dilation*.
Given a set X and a structuring element B with an origin, the erosion of X, denoted X0B (resp. the dilation of X, denoted X/B), is the set of all points x of the space where B_x (B translated in x) is included in (resp. hits) the set X.

II.2. Greytone transforms

When dealing with greytone images, the most suitable mathematical notion which describes them seems to be the notion of a function: with every point x of the image, we can associate a grey value f(x). So, how to extend the binary operators to greytone images? In fact, it is simple if we consider *the set under the graph* of the function f. The graph G(f) of the function can be viewed as a topographic surface. All the points of the 3D space belonging to the graph or under the graph define a 3D set, the set under the graph D(f). It is therefore easy to extend the basic morphological transforms to functions. It suffices to erode or dilate the set D(f) to produce a new set D(f)0B or D(f)/B. It can be shown that these sets correspond to the sets under the graph of two new functions, namely f0B and f/B.
Most of the time, in greytone morphology, the structuring elements are flat (defined in R^2): thus, the corresponding operators are compatible with affinity (modification of the light intensity of the image) and the transformed images remain in the same range (no overflow).
The main advantage of morphological transforms is the fact that there is no change of space (starting with sets, they produce sets). This allows to see immediately the effect of a transformation. Moreover, it is possible to define more sophisticated transformations by iterating basic ones. This technique is used in particular to define *openings* and *closings*.

III. Openings, closings, morphological filters and granulometry

III.1. Opening and closing

The opening (in its basic morphological meaning and with symmetric structuring elements) is simply an erosion followed by a dilation:

$$X_B=(X0B)/B \; ; \; f_B=(f0B)/B$$

Conversely, a closing (X^B or f^B) is a dilation followed by an erosion.
Applied to functions, the effects of openings and closings are straightforward: an opening erases the domes, peaks and ridges of the corresponding topographic surface whereas a closing fills in the valleys and the pits. Openings and closings belong to a specific category of transformations, the *morphological filters*. These non linear filters have been widely studied recently. Among their interesting properties, the most important is the *idempotence* (iterations of the transform do not modify the initial result).

III.2. Size distributions for sets and functions

Openings and closings share another interesting feature: when used with convex structuring elements, they have sieving properties. This leads to the notion of *granulometry* for sets but also for functions. Let B_λ be a sequence of structuring elements depending on a size parameter λ. The successive openings and closings of a set X (or a function f) sieve X or f according to the size λ of the structuring elements. Note that these size distributions are not related to the notion of particle since they apply to grains as well as pores and even greytone images.

III.3. Residues and derived concepts

All the parts of the initial set or function which have been removed by an opening or added by a closing are called *residues*. The residues extraction allows to introduce new operators (especially for functions) called *top-hat transformations*. These operators are powerful extractors of salient features of the topography visible in the image: edges, peaks, crest lines, etc. Other image analysis concepts such as *skeletons* are also derived from these residues.

IV. The notion of geodesy

The notion of *geodesy* and of *geodesic transformations* is another powerful concept in MM. When using isotropic structuring elements (balls or disks in R^2), it is equivalent to defining the erosion and the dilation by means of boolean operators or by means of distances. The erosion of X by a ball B of radius λ can be written:

$$X0B=\{x: \ldots y,\ d(x,y) [\ \square\ \varepsilon\ y\ \chi\ X\}$$

In this case, the distance d is the *euclidean distance*. However, another distance can be used, in particular the *geodesic distance*, which leads to new morphological operators, the geodesic transforms.

IV.1. Geodesic distance and geodesic transforms for sets

Given any set X (called the geodesic space), one can define the geodesic distance $d_X(x,y)$ between two points x and y belonging to X as the length of the shortest path included in X linking x and y. Using this distance in the previous definitions produces two new transformations, the *geodesic erosion* $E_X^\lambda(Y)$ and the *geodesic dilation* $D_X^\lambda(Y)$. For instance:

$$D_X^\lambda(Y)=\{x\chi X: \ |y\chi X,\ d_X(x,y)[\lambda\ \varepsilon\ y\chi Y\}$$

Compared to the euclidean operators, these transformations re-introduce the concept of a connected component and they can be seen as morphological transformations using soft structuring elements which follow the local «curvatures» of the geodesic space X.

IV.2. Reconstruction

What happens when we iterate indefinitely the geodesic dilation of a set Y in the geodesic space X? The final result is made of all the connected components of X that contain points of Y. This transformation is called a *reconstruction*, because the connected components of X which are marked by Y are reconstructed.

IV.3. Geodesic transformations for functions

When applied to functions (or equivalently to the 3D sets under the graph), the geodesic transformations may be trivial. It is in particular the case for the reconstruction because the reconstruction of any function f by any function g (g[f) always gives f if 3D structuring elements are used. This is why we use *flat structuring elements* in the definition of the geodesic operators for functions. It can be shown that it is equivalent to modifying the definition of the geodesic distance in this special case: the projected length of the shortest *monotonous* path (never ascending or never descending) between two points of D(f) gives the geodesic distance between these two points. The geodesic reconstruction of functions is widely used to erase the summits or fill the basins of a function whatever their size. It is also an efficient and simple tool to extract the *minima* or the *maxima* of a function.

IV.4. Geodesic filters by reconstruction

Very powerful geodesic filters can also be defined with the reconstruction. The most common is called *opening by reconstruction*. It is made of an erosion followed by a geodesic reconstruction. Applied to sets, these filters often give better results mainly because they take into account the different connected components of the sets. Therefore, their behavior seems more natural. Applied to functions, they act as efficient tools for filling in the deep and basins of the topographic surface drawn by the function.

V. Watershed transform and segmentation

V.1. Definition

The *watershed transformation* is a morphological tool whose definition is directly derived from the topographic representation of a function. Considering a function, its topographic surface can be flooded, starting from its minima (imagine we «cut» the minima of this topographic surface and that we plunge this «open» surface into water). The watershed lines correspond to the dams which are built to separate waters coming from different minima as the flooding process goes on. This transformation partitions the function into two sets: the *catchment basins* and the *watershed lines*. Many algorithms have been designed to achieve this transformation, but the most efficient and robust use geodesic transformations.

V.2. Use of watershed in segmentation

The watershed transformation is a highly efficient tool in image segmentation. Segmenting a picture is a task which in most cases (but not always) consists in partitioning the initial picture into more or less homogeneous regions. These regions are characterized by a minimum value of the *gradient modulus*. Therefore, when considering the topographic surface of this gradient modulus, the homogeneous regions correspond to the catchment basins of this gradient. This is why the *gradient watershed* is the basic morphological tool for segmentation.

V.2.1. The segmentation paradigm

Unfortunately and because images are noisy and homogeneous regions not so homogeneous, the watershed transformation often produces a severe over-segmentation. To avoid it, instead of flooding the gradient starting from its minima, we prefer to use selected regions called *markers* as initial sources of flooding. A *marker-controlled watershed* applied to the gradient leads to a segmentation where only the marked regions are segmented. The markers themselves must be designed or chosen before applying the watershed transformation. This general procedure constitutes the *morphological segmentation paradigm.*

V.2.2. Segmentation and hierarchisation

However, finding appropriate markers is sometimes difficult and over-segmentation cannot be avoided. But it is then possible to merge the various catchment basins according to some criteria. This merging can be iterated, each level of iteration providing a level of *hierarchy* of the segmentation. Among these criteria, let us quote two of them: the *dynamics* and the *waterfall algorithm*. Here again, both can be explained in terms of paths on the topographic representation of a function.

VI. Some applications of MM in Earth sciences

Three applications of morphological segmentation are presented: segmentation of a petrographic microscopic plate, segmentation of electrical loggings and an example illustrating the use of MM in seismic data processing.

VI.1. Color segmentation of cristalline components in a petrographic plate

In this study, the initial data consists in a sequence of images of a petrographic section taken with a polarized microscope. The combination of the various gradient images and the use of the three color channels are associated to design an automatic segmentation algorithm based on the watershed transform.

VI.2. Electrical resistivity loggings analysis

The pictures under study represent resistivity measurements along borings. The purpose of the segmentation process is to emphasize some features appearing in this logs, in particular the «vugs» (holes filled by clay) and the bedding. The reconstruction and other transformations based on directional gradients are very helpful to achieve this segmentation.

VI.3. Seismic data processing

Some feasibility studies have been performed on these data. It is shown in particular how some semi-automatic profile pickings could be achieved. The use of openings and closings by reconstruction provides also efficient techniques to extract the most important features. It is also possible to iterate these tools in order to build a hierarchy of the visible structures. These tools applied here on 2D seismic data could also be applied on 3D data with minor modifications.

VII. Conclusions

Many factors contribute to make the use of morphological tools very intuitive:
- the effects and properties of the operators can be described in natural language.
- the visualisation of the transformations is made easy thanks to the topographic analogy.
- the fact that no change of space occurs allows the iteration of the transforms.

All these factors explain undoubtedly the increasing success of MM as an image-based investigation tool in natural sciences. Furthermore, efficient and cheap implementations of these tools now exist. For all these reasons, MM should continue to provide elegant solutions to image analysis problems in particular in the Earth sciences field.

VIII. References

BEUCHER, S. 1990. Segmentation d'images et Morphologie Mathématique. Doctorat thesis, School of Mines, Paris, 295 pp.

BEUCHER, S. 1994. Watershed, hierarchical segmentation and waterfall algorithm. In: Serra, J. & Soille, P. (Eds.) Proc. Mathematical Morphology and its Applications to Image Processing, Kluwer Ac. Publ., Nld, 69-76.

BEUCHER, S. 1995. Micromorph, un logiciel d'apprentissage de la Morphologie Mathématique. Journées d'Etudes INRP/CNAM, Images numériques dans l'enseignement des sciences, 77-80.

BEUCHER, S., LANTUEJOUL, C. 1979. Use of watersheds in contour detection. International Workshop on image processing, real-time edge and motion detection & estimation. Rennes (France).

BEUCHER, S., MEYER, F. 1992. The morphological approach to segmentation: the Watershed transformation. In: Dougherty, E. (Ed.) Mathematical Morphology in Image Processing. Marcel Dekker, N.Y., 433-481.

GRIMAUD, M. 1992. A new measure of contrast: the dynamics. Proc. SPIE, Image Algebra and Morphological Image Processing III, San Diego, 1769, 292-305.

LANTUEJOUL, C., BEUCHER, S. 1981. On the use of the geodesic metric in image analysis. Journal of Microscopy, 121-1, 39-49.

MATHERON, G. 1967. Eléments pour une théorie des milieux poreux. Masson, Paris.

RIVEST, J.F., BEUCHER, S., DELHOMME, J.P. 1992. Marker-controlled segmentation: an application to electrical borehole. Journal of Electronic Imaging. 1-2, 136-142.

SERRA, J. 1982. Image analysis and Mathematical Morphology. Academic Press, London, 610 pp.

SERRA. J. 1988. Introduction to Morphological Filters. In: Serra, J. (Ed.) Image Analysis and Mathematical Morphology, Theoretical Advances. Academic Press, London, 101-114.