SETS, PARTITIONS & FUNCTIONS INTERPOLATIONS

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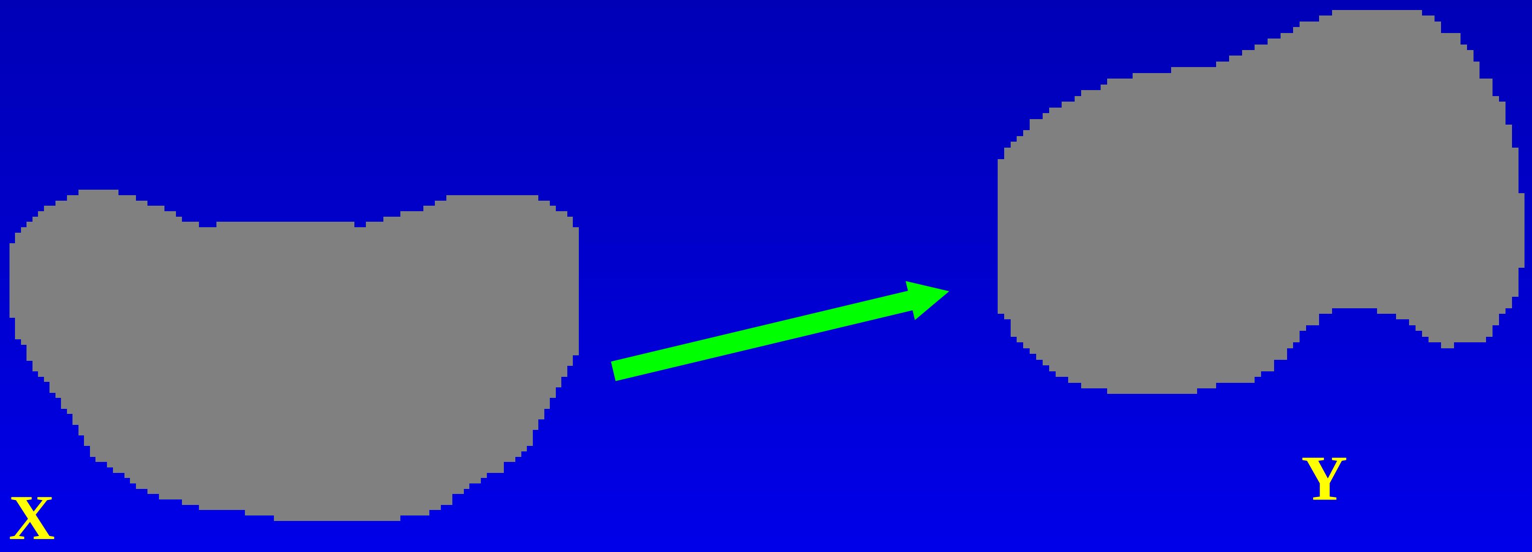
Introduction

Purpose of the interpolation:
Finding ways to continuously map a set X on a set Y:



This study belongs to a triptych:
- **Interpolation and Haussdorff distance J. SERRA (ISMM98)**
- **Interpolation by means of geodesic distances F. MEYER**
- **Interpolation using the SKIZ (Skeleton by Influence zones)**

Sets interpolation (1)

The SKIZ

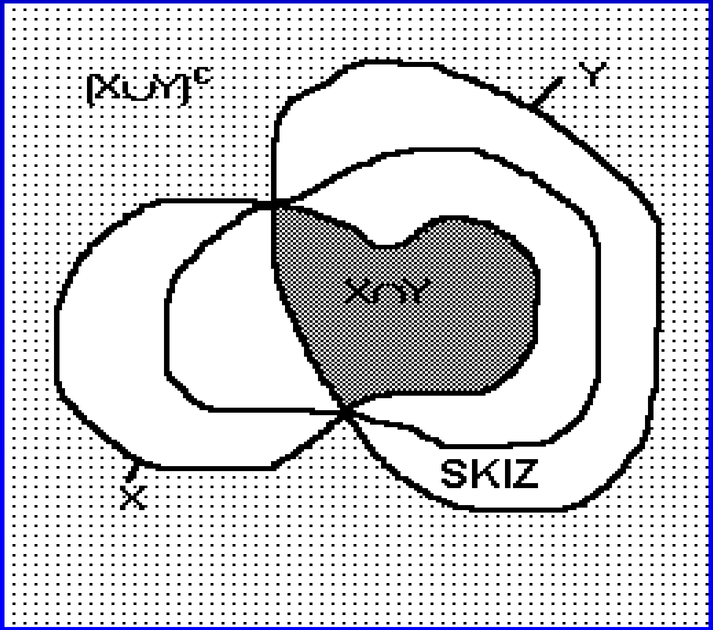
Let X and Y be two sets with $X \cap Y \neq \emptyset$

Consider the set W made of two components:

$$W = \{ X \cap Y , (X \cup Y) \}$$

Let us build SKIZ (W)

M(X , Y) = IZ ($X \cap Y$) is the zone of influence of $X \cap Y$



M (X , Y) is the interpolated set (or median set) between X and Y by SKIZ.

Sets Interpolation (2)

Sequence of deformations

$$K_0 = X \quad ; \quad K_n = Y \quad \textbf{with} \quad n = 2^i + 1$$

$$K_{n/2} = M(X, Y)$$

$$K_{n/4} = M(X, K_{n/2}) \; ; \; K_{3n/4} = M(K_{n/2}, Y)$$

The SKIZ is not performed by thickenings but by simple dilations:

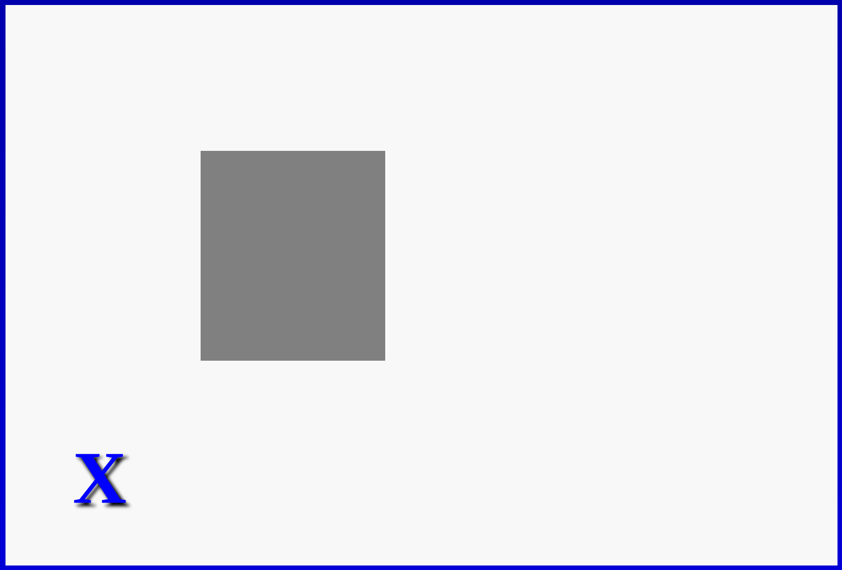
$$Z_0 = X \cap Y \quad \textbf{and} \quad W_0 = (X \cup Y)^c$$

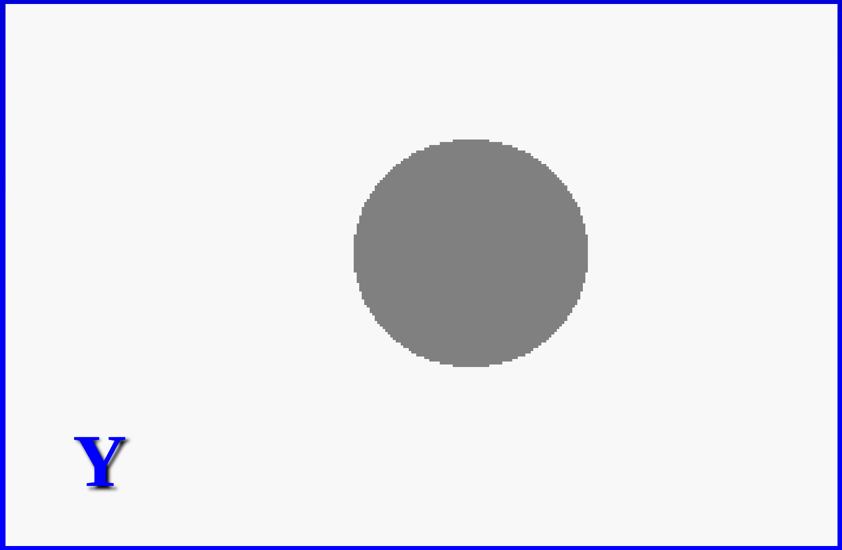
$$Z_i = [(Z_{i-1} \oplus B)/(W_{i-1} \oplus B)] \cup Z_{i-1}$$

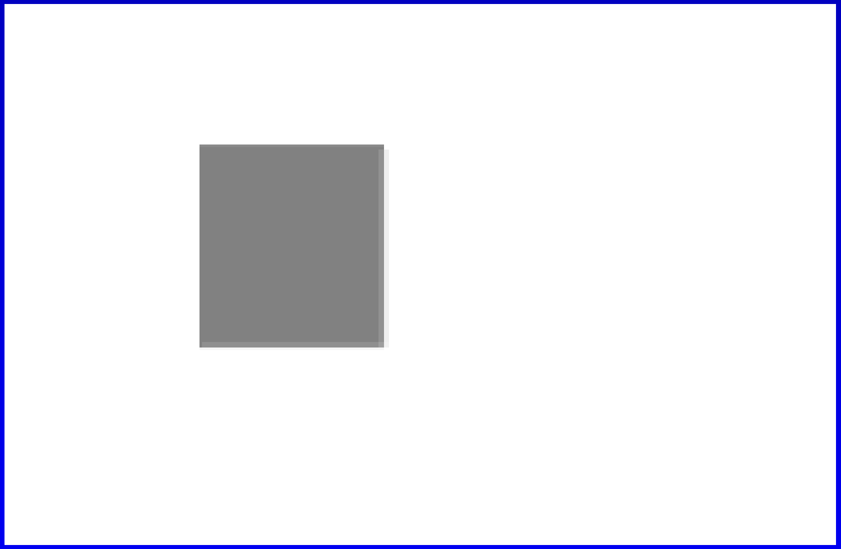
$$W_i = [(W_{i-1} \oplus B)/(Z_{i-1} \oplus B)] \cup W_{i-1}$$

Then $\boxed{M(X, Y) = Z_\infty}$

Example





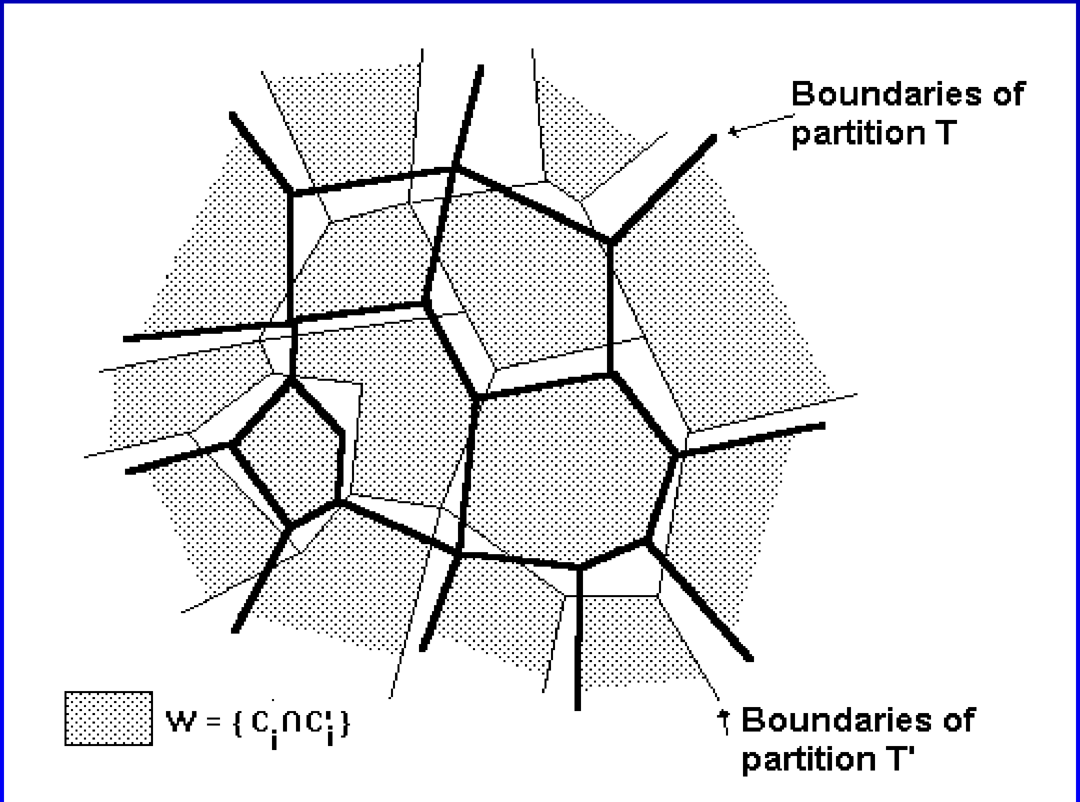


Interpolated sequence

Interpolation of partitions

Let T and T' be two partitions: $T = \{C_i\}; T' = \{C'_i\}$

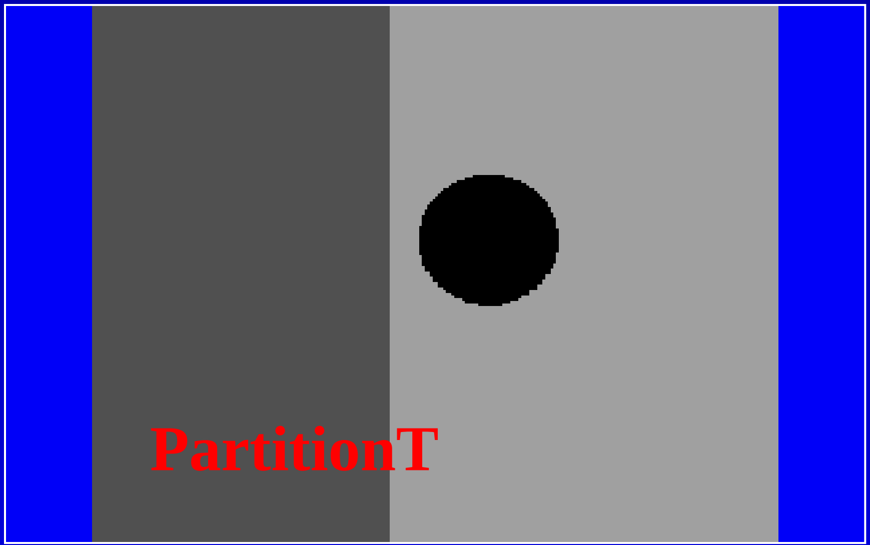
$$\forall C_i \in T, \exists C'_i \in T' : C_i \cap C'_i \neq \varnothing$$

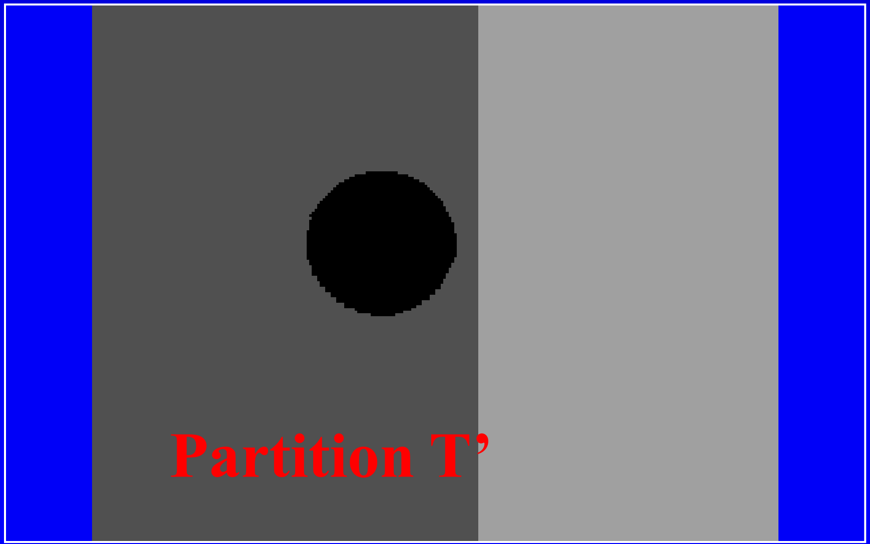


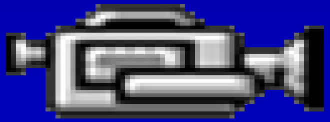
$W = \{C_i \cap C'_i\}$ **and** $M(T, T') = \{IZ(C_i \cap C'_i)\}$

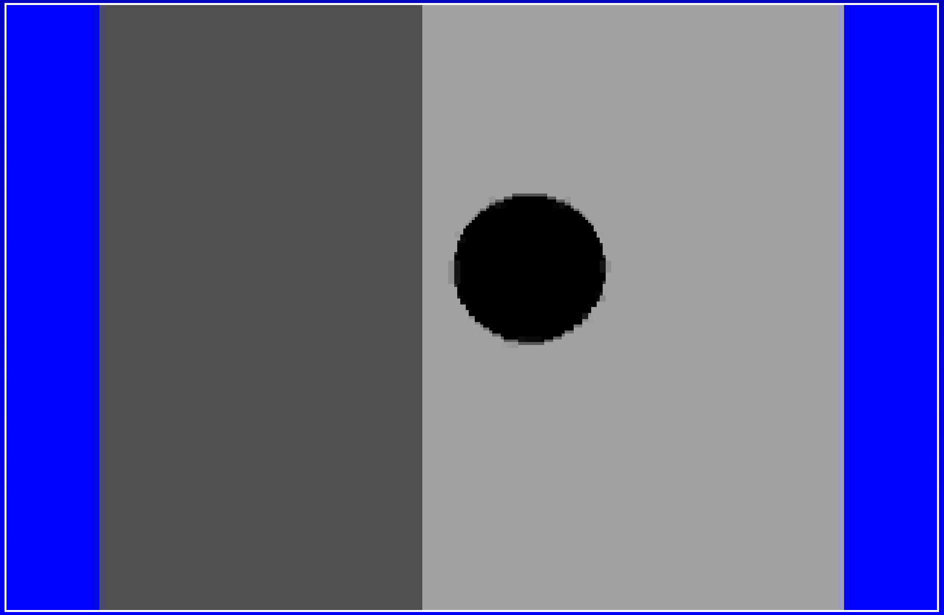
We can generate partitions with or without boundaries.

Examples (1)



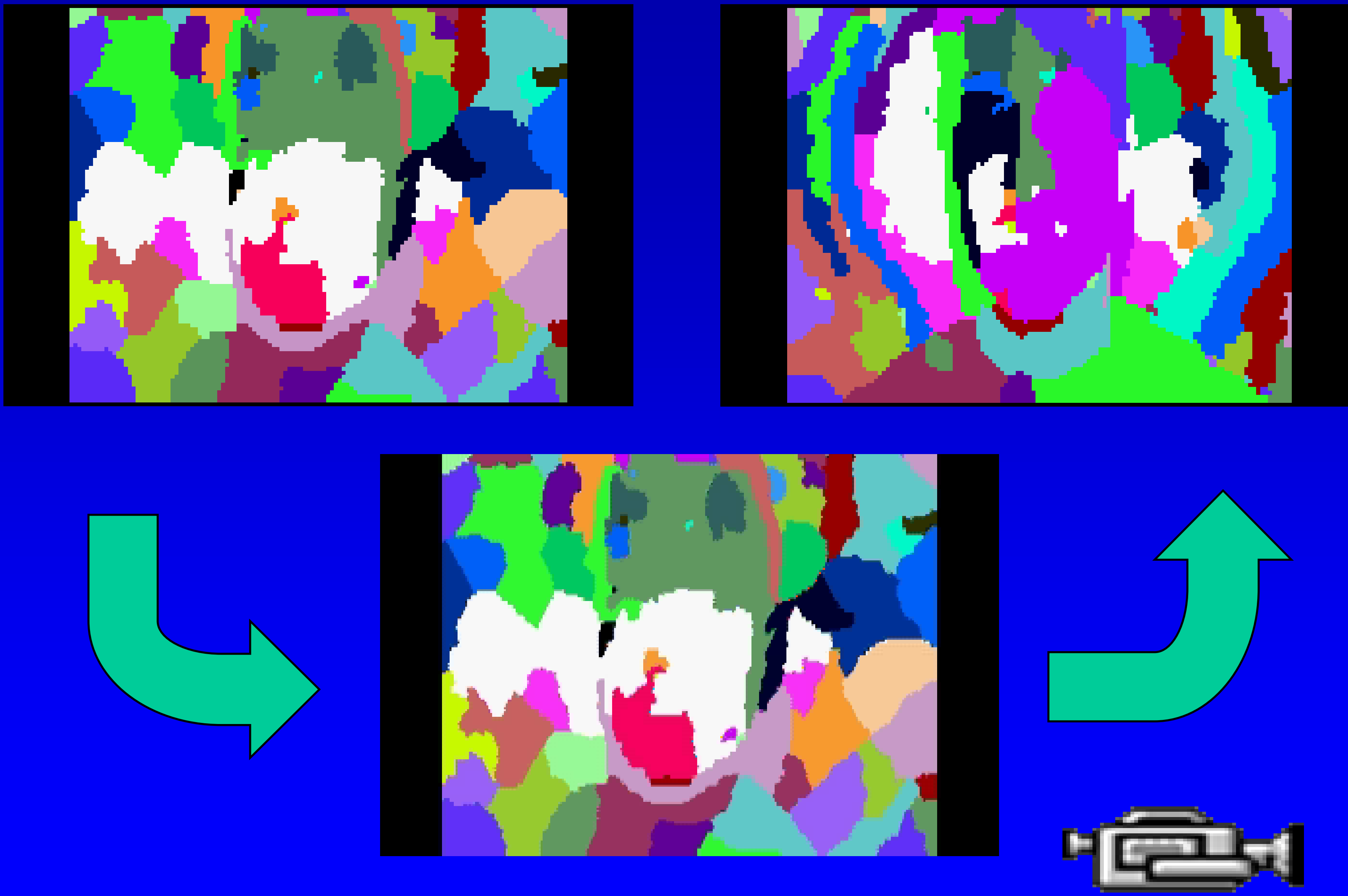






Interpolated sequence

Examples (2)



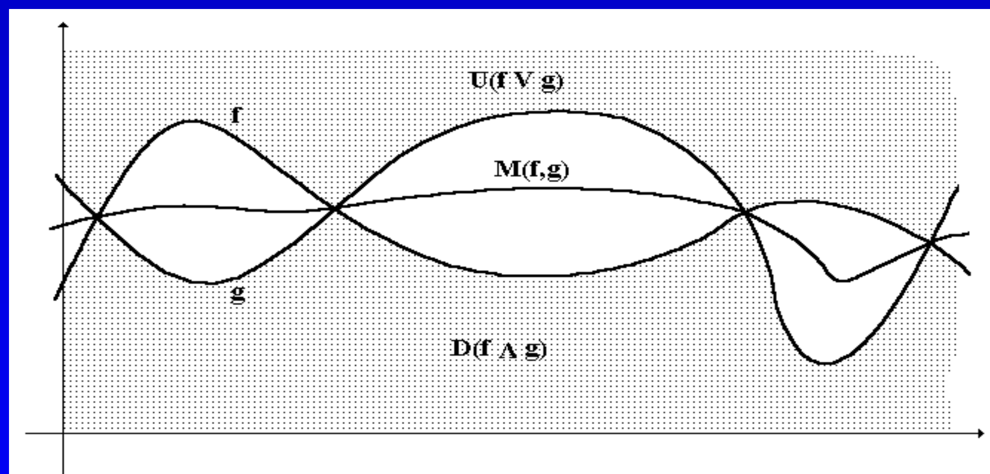
Interpolation of functions

Let f and g be two functions:

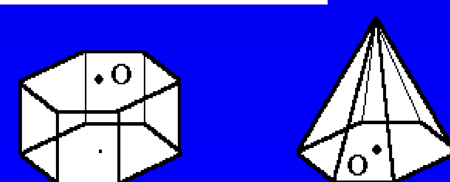
$D(f \wedge g)$ **Set under the graph of Inf(f,g)**

$U(f \vee g)$ **Set over the graph of Sup(f,g)**

$$W = \{D(f \wedge g), U(f \vee g)\} \quad D[M(f,g)] = IZ[D(f \wedge g)]$$

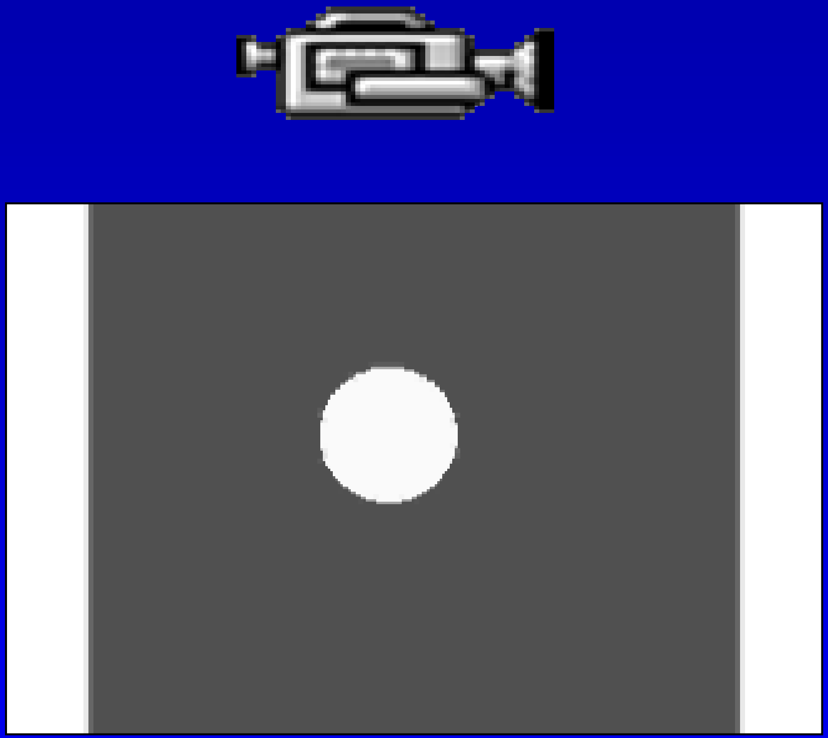


The transformation uses dilations by 3D structuring elements.

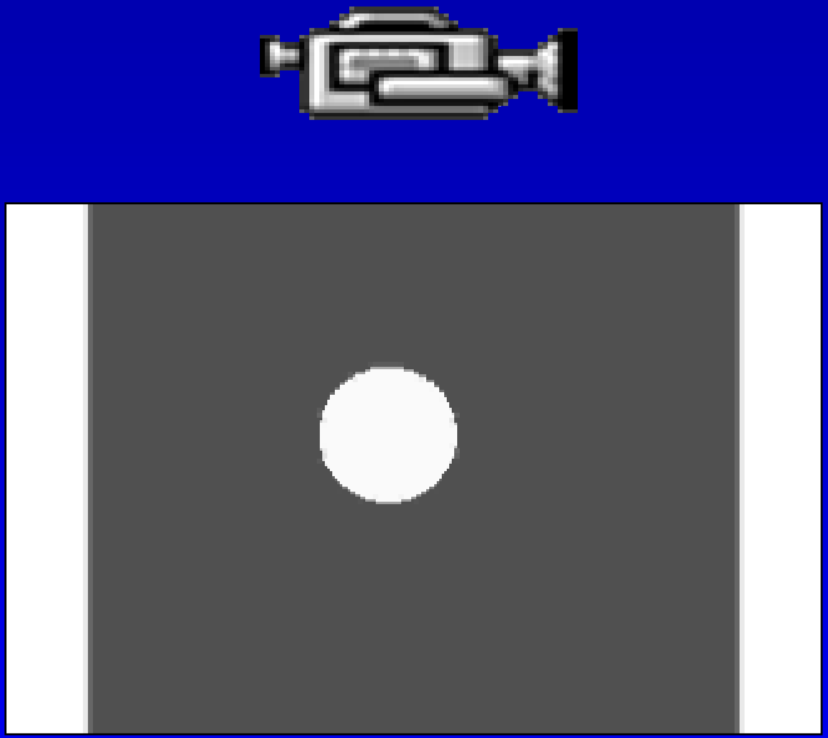


→ **Smoothing effect.**

Examples (1)



Linear (mean)



SKIZ

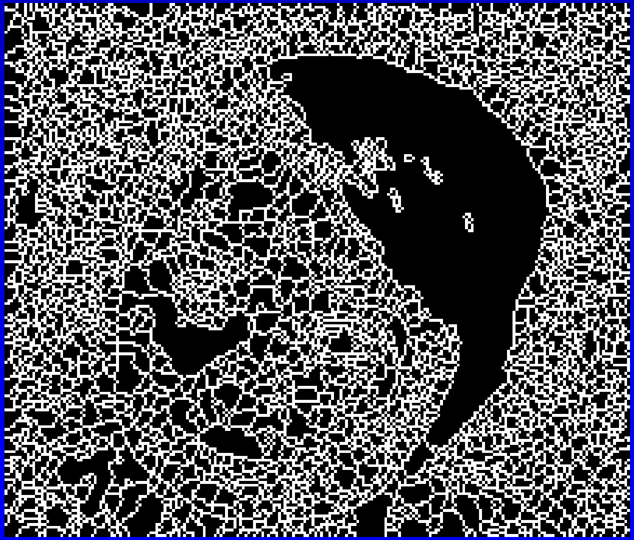
Comparison between the SKIZ-based and the linear interpolation

Examples (2)

Partitions & mosaic images

Another technique to interpolate greytone images is to use the mosaic representation and to interpolate these two mosaic images as partitions.

Original image



Watershed of the gradient

Mosaic image

Examples in compression

Original Miss America sequence (of mosaic images)

Only 1 image over 8 is kept, the others are interpolated.

Generalisation: deformation fields (1)

From X and Y, we have defined a sequence $M_\lambda(X,Y), 0 \le \lambda \le 1$ **of intermediary sets.**

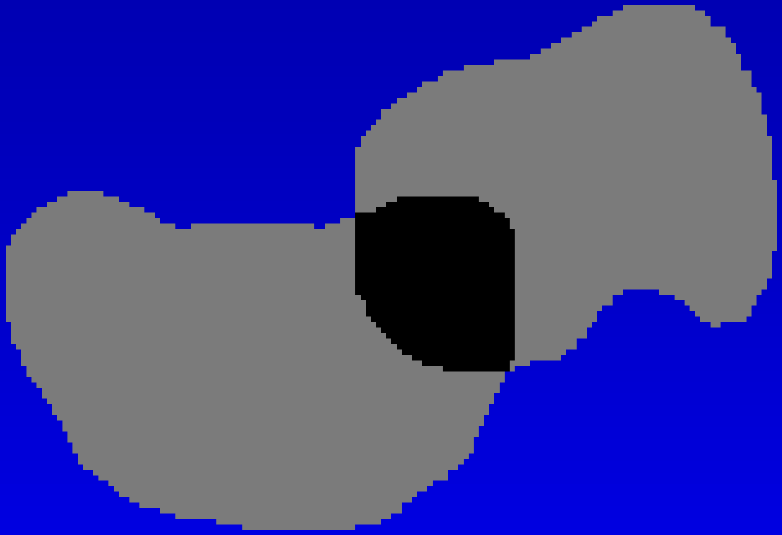
These sets are made of two parts:

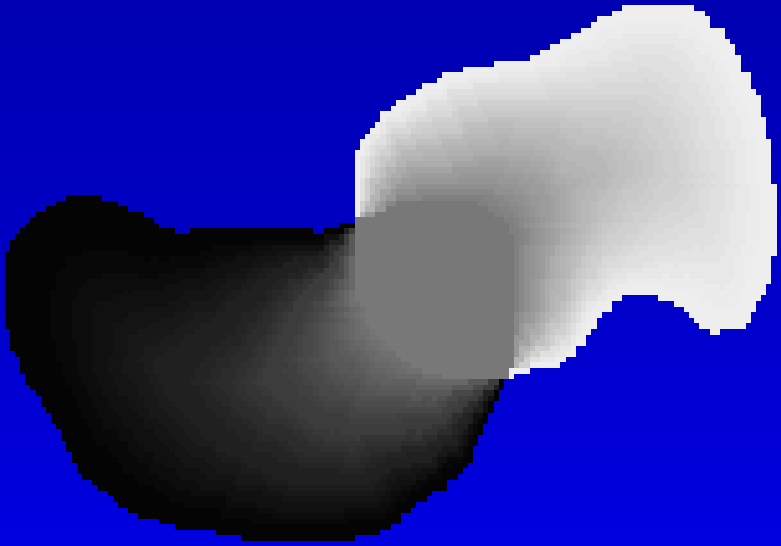
- **An anti-extensive one** $M_\lambda^-(X,Y)$
- **An extensive one** $M_\lambda^+(X,Y)$

These two sequences can be considered as thresholds of two distance functions:

$$M_\lambda^- = \{x \in X : d^-(x) \ge \lambda\}$$
$$M_\lambda^+ = \{x \in Y : d^+(x) \le \lambda\}$$

Deformation fields (2)



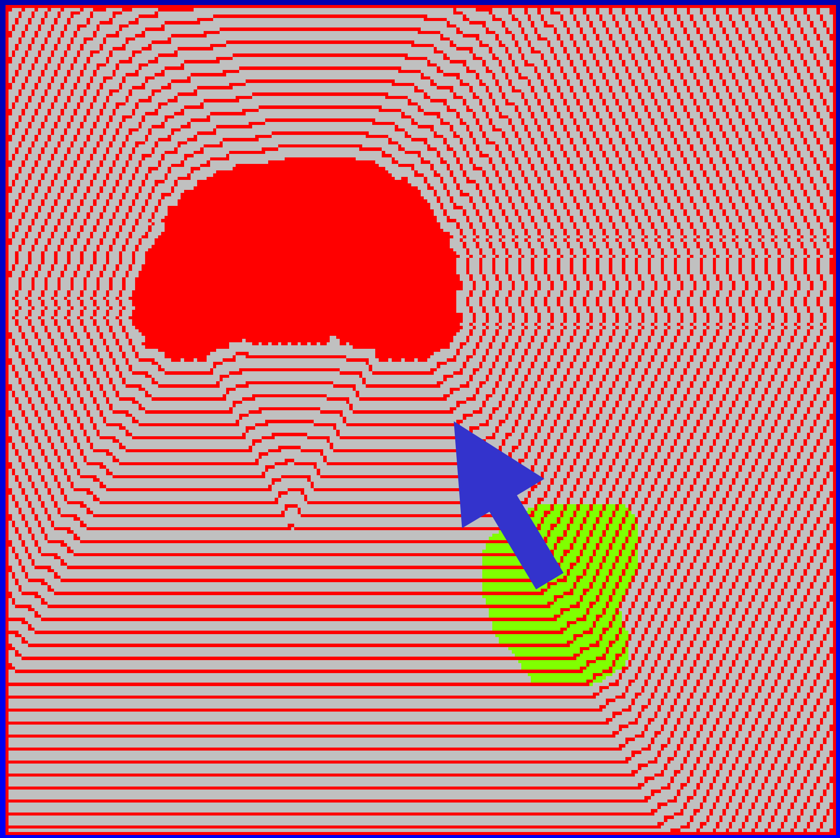


Example of such a distance function (and the corresponding field)

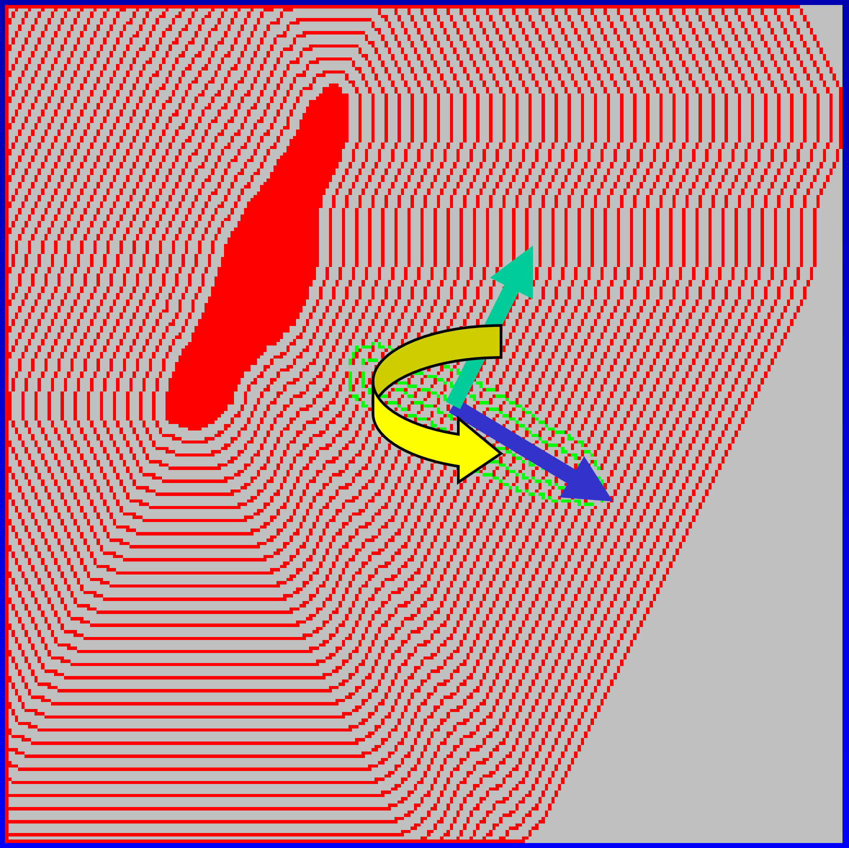
The SKIZ can be considered as a generalized geodesic transformation controlled by the field.

Deformation fields (3)

Other deformations could be controlled by a generated field.



Translation and distance function



Rotation and vector product of two distance fields