DIRECTIONAL FIBRE ANALYSIS BY MATHEMATICAL MORPHOLOGY

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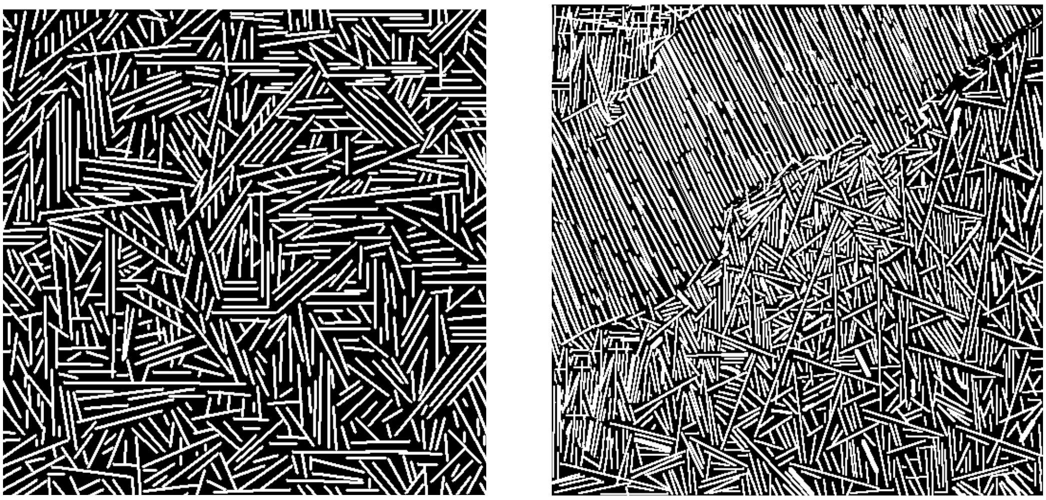
ABSTRACT

One of the problems faced by image analysis and mathematical morphology is the analysis of the directions of fibres. This concept is to be understood both as the determination of the direction of individual fibres as well as the detection of structure areas with uniform orientation of fibres (Kurdy, 1990). Fibre analysis is of vital importance in those fields of study in which materials of fibrous structure occur, e.g. in medicine, biology, metallurgy, geology etc. It plays also a substantial role in the process of quality inspection (e.g. of metallurgical products), as the direction of fibres often essentially affects the physical properties of the material. The authors of the present study suggest the application of mathematical morphology tools for the description of the direction of individual fibres, as well as for the detection of areas of similar fibre orientation. The investigations were conducted using binary source images. These images were subjected to preliminary filtering and thinning. A directional erosion by a pair of points oriented in 6 or 9 directions was performed. The corresponding grey level was assigned to the result of each erosion. A grey image was obtained, encoded in such a way that each grey level corresponded to a different object direction. However, this process does not allow to obtain fibres coded uniformly along their entire length. Therefore algorithms for the description of each fibre through one variable have been developed. These algorithms allow to analyse individual fibres as well as to detect the areas of similar orientation of fibres. Algorithms for such cases are presented and the results illustrated on several examples.

Key words: mathematical morphology, image analysis, fibre analysis, segmentation.

OBJECTIVE OF THE INVESTIGATION.

The starting points for studying the problem was the necessity to analyse fibrous structures shown in Fig. 1a, 1b. In these figures one can observe rectilinear fibres running at random - without any preferred direction (Fig. 1a), besides fibres which, on some parts of the image, are arranged in parallel (Fig. 1b). Such arrangement of the fibres is the evidence of defects and may lead to a reduced strength of the material. The objective of the study was the design of a morphological algorithm which could enable automatic identification of these areas.



a) b)

Fig.1. Examples of fibrous structures.

OUTLINE OF THE PROPOSED METHOD.

In the algorithm proposed in this paper, it is assumed that after a preliminary filtering and thresholding of the fibres, the skeleton of the image is calculated (Serra 1982). As a result a line, one pixel thick, is obtained, the direction of which corresponds to the direction of the object. Then, an erosion by a pair of points of direction *d* and size *s* of the resulting image is performed. As a result of this erosion parts of the object with a direction different from the direction *d* of the structuring element are removed. Ideally, if the analysed fibre is in the direction *d*, then each erosion by a pair of points in a direction k, k≠d, gives an empty set. Only the erosion in direction *d* will leave the fibre almost unchanged (only shortened by *s*).

DIRECTIONAL EROSION.

When performing an automatic image analysis one must be aware of the limitations imposed by the method on the user. One of these limitations is the number of basic directions which can be distinguished on a given grid. The most frequently used grids are the square or hexagonal ones. In the case of a hexagonal grid for instance, 3 main directions can be defined. In other words, when using a structuring element of size 1, 3 independent erosions only can be performed (the 3 remaining eroded sets are reduced by 1 pixel with respect to the previous ones, the edge effects being neglected). In order to obtain a greater number of independent erosions, a structuring element of size greater than 1 must be used. The maximum number of independent erosions depends thus on the size *s* of the chosen structuring element and is equal respectively to 3*s* for an hexagonal grid and 4*s* for a square one.

All the applications presented in this paper were performed on an hexagonal grid.

CODING OF DIRECTIONS.

The aim of this section of the process is to ascribe to each fibre a single value which will characterise its direction in the plane. The most obvious variable which can be used for this purpose is the grey level. It is to be noticed that encoding of the fibre direction by means of a single variable is justified only in the case when this direction is strictly defined. Such an approach is useless when the shape of objects (circle, ellipse, curved fibres, etc.) does not allow such a definition. Let us assume that the direction of an arbitrary fibre is to be analysed. After thresholding and skeletonizing (Salambier, Serra 1994), erosions by a pair of points of size *s*, oriented in all independent directions are performed. As a result, $3s$ independent binary sets are obtained. Then, each set is assigned its individual grey level and is merged in one greytone image. In the ideal case, the image obtained in that way will show the skeleton of the fibre in one greytone level corresponding to the orientation of this fibre. Most often, however, the obtained skeleton is composed of pieces of different grey levels. The next step in the procedure is to assign only one grey level to the object. The most obvious method to attain this is to ascribe to the object a grey level representing some kind of mean from the grey levels of all pixels of the object. But such an approach has some disadvantages, the most significant being the computation time. When applying this method each object must be analysed separately, which in the case of objects in Fig. 1b takes about 1300 sec when the process is performed on a microcomputer. Such a long time required for calculations makes it necessary to propose another, faster algorithm.

A NEW ALGORITHM FOR CODING OF DIRECTIONS

Let us note that each object in the greytone image, occurring as the result of a series of directional erosions, is made up of pixels of various grey levels. Pixels of the same grey level, if they appear one beside the other, form sections of various length (Fig. 2a). Based on the assumption that the analysed fibres with respect to their shape may be treated as rectilinear objects, it can be assumed that the direction of the fibre corresponds to the direction of the longest of these sections. In order to identify it, the distance function will be calculated for each of these sections. In the case of skeletons, which are one pixel thick, the distance function is determined through successive removal of ending points, the addition of the successive sets gives the final result shown in Fig. 2b. The number of required distance functions is equal to the number of grey levels in the image, and it is not greater than the number $3s$ of erosions used to build the greytone image. A geodesic reconstruction (Beucher, 1990) is then performed to label each object with the grey level equal to the maximal value of the distance functions computed in the whole fibre (Fig. 2c). When comparing the results in Fig. 2b and 2c, some remarkable points can be depicted in each figure. In our example, such a point is the point D_B (Fig. 2c). It corresponds also to the centre D_A of the longest section in Fig.2a. By means of another geodesic reconstruction, using D_A as marker and the whole fibre as mask, we get a labelled image of the fibre skeleton, its unique grey level corresponding to its direction in the plane (Fig.2d). The proposed algorithm does not require individual analysis of each object, which dramatically reduces the time needed for analysis. In the investigations conducted by the authors this time was about 50 sec in comparison with the 1300 sec needed by the previous method.

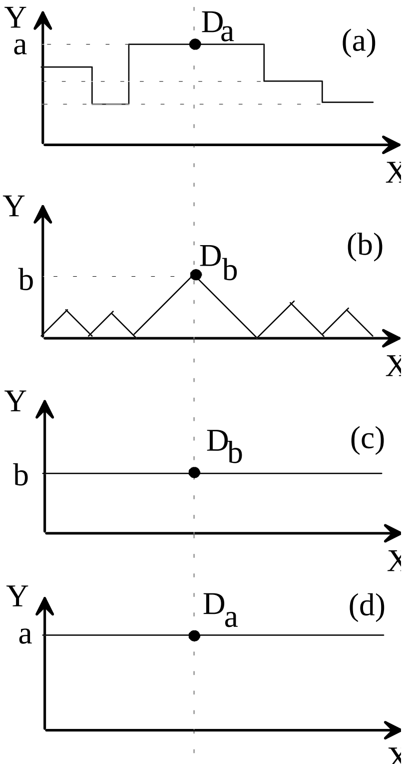


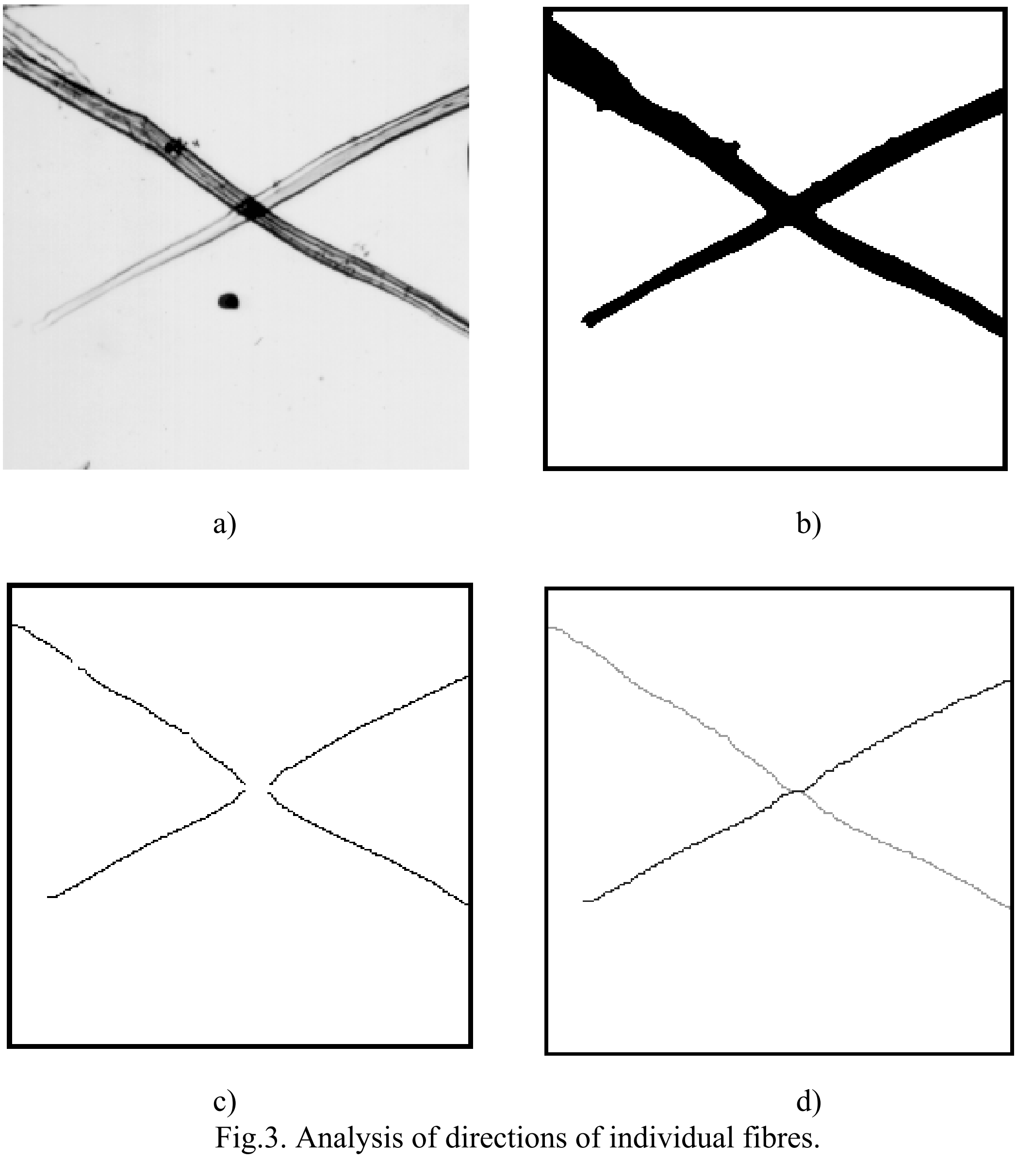
Fig.2 Coding of directions for a single fibre.

The proposed algorithm is connected with some assumptions. The assumption that the direction of the longest section of the fibre is consistent with the direction of the longest section given by directional erosions is correct only in the case of fairly rectilinear objects. If, on the fibre, several sections of equal length occur, but of different grey level, the algorithm will favour the section of higher level. Moreover, we must be aware that, as a result of series of 3s erosions, some points will be duplicated in several binary results. When all these binary eroded sets are merged in one greytone image, some information may be lost. However, from the experiments performed by the authors such a case occurred only for a small number of points and had no influence on the result.

The proposed method can be slightly improved by eliminating the too short sections (made of one or a few pixels) obtained after erosions (e.g. Fig. 2a), and by filling by reconstruction the voids produced by this removal. By this means, local disturbances are removed. However, this technique must be used carefully as it may result in the removal of significant objects, particularly the small ones.

ANALYSIS OF INDIVIDUAL FIBRES.

Let us apply the above algorithm on the grass fibres from Fig. 3a. The initial greytone image is firstly filtered, thresholded (Fig. 3b), and skeletonized. The next, very important step in the procedure, is to remove the points of intersection of the lines (the so-called multipoints). The removal of the intersection points of the fibres allows to split the image from one object of indefinite orientation into a set of four distinct objects, each one having a definite orientation (Fig. 3c).



a) b)

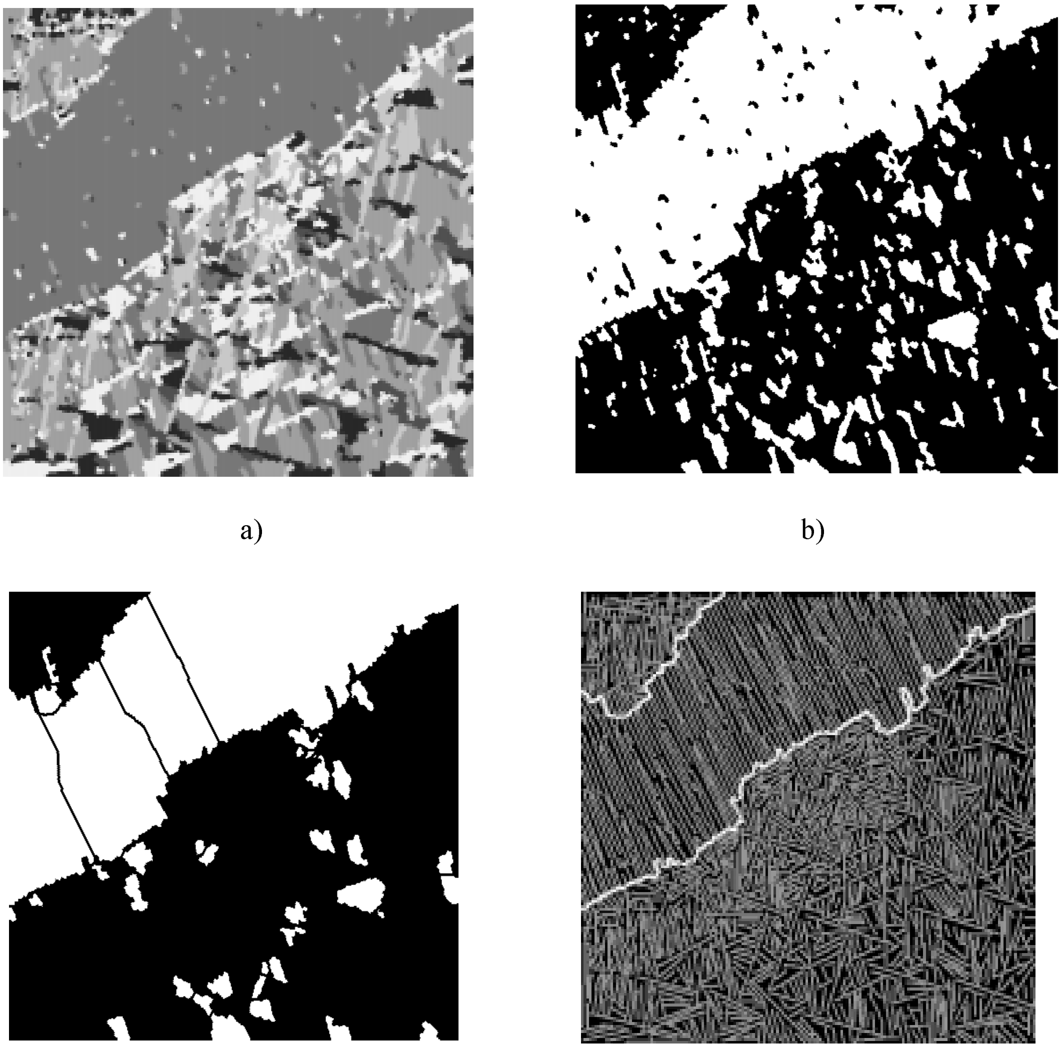
c) d)

Fig.3. Analysis of directions of individual fibres.

Erosions by a pair of points of size 2 in 6 directions are then performed on this latter image and the directions coding algorithm, presented in the preceding section, is applied. The final stage of the treatment is made of a propagation of the direction labels on the whole set of fibres (Kurdy, Jeulin 1991) which allows to reconnect the fibres which have been cut by the removal of multiple points (Fig. 3d).

DETECTION OF AREAS OF SIMILAR ORIENTATION OF FIBRES.

The algorithm presented so far enables to solve the major problem presented in the introduction of this paper, namely the analysis of the structure shown in Fig.1b. The first stage of the process consists in applying the direction coding algorithm described in the previous example. The greytone image is then expanded in the whole space by successive geodesic dilations. doing so, we assign a direction not only to each fibre but also to the zone of influence of the fibre (Fig. 4a). In this example, six directions have been coded. If the previous image is thresholded, the resulting binary image emphasises the regions where fibres extend in the selected direction (Fig. 4b). In this example, one evidently large area appears, connected with smaller areas, constituting a fairly extended object.



a) b)

c) d)

Fig. 4 Segmentation of areas of similar orientation of fibres

These connections and the small areas, however, do not corresponds to significant orientations. So, in the next step, the small disjoined areas can be removed by an opening by reconstruction (Salambier, Serra 1994). Then, in order to break the connections between the large regions remaining after the opening and the small areas which are connected to them, a segmentation by a watershed transformation will be used (Beucher 1990). The distance function of the opened image is computed and the watershed transform of the inverted distance function is performed. The result is given in Fig.4c. Another opening by reconstruction of size *k* eliminates the smallest objects. Then an elementary closing removes the divide lines between the remaining objects. This algorithm is repeated for all the grey levels in Fig. 4a. The final result of such a treatment is shown in Fig. 4d. Let us note that the key parameter of the proposed algorithm is the size *k* of the opening by reconstruction. It corresponds to the size of those areas of similar orientation of fibres, which are of no significant importance in regards to the physical properties of the material and therefore which can be neglected.

CONCLUSIONS.

Some tools of mathematical morphology applicable to the analysis of the orientation of fibres have been presented. The main advantage of the proposed method is the possibility of a simultaneous analysis of a great number of fibres present in the image. This result in a huge reduction of the computation time required for such an analysis. By applying efficient morphological operations, especially the watershed transform, accurate segmentations of areas of similar orientation of fibres are obtained. On behalf of the available results, the method described here can be considered as a helpful and reliable tool for directional analysis of fibres.

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