WATERSHEDS OF FUNCTIONS AND PICTURE SEGMENTATION

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ABSTRACT

The watersheds of a function are geometric features which are very useful in picture segmentation. We briefly and intuitively define the notion of watersheds, and we state that this transformation can be performed starting from the skeleton of the function. This skeleton is a particular case of a general morphological transformation called Thinning.

Two examples of use of the watersheds are then given : the first one drawn from contour detection of defects in weld radiographs and the other one from the segmentation of bubbles on an electrophoresis gel.

Introduction

The aim of this paper is to present the notion of watershed of a function in a very practical way. For this purpose, we shall define and compute watersheds for digitized functions. The digitization of these functions is performed according to an hexagonal frame.

Watersheds of a function

Definition

Let f be a function. Its graph may be considered as a topographic surface, on which a number of features may be defined. Among those, the minima of the function, and its watersheds are of some interest.

Let us consider a drop of water on this topographic surface. The water streams down, reaches a minimum height where it stops. With each minimum, we associate the set of all the points of the graph from which the water may come. Such a set is called a catchment basin. Several catchment basins can overlap : Their common points form the watersheds of the function. (see Figure 1).

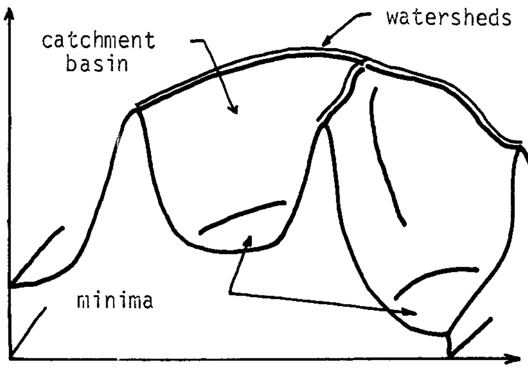


Figure 1 : Remarkable features of a function : Minima, catchment basins, watersheds.

This intuitive definition of a watershed must be refined;to do this, we need the following definitions : Let $\mathcal{C}$ be the sampling hexagonal frame of the function, and put :

$$X_i = \{ x \in \mathcal{C},\ f(x) \geq i \}$$

X_i is the set of all points of the frame such that f(x) is less or equal to i. Let us consider two distinct points x and y embedded in X_i. We can define the geodesic distance between these two points as the length of the smallest arc, provided it is contained in X_i and join x to y. Let us now consider two sets X and Y, $Y \subset X$. Y is assumed to be the union of n connected components K_i. We can define the zone of influence of a component K_j as the set of all points of X at a finite geodesic distance from K_j (that is, at a finite distance from the nearest point of K_j) and closer to K_j than to any other K_i (Figure 2). The set of points of X which do not belong to any zone of influence is called skeleton by zone of influence of Y with respect to X and denote SKIZ (Y;X) [2]

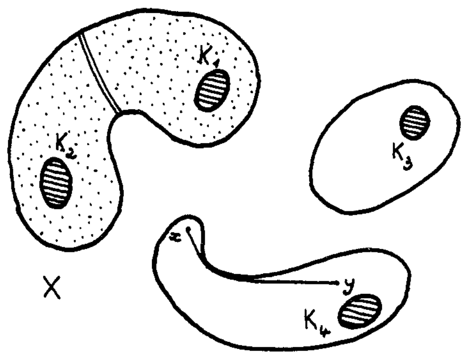


Figure 2 : Geodesic distance and skeleton by zone of influence (SKIZ)

Let Z be the set of watersheds and Z_j the subset of Z of those points which are at height j. Let us assume Z_k is known, for $k \leq j-1$, X_{j-1} being the threshold of function f at level j-1 ; $X_{j-1} - Z_{j-1}$ is then the set of points whose height is less than j and which belong to only one catchment basin. It is then clear that the points of X_j equally distant from two different connected components belong to the watersheds at level j. Then :

$$Z_j = \mathrm{SKIZ}\,(X_{j-1} - Z_{j-1} \; ; \; X_j)$$

and $$Z = \bigcup_j Z_j$$

This definition provides a method for building the watersheds, in an iterative process.

Computation of watersheds

There exists a very tight relationship between the transformation which provides the watersheds of a function and a morphological transformation of functions called Thinning [3].

Let us consider a function f at point x, and an hexagon centered at this point. Let T_1 and T_2 be two disjoint sub-sets of the hexagon (T_1 and T_2 need not be a partition of the hexagon). We define the thinning of f by $T = (T_1, T_2)$ as a function g defined at point x as follows :

$$g(x) = \sup_{y \in T_2} f(y)$$

$$\text{iff} \quad \sup_{y \in T_2} [f(y)] < f(x) \leq \inf_{z \in T_1} [f(z)]$$

$$g(x) = f(x) \quad \text{if not}$$

This function is denoted by $g = f \bigcirc T$. It is possible to use many elements T. In particular, we can take the successive rotations of one element. The transformation is called iterative thinning and denoted by :

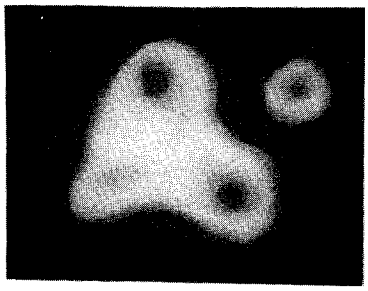
$$f \bigcirc \{T\} = (((f \bigcirc T^{(1)}) \bigcirc T^{(2)}) \ldots \bigcirc T^{(6)})$$

When the structuring element T is the following :

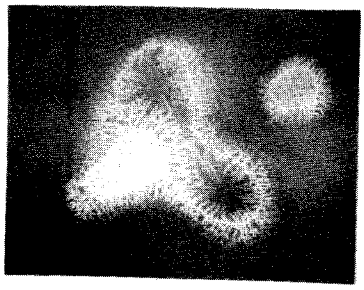
$$\begin{array}{ccc} 1 & \cdot & \\ \cdot & 1 & \emptyset \\ \emptyset & \emptyset & \end{array}$$

; (where the 1's represent T_1 and the 0's, T_2)

Then, this particular thinning is called skeleton of the function [1] (Figure 3)



(a)



(b)

Figure 3 : a) grey-tone function
b) skeleton of the grey-tone function

We can prove that the watersheds of a function are nothing other than the closed arcs of the skeleton [4] . In other words, if we eliminate the barbs of the skeleton, we obtain the waterheds, as shown in Figure 4.

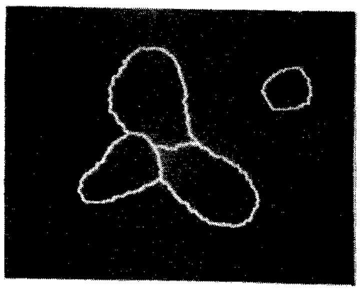


Figure 4 : Computation of watersheds by elimination of barbs in the skeleton.

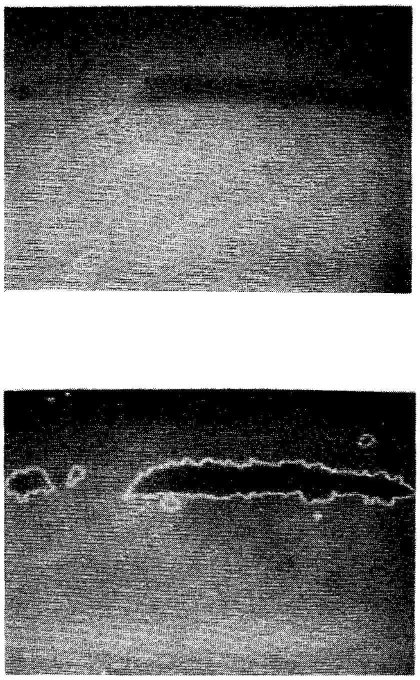
This interesting property provides a powerful method of computation of the watersheds.

Watersheds and picture segmentation

The watersheds transformation is very effective in picture segmentation and in contour detection. Let us illustrate these transforms with two examples.

Weld Radiographs

Many weld radiographs show various defects, different in size and in contrast. In order to detect the contours of these defects, we can perform the watersheds transformation on the gradient function of the picture. In actual fact, the defects correspond to points with a low gradient, as is the case for the background (Figure 5).



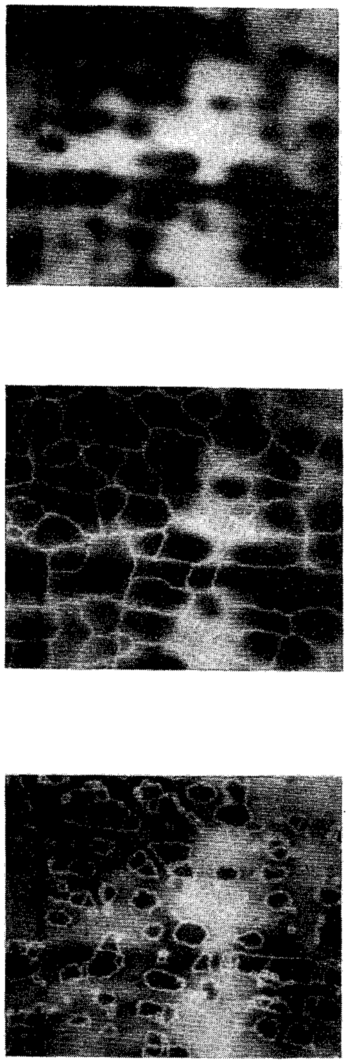
(a)

(b)

Figure 5 : a) Weld radiographs with defects
b) watersheds of the gradient function

Electrophoresis gel

In this case, the problem is to detect contours and to separate bubbles produced by proteins during their migration on a gel. Two watersheds have been performed : the first one on the initial picture, in order to segment the bubbles, and the second one on the gradient function to detect their contours. The first transformation is also used to eliminate over-segmentation which may occur during contour detection (Figure 6).



(a)

(b)

(c)

Figure 6 : a) electrophoresis gel
b) watersheds of the grey-tone function
c) contour detection

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