APPLICATIONS OF MATHEMATICAL MORPHOLOGY IN MATERIAL SCIENCES: A REVIEW OF RECENT DEVELOPMENTS

Serge BEUCHER

Centre de Morphologie Mathématique
Ecole des Mines de Paris
Fontainebleau, France

ABSTRACT

Image analysis applied to material sciences aims at quantifying the objects or structures under study. With the increasing complexity of images provided by more and more sophisticated sensors, two major and almost contradictory approaches are available: a probabilistic one, where individual or local information can be obtained through the design of a stochastic model and by means of global measurements performed on the image, or a deterministic approach consisting in the extraction of the interesting features in the image, each one being analysed and measured in a second step. The latter technique requires efficient segmentation tools.

The purpose of this presentation is to review the various operators available in mathematical morphology to deal with the second approach. We first introduce a general methodology for image segmentation, based on the watershed concept. As the marking of objects to be extracted is necessary, various morphological tools are introduced: morphological filters, selection of extrema in the image (contrast and/or shape), dynamics and hierarchical segmentation. Many of these tools use a very powerful morphological transformation: geodesic image reconstruction.

Examples of this methodology are given both in the microscopic (SEM, TEM) and macroscopic (industrial control) domains. Finally, some algorithms and hardware implementation for a greater computation speed are introduced.

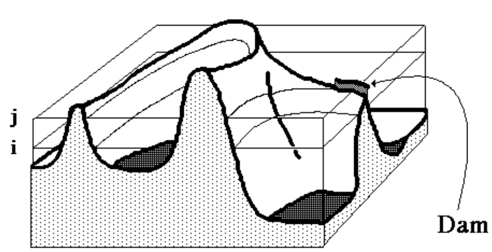
1. INTRODUCTION

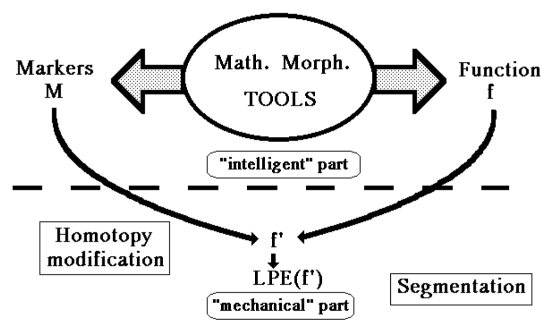
Image analysis in material sciences is mainly used for measuring structures or objects of interest. However, faced to the increasing complexity of the sensors and of the images delivered by these sensors, the extraction of the features of interest is more and more difficult. A great amount of transformations must be applied to the original image to obtain a sufficiently accurate binary image of the objects under study. Two different approaches are available for achieving this goal. The first one is a probabilistic approach where individual or local information can be obtained through the design of a stochastic model and by means of global measurements performed on the image [9]. This approach is particularly useful when the analysed structures are too complex or too fuzzy to be adequately segmented. The main problems raised by this technique firstly come from the fact that the real sample must fit in the probabilistic model. Secondly, some information on the elementary brick of the model (the "primary grain") is necessary, information which leads, sooner or later, to a segmentation problem. This approach will not be discussed in this paper. We shall introduce on the contrary the second methodology based on the direct extraction of the interesting features from the image. Various morphological transformations can be used to build efficient segmentation tools for achieving this task. Furthermore, mathematical morphology provides a general methodology for image segmentation based on the concept of the marker-controlled watershed [1]. This "object oriented" technique requires that regions to be segmented be marked and that a segmentation criterion be defined. In most cases, these two steps are not easily achieved. Fortunately, many helpful morphological tools are available. Among them are morphological filters, extrema extraction, dynamics and hierarchical segmentation. Some of these tools are first briefly introduced, then examples of this methodology are given in microscopic imagery and industrial control. Finally, some information about the computational speed and the hardware and software implementation is given.

2. PRINCIPLE OF THE MORPHOLOGICAL SEGMENTATION

2.1. Watersheds

Watershed transformation is the basic tool for image segmentation in mathematical morphology. An extended presentation of this transformation can be found in [6]. Let f be an image and consider it as a topographic surface. The catchment basins of f and its watershed lines can be defined by means of a flooding process starting from the minima (or sinks) of f. Floods coming from different minima are separated by dams or watershed lines. These watershed lines surround the various catchment basins of f, each one containing one and only one minimum (figure 1a). Watershed transformation in image segmentation is often applied to the gradient image because the contours of the different regions correspond to the watershed lines of the gradient.





(a) (b)

Figure 1: Flooding of a function and watershed (a), synopsis of the segmentation methodology (b).

2.2. Marker-controlled watersheds

Unfortunately, the real watershed of a function f often presents a lot of catchment basins produced by small variations in the grey values. This over-segmentation can be avoided if the patterns to be segmented are marked before applying the watershed transform. In this case, the flooding process is only performed from the marker set and produces as many catchment basins as there are markers in the marker set. Moreover, the watershed lines will correspond to the crest lines of f, that is to the contours of the marked regions if the function is a gradient.

2.3. Segmentation paradigm

The segmentation procedure may then be split into two steps. The first one consists in modifying the initial function (the gradient function for instance) in order to produce a new function, similar to the original one, except that its initial minima have been replaced by a set of markers. The second step is simply the watershed transformation of the new function. This approach leads to a general methodology of the segmentation consisting in selecting first a marker set M pointing out the objects to be extracted, then a function f assessing a segmentation criterion (which can be, but not necessary, the variation of the grey values). This function is then modified to produce a new function f' having as minima the set of markers M. The segmentation of the initial image is performed by the watershed transform of f'. The segmentation process is therefore divided into two steps: an "intelligent" part whose purpose is the selection of M and f and a "straightforward" part consisting in the use of the watershed transformation and the image modification (figure 1b). The latter transformation can be achieved by means of a very powerful operation: the geodesic reconstruction [1].

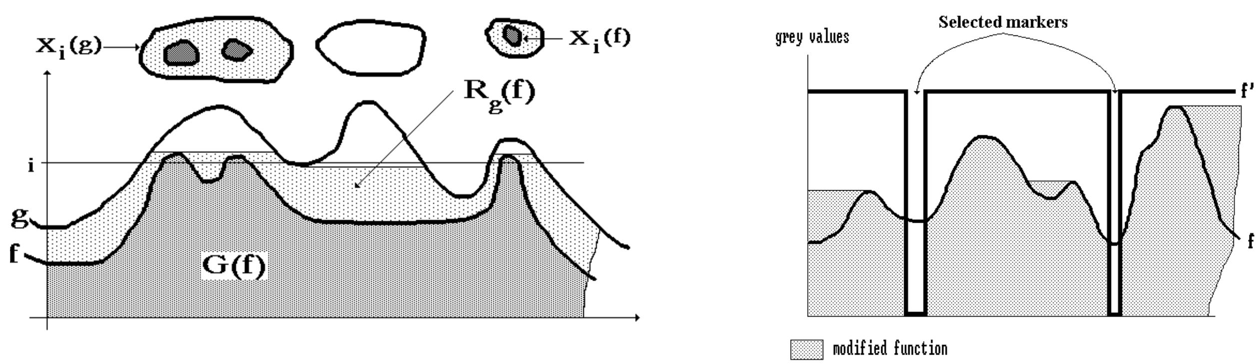
2.4. Geodesic reconstruction

The geodesic reconstruction is the extension of the geodesic transformation [12] to greytone images. Let f and g be two greytone images, with $g \leq f$. The reconstruction of f by g is given by successive dilations of g "under" f. This reconstruction $R_g(f)$ can be performed by iterating the following formula:

$$g = \mathrm{Inf}(\mathrm{Dil}(g), f) \tag{1}$$

where Dil(g) is the dilation by the elementary disk, until idempotence (figure 2a).
A dual reconstruction $R^*_g(f)$ can also be defined by using successive erosions of g "over" f:

$$g = \mathrm{Sup}(\mathrm{Ero}(g), f) \tag{2}$$



(a) (b)

Figure 2: Reconstruction of a function (a), modification by means of a dual reconstruction of a function by a set of selected markers (b).

This reconstruction is used in the image modification process introduced above. As illustrated in figure 2b, this dual reconstruction uses the characteristic function of the marker set M and produces a new function where the initial minima are filled and replaced by new ones corresponding to the marker set M.

3. MORPHOLOGICAL TOOLS USED IN THE SEGMENTATION PROCESS

Mathematical morphology provides many tools for handling the intelligent part of the segmentation, both with regard to the choice of the segmentation criterion as to the marker selection. In this latter case, many transformations use the geodesic reconstruction.

3.1. The segmentation criteria

Two major criteria are used in the image segmentation process. The first one is the variation in grey levels in the image. This variation is represented by the morphological gradient [18] defined by:

$$g(f) = Dil(f) - Ero(f) \tag{3}$$

Other contrast enhancers can be used alternately. One of the most useful is the top-hat transform [13] defined by:

$$TH(f) = f - Dil_n(Ero_n(f)) \tag{4}$$

where Dil_n and Ero_n are respectively the size n dilation and erosion.
This transformation enhances bright and narrow features in the image. A dual top-hat transform enhancing the dark and narrow structures can also be defined.
Conversely to these contrast enhancers, it is sometimes useful to smooth the image in order to reduce over-segmentation. To achieve this, various morphological filters can be designed [19]. One of the most interesting is the reconstruction-opening filter built from a geodesic reconstruction of a function by its erosion.
Another important criterion for the segmentation is the shape of the objects. It is particularly helpful in the separation of intricate binary objects. This criterion can be formalised and manipulated by means of the distance function of a set [2]. More sophisticated criteria can be designed based on a combination of shapes and contrast [15].

3.2. The marking tools

Markers can be obtained by various means. However, in many cases, they correspond to the extrema (minima or maxima) of a function (a criterion function or any other function). For instance, as seen above, the homogeneous regions can be marked by the minima of the gradient. The extrema of a function f can be obtained by a geodesic reconstruction [1]. The minima m(f) and maxima M(f) are given by:

$$m(f) = R^*_{f+1}(f) - f\ ;\ M(f) = f - R_{f-1}(f) \tag{5}$$

It is also possible to select sufficiently significant extrema by deleting those whose depth, or height, is lower than a given value h. These h-minima $m_h(f)$ or h-maxima $M_h(f)$ are given by:

$$m_h(f) = R^*_{f+h}(f) - f\ ;\ M_h(f) = f - R_{f-h}(f) \tag{6}$$

Another powerful characterisation of the extrema is given by their dynamics [8]. The dynamics of a minimum corresponds to the height which must be climbed along a path on the topographic surface associated with the function to reach a point at a lower altitude than the minimum. This parameter characterises the significance of a minimum in relation to the surrounding (but not necessarily adjacent) ones.
The main drawback of these transformations is the need for an appropriate selection of the value of the parameter (height or dynamics) able to detect the relevant extrema. Another approach named "waterfall transformation" avoids this drawback because it is non parametric [4]. Starting from the initial watershed of the function f, which produces over-segmentation, another watershed transform is performed on the function f' defined by:

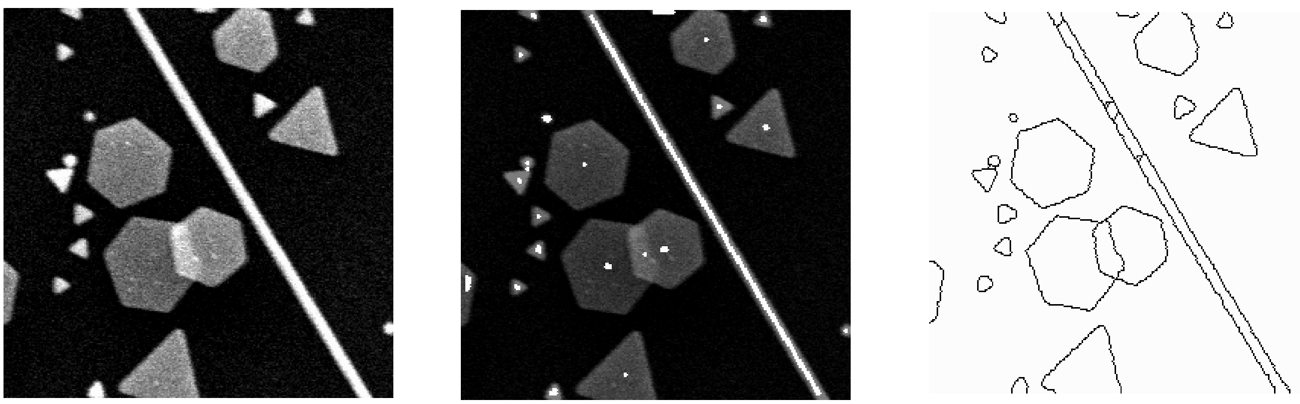
$$f' = R^*_w(f) \tag{7}$$

where w is the characteristic function of the catchment basins of f.
This non parametric approach leads to a very efficient hierarchical segmentation technique [3].

4. APPLICATION EXAMPLES

Two examples will illustrate the use of these segmentation tools.

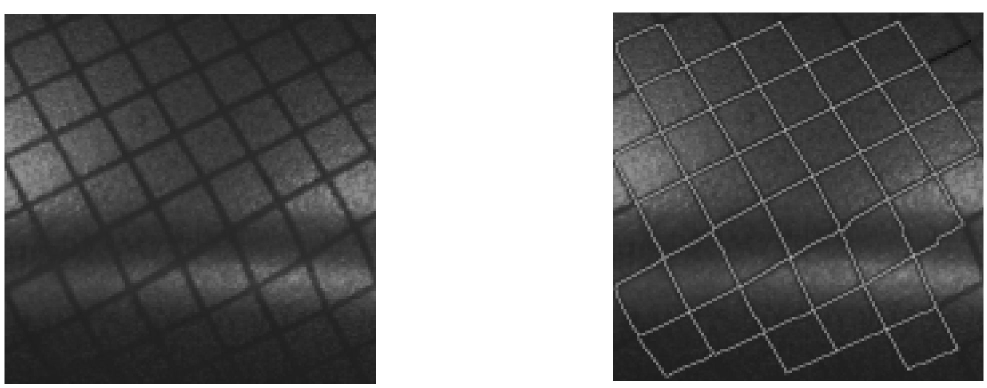
The first one concerns the segmentation of overlapping grains of silver nitrate scattered on a photographic plate (figure 3a). To segment this image, the background, the grains and the overlapping regions must be marked. A binary mask X of the grains is obtained by an automatic thresholding. The maxima of the distance function d(X) provide the markers of the grains. The watershed transformation of the inverted distance function -d(X) produces divide lines passing through the overlapping regions. These lines therefore can be used to mark these overlapping regions. The marker of the background is obtained by eroding X^c (figure 3b). The function controlling the segmentation is the gradient. The final segmentation is given in figure 3c.



(a) (b) (c)

Figure 3: TEM image of silver grains (a), set M of markers (b), final segmentation (c).

The second example is a macroscopic application. Its purpose is to extract a deformation grid in a stamped metal sheet (figure 4a) [17]. In this case, the markers of the regions between the grid are obtained by selection of the maxima of high dynamics and the criterion function is the black top-hat transform (figure 4b).



(a) (b)

Figure 4: Close view of a stamped metal sheet with its deformation grid (a), segmentation of the grid (b).

Many other examples of segmentation performed by means of this methodology can be found in [3], [11] and [1].

5. PERFORMANCES AND CONCLUSIONS

This powerful approach of segmentation needs fast and efficient algorithms. Until recently, the computation time of a watershed transformation was the major drawback of this methodology. It is no longer the case today, as many new algorithms and processors have been designed which dramatically increase the speed of these transformations. Recursive algorithms, arrowing algorithms [1], hierarchical queues [14], the use of anamorphosis [5] coupled with fast hardware processors [16] allow to segment a picture in a few hundred milliseconds and no there is doubt that this speed is going to increase in the near future. These tools are more and more adapted to the complexity of the new sensors and of the images they provide. This technique can be applied successfully to 3D images [7] as well as to colour ones [14]. Finally, the gap between this approach and the stochastic methods is not as deep as it used to be and, in many examples, these two methods efficiently collaborate [10]: the segmentation allows the extraction of the primary grain, while modeling enables dealing with highly intricate structures unreachable to segmentation.

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