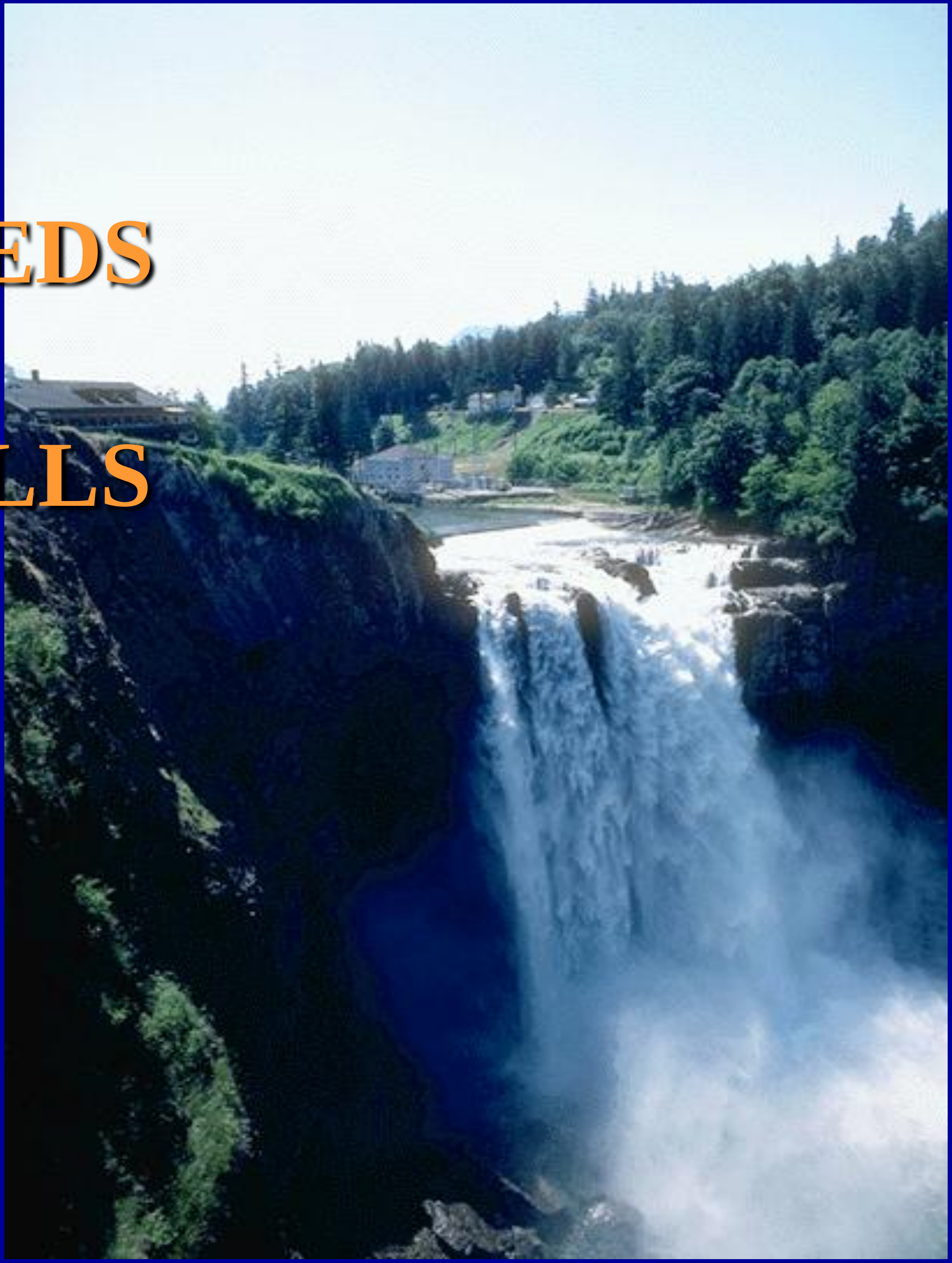
WATERSHEDS
&
WATERFALLS



Serge BEUCHER

CMM / ENSMP
February 2000

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- **The watershed transform**
  - Algorithm, properties, etc...
  - Geodesy, reconstruction
  - Use of watershed, mosaic image
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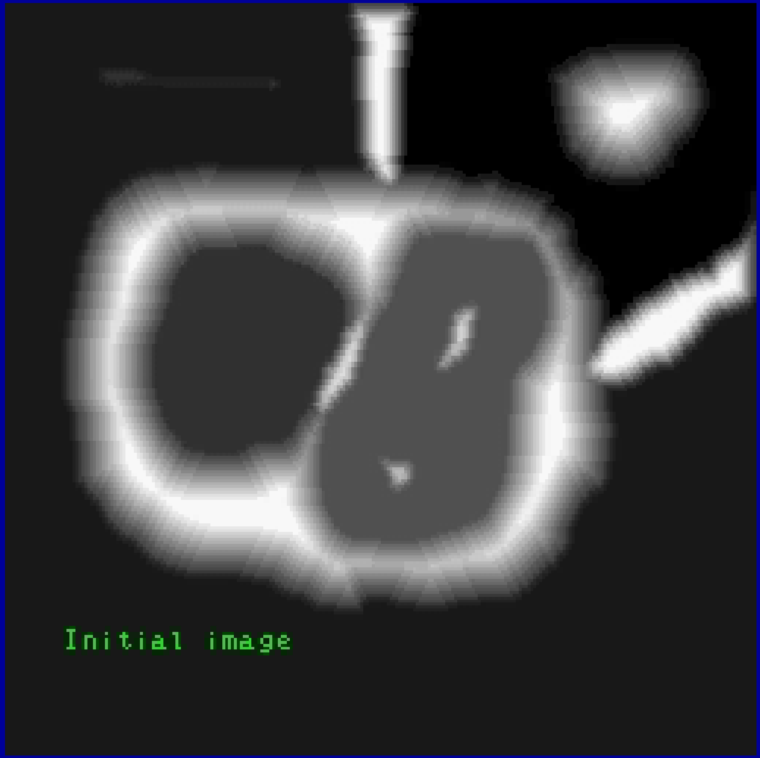
- **Marker-controlled watershed**
  - Algorithms
  - Swamping
  - Position of markers
  - Applications
- **Hierarchical segmentation, waterfalls**
  - Mosaic, gradient mosaic
  - Graph and hierarchical image
  - Waterfalls, introduction
  - Algorithm, applications
- **Detailed applications**

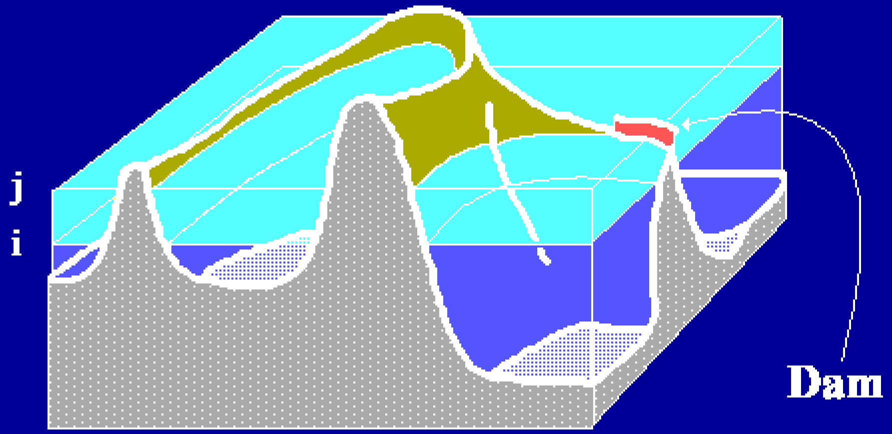
THE CLASSICAL WATERSHED ALGORITHM



- It's a flooding process
- Flooding sources are the minima of the function

Two hierarchies are used:

- flood progression with the altitude (sequential process)
- flood on the plateaus/flat zones (parallel process)



The result is a partition of the image into catchment basins and watershed lines (dams)

THE CLASSICAL WATERSHED ALGORITHM (2)

The transformation can be expressed on the successive levels Z_i of the function f:

$$W_0 = m_0(f)$$

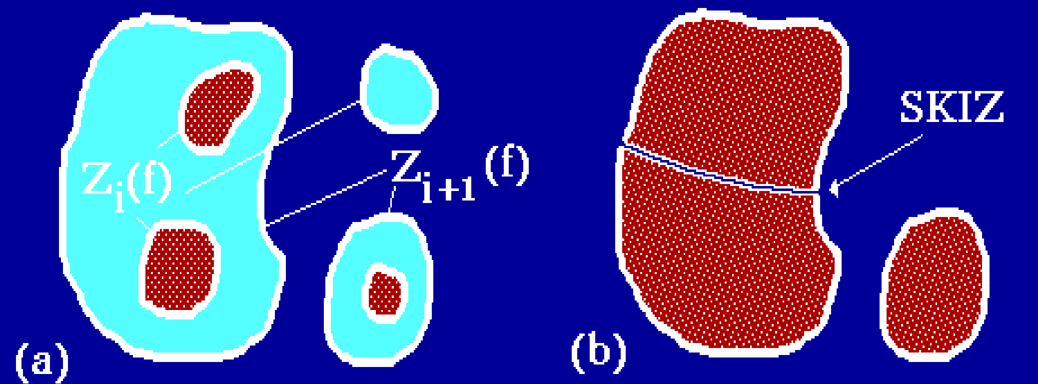
The catchment basins at level 0 are the minima at this level

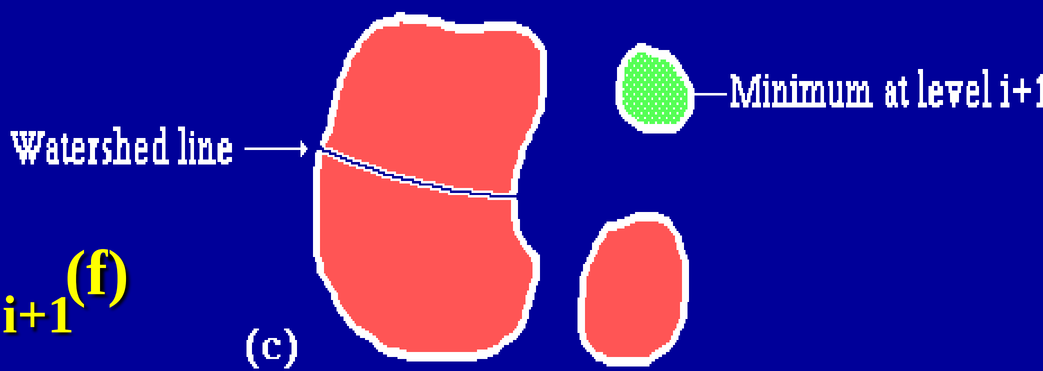
$$W_{i+1} = [\,SKIZ_{Z_{i+1}(f)}(W_i)\,] \cup m_{i+1}(f)$$

with:

$$m_{i+1}(f) = Z_{i+1}(f) \,/\, R_{Z_{i+1}(f)}(Z_i(f))$$

R is the geodesic reconstruction



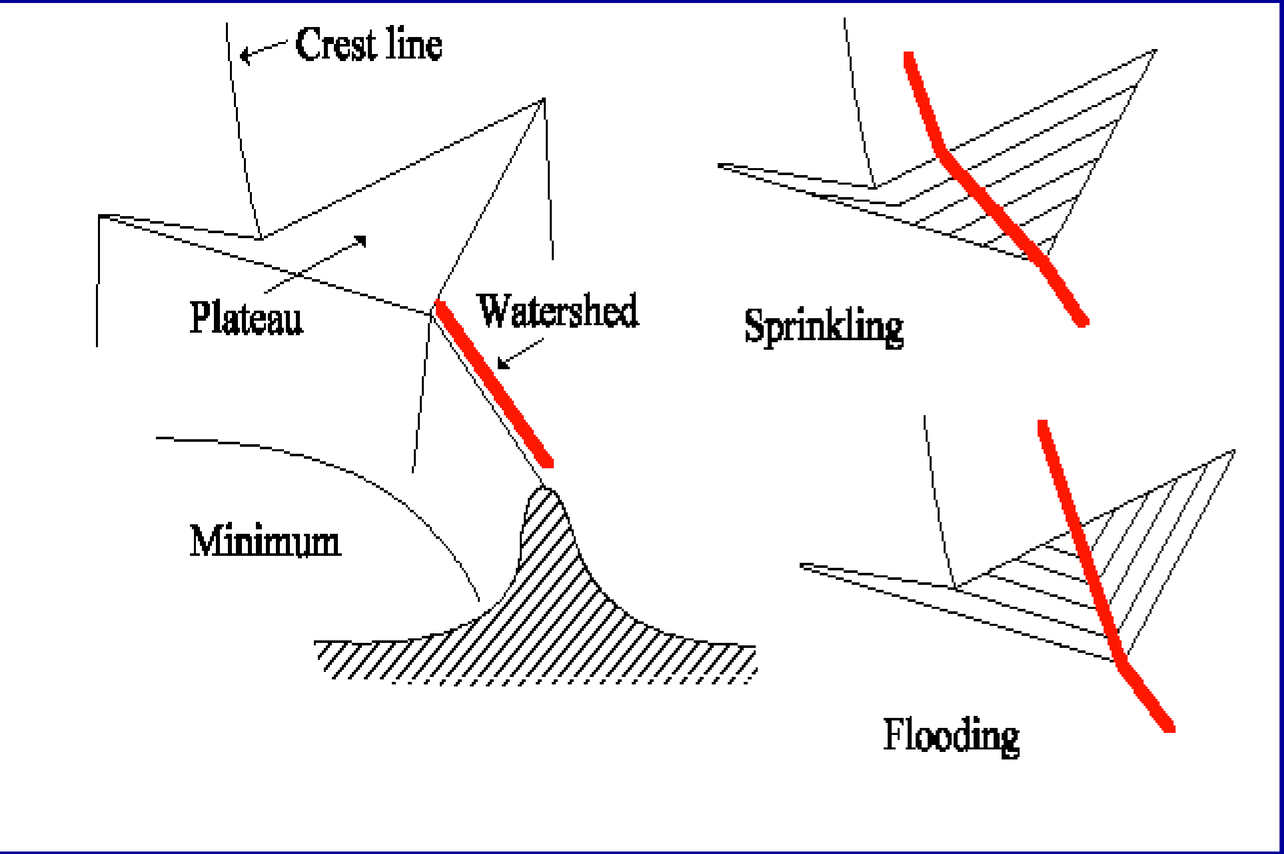


Use of the geodesic SKIZ transform to simulate the propagation without merging

BIASES AND FALSITIES WITH THE WATERSHED

The watershed cannot be built by simulating rain drops falling on the topographic surface (water sprinkling). FORGET IT!

The flooding on the plateaus is based on a MODEL (constant speed). It has mainly two advantages: it is simple and it has a physical meaning.



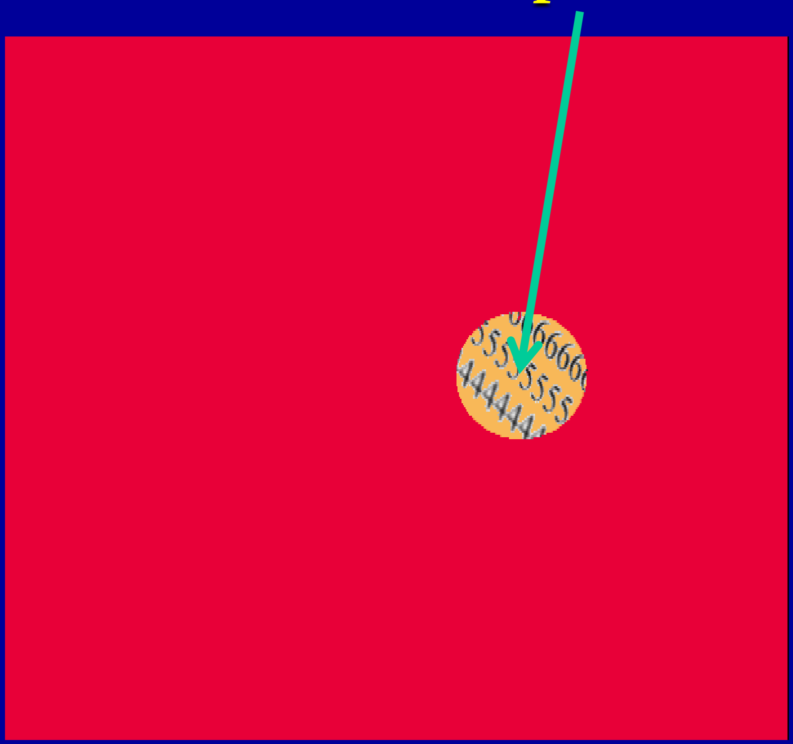
In any case, the results could not be identical (due to the propagation on the flat zones).

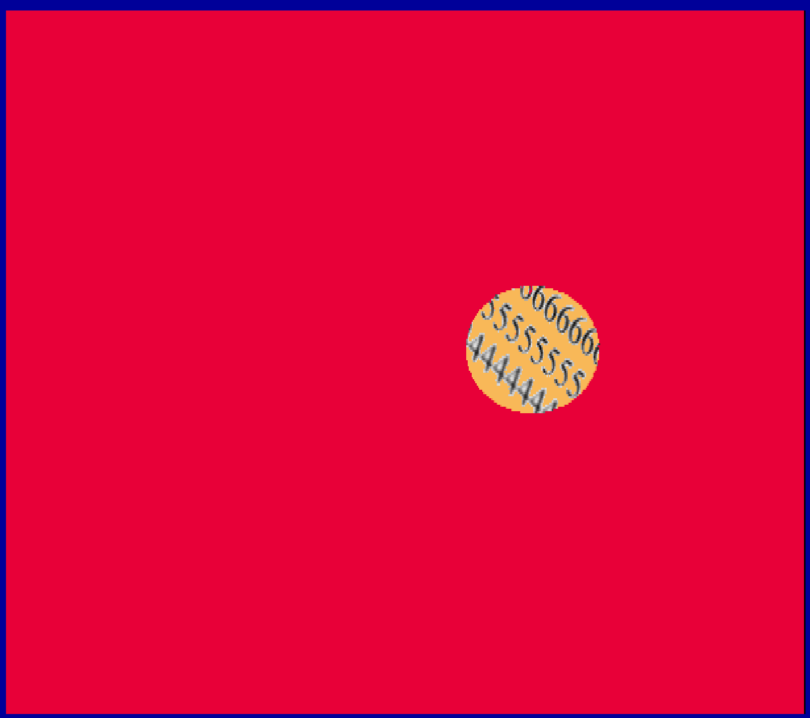
BIASES AND FALSITIES WITH THE WATERSHED (2)

The watershed lines are not local. In particular, they are not related with local features (crest lines, ridges,…). The watershed is not a LOCAL concept.

You cannot, having only a local knowledge of the neighbourhood of a point, answer the question:

Does this point belong to a watershed line?





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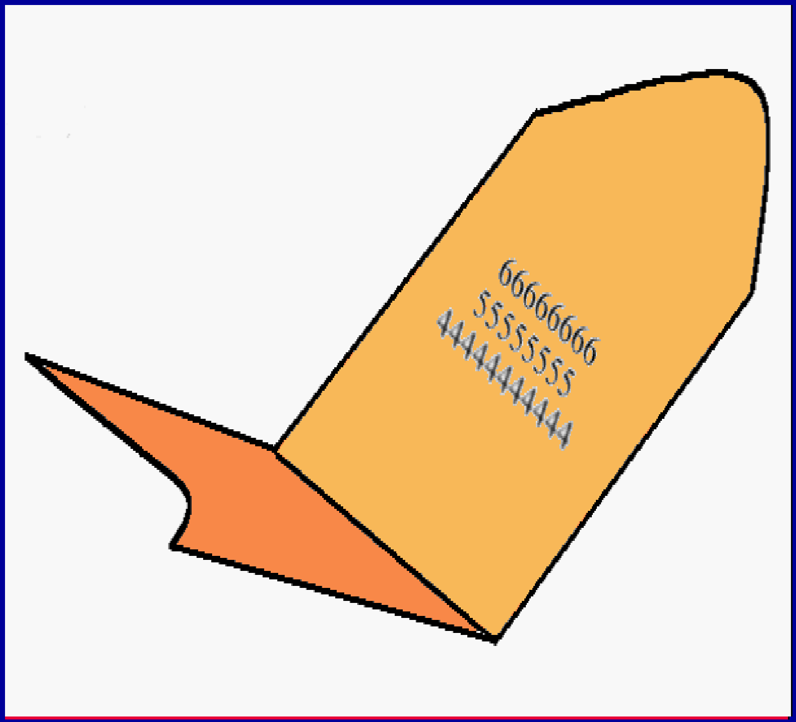
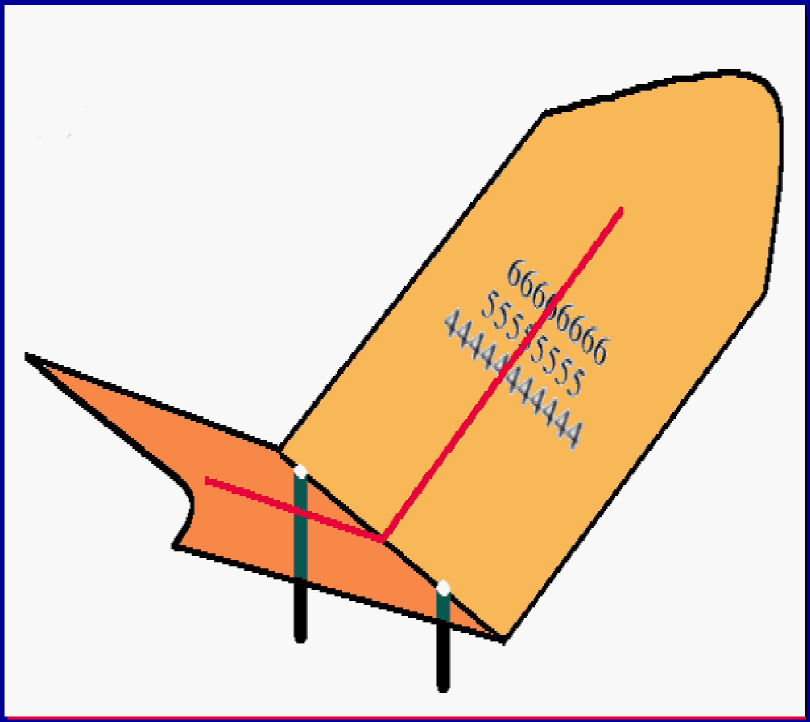
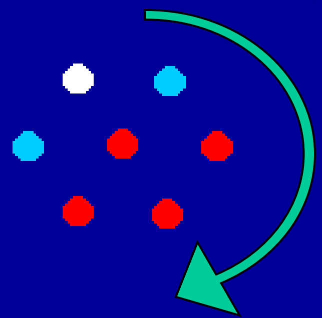




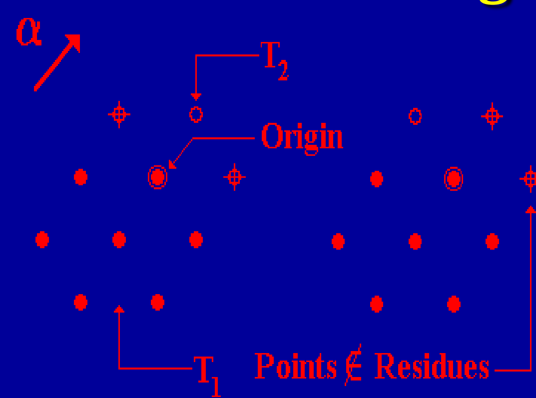
BIASES AND FALSITIES WITH THE WATERSHED (3)

Most of the watershed algorithms are biased:

- **classical one (SKIZ using rotation thickenings)**

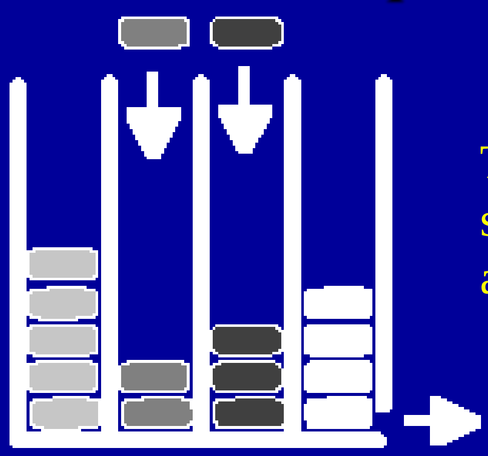


The use of rotating structuring elements in the SKIZ generates a non parallel flooding on the plateaus

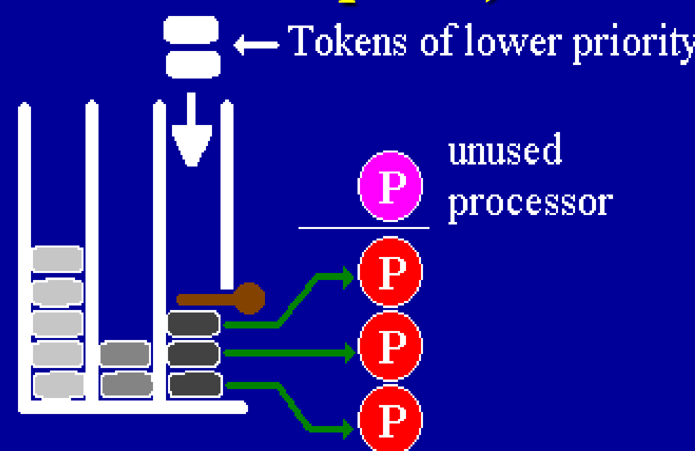


Solution: Union of structuring elements defined on a larger neighbourhood

- **hierarchical queues (a priori order defined in the queue)**

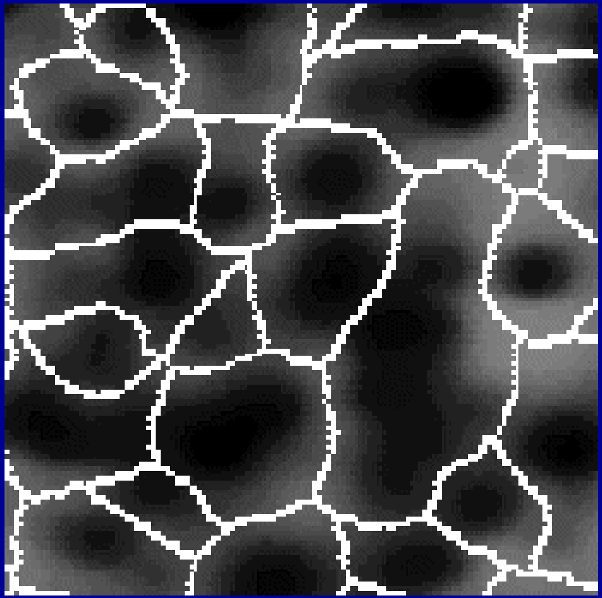


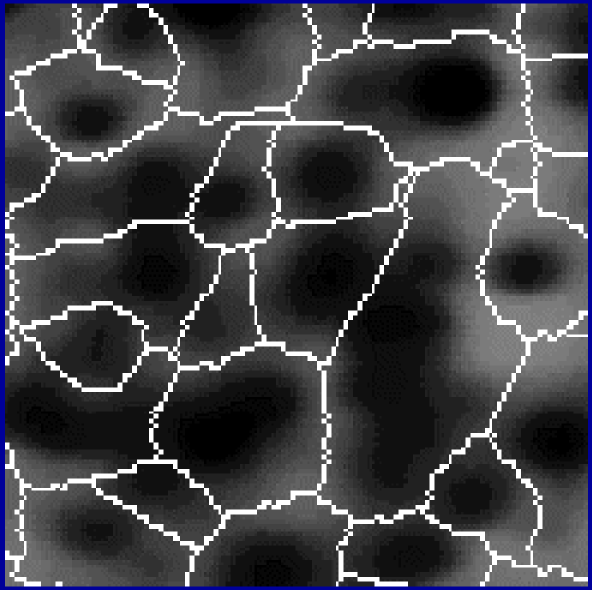
The tokens in the same stack should be processed at the same time



BIASES AND FALSITIES WITH THE WATERSHED (4)

For various reasons (complexity, computation speed, laziness…), the unbiased watershed transforms are seldom used.





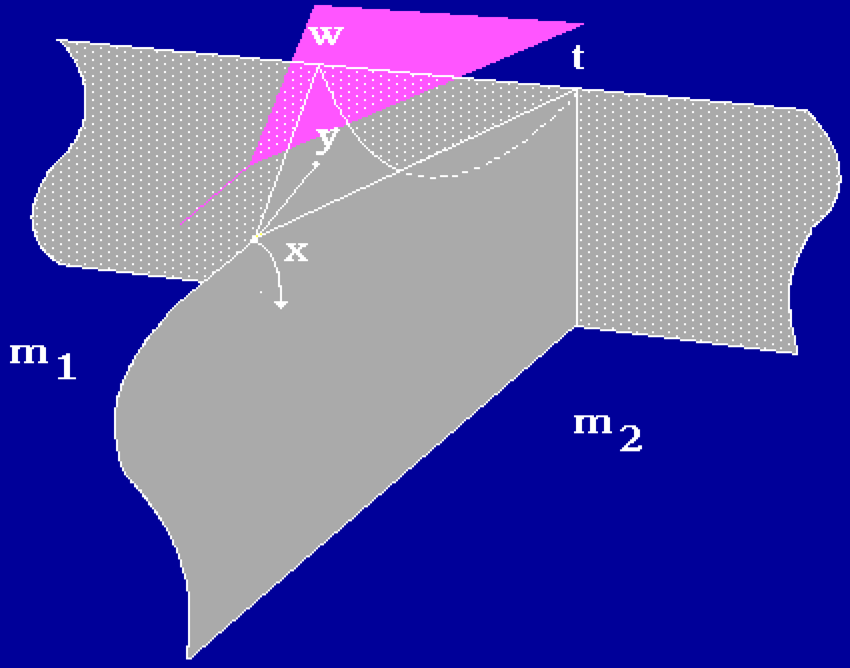
Comparison between a true watershed (left) and the result of a “classical” algorithm.

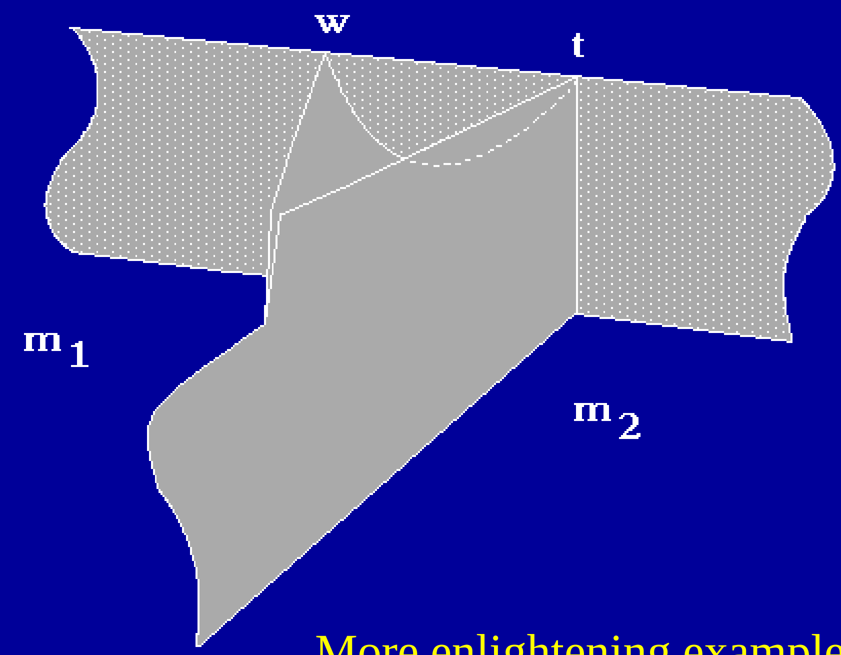
These biases may have dramatic consequences in hierarchical approaches based on the comparison of adjacent catchment basins.

BIASES AND FALSITIES WITH THE WATERSHED (5)

Is flooding always an upstream process?
That is, when the flood is at height h, is it true that ALL the points at a lower altitude have been processed?

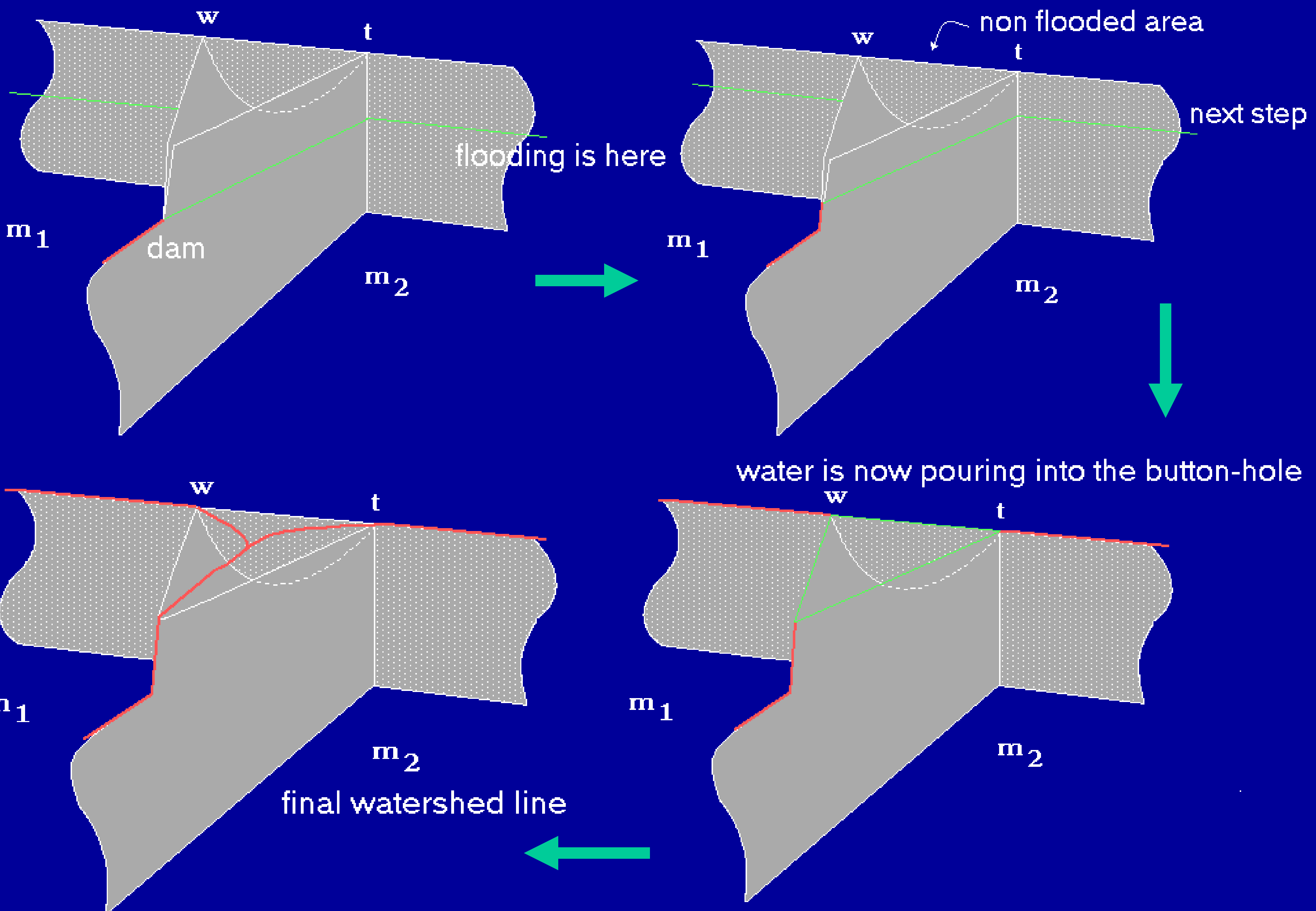
The answer is NO!
Counter-example: the button-hole case





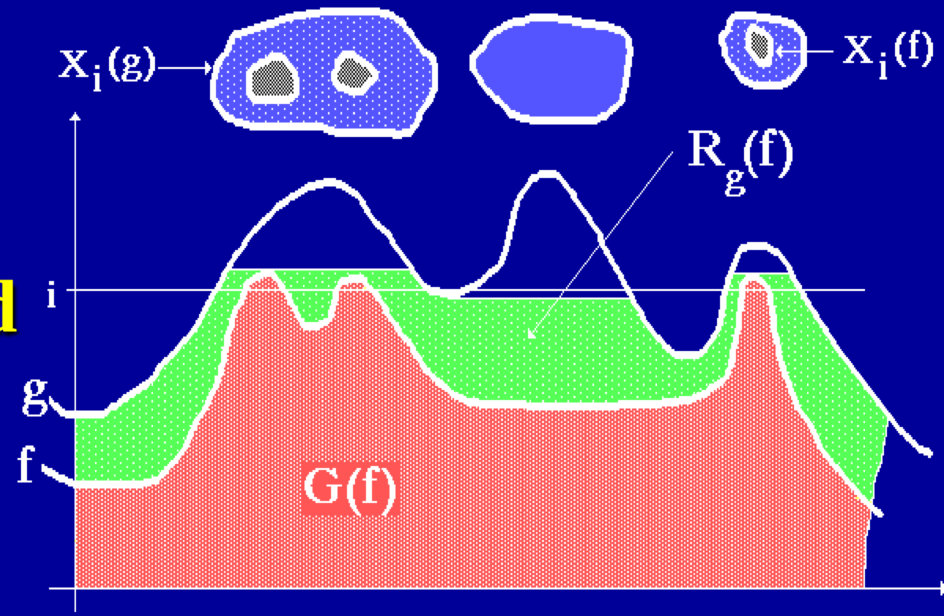
More enlightening example...

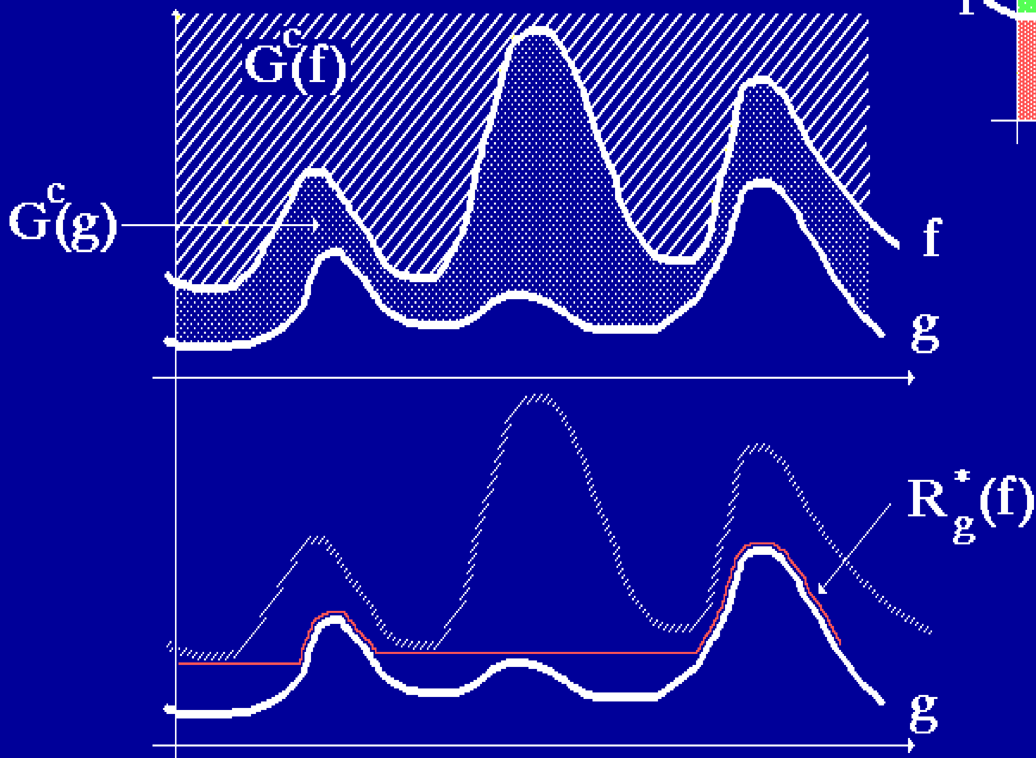
THE BUTTON-HOLE CASE



GEODESIC RECONSTRUCTION

The geodesic reconstruction is of outmost importance for performing and understanding the watershed transformation.



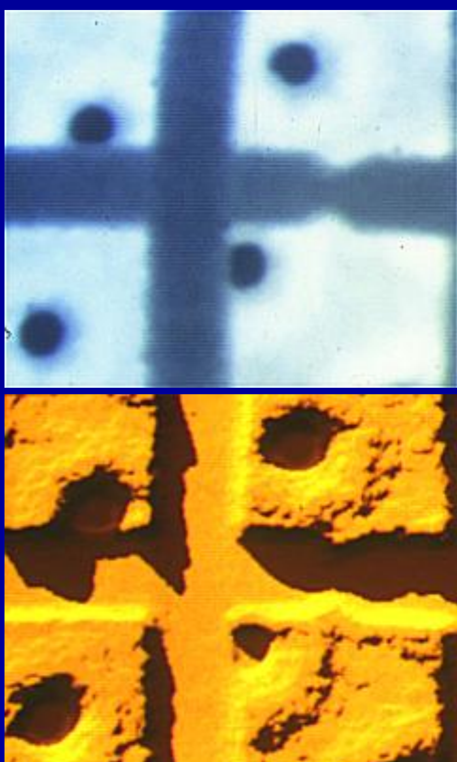


A dual reconstruction can also be defined (it uses geodesic erosions).

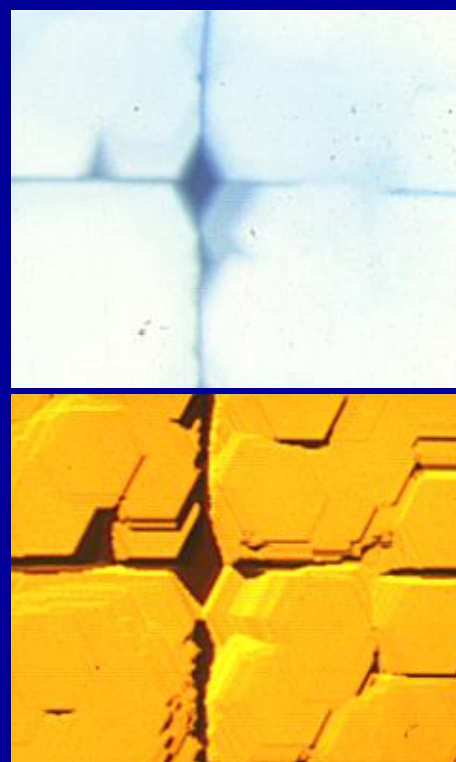
GEODESIC RECONSTRUCTION

The geodesic reconstruction is widely used in mathematical morphology:

- **detection of extrema (minima, maxima)**
- **filtering (openings and closings by reconstruction)**
- **watersheds (swamping, homotopy modification)**
- **waterfalls**



g



f

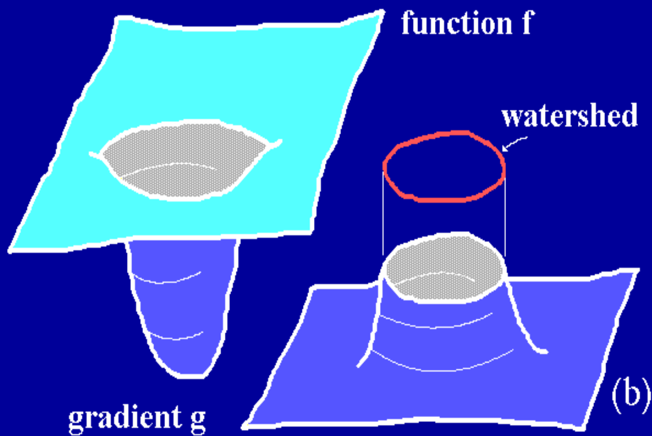
$R^*_g(f)$

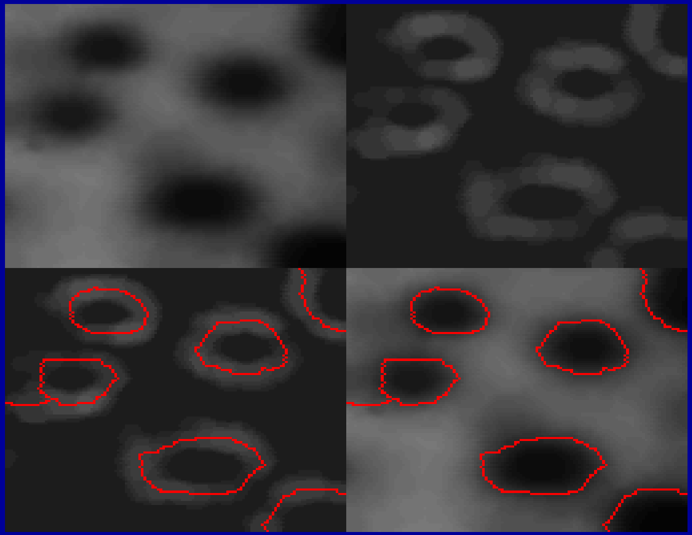
USE OF WATERSHED

The watershed transform is used for image segmentation

• Greytone segmentation

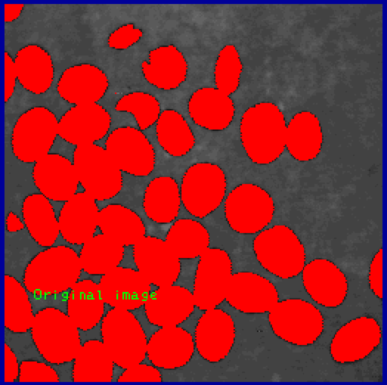
The watershed of the gradient corresponds to the contour lines

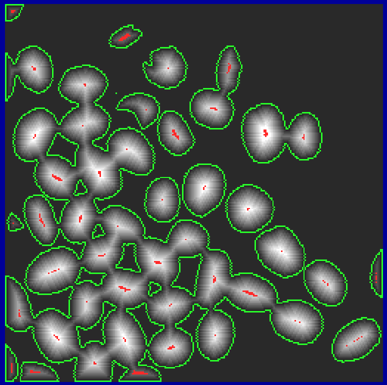




• Shape segmentation

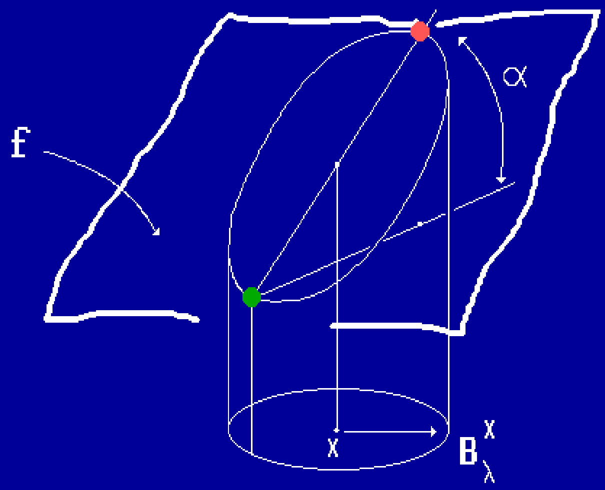
Cutting objects into a union of “convex” sets by means of the watershed of the distance function





THE GRADIENT: A REMINDER

Morphological gradient



$$g(f) = (f \oplus B) - (f \ominus B)$$

Other morphological gradients (half-gradients) can also be defined:

$g-(f) = f - (f \ominus B)$

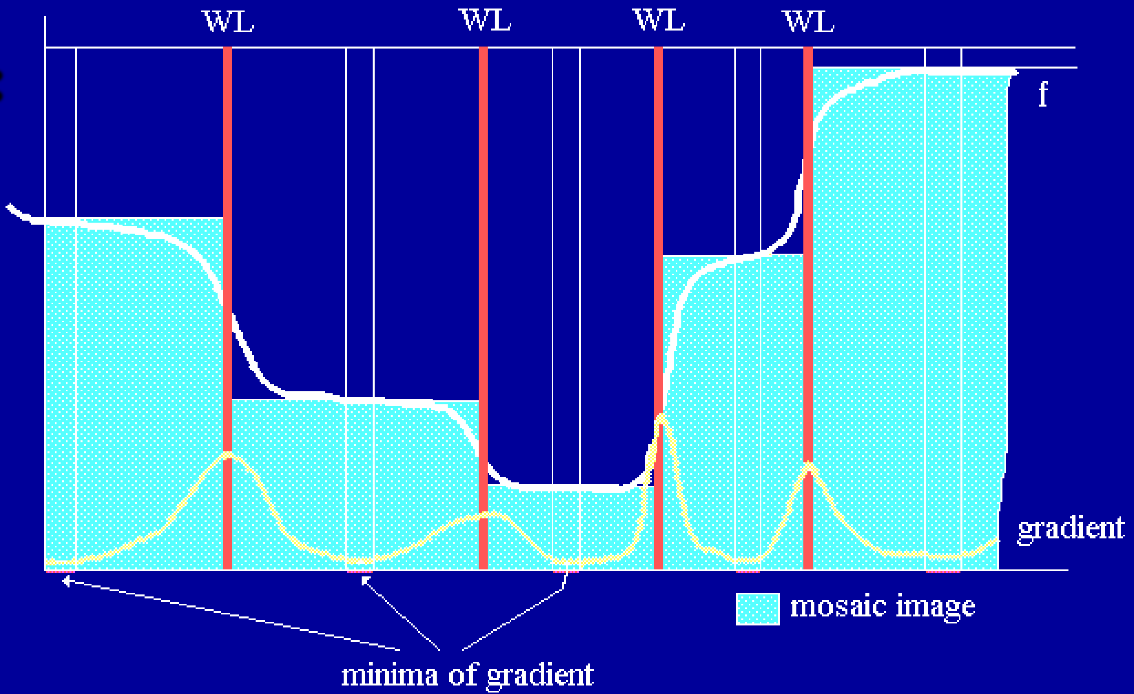
$g+(f) = (f \oplus B) - f$

Gradient defined on the watershed lines: the mosaic-image concept

MOSAIC IMAGE AND ITS GRADIENT

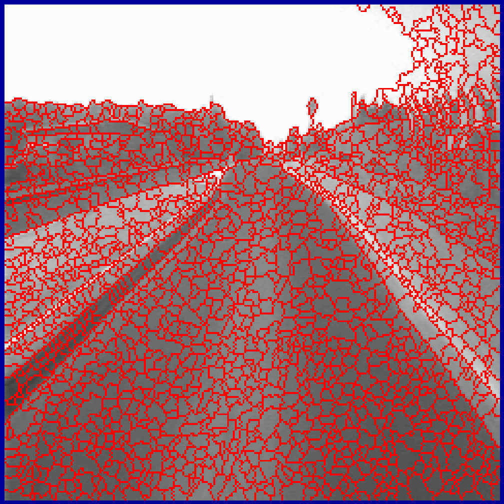
Building the mosaic image:

- **Watershed of gradient**
- **For each minimum of gradient, compute the corresponding grey value**
- **Fill in the catchment basin with this grey value**

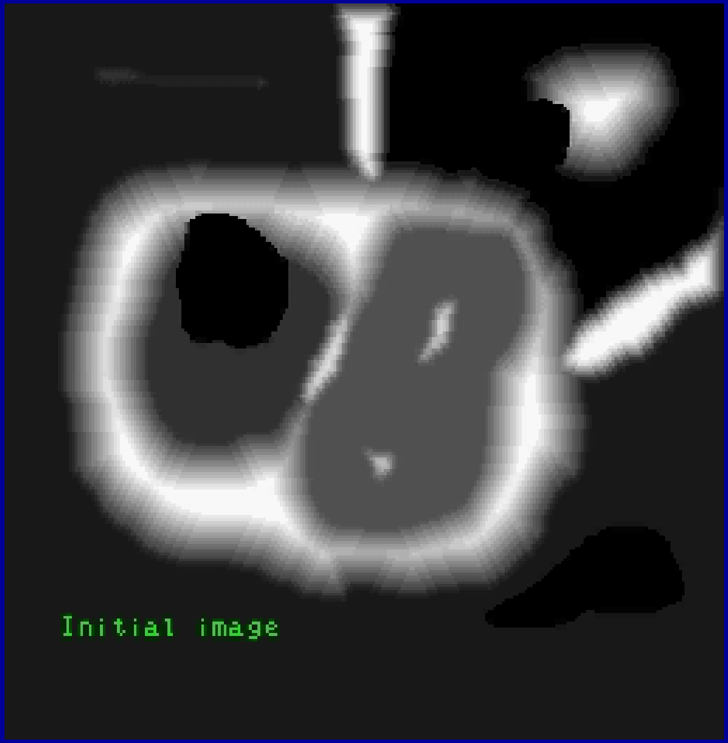


THE MARKER-CONTROLLED WATERSHED

The gradient watershed is over-segmented.



Gradient images are often noisy and contain a large number of minima. Each minimum generates a catchment basin in the WT.



To avoid this over-segmentation due to numerous sources of flooding, one can select some of then (the markers) and perform the watershed transform controlled by these markers.

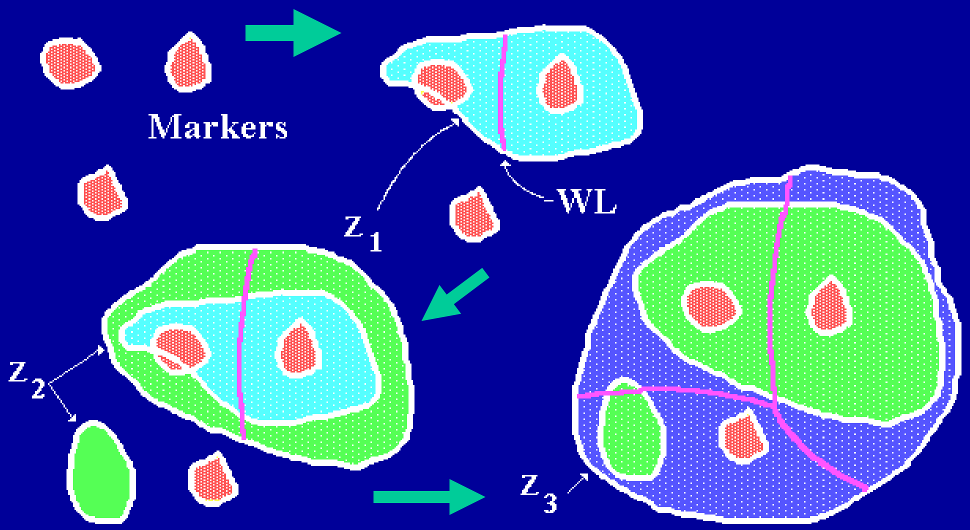
MARKER-CONTROLLED WATERSHED ALGORITHMS

- **Level by level flooding**

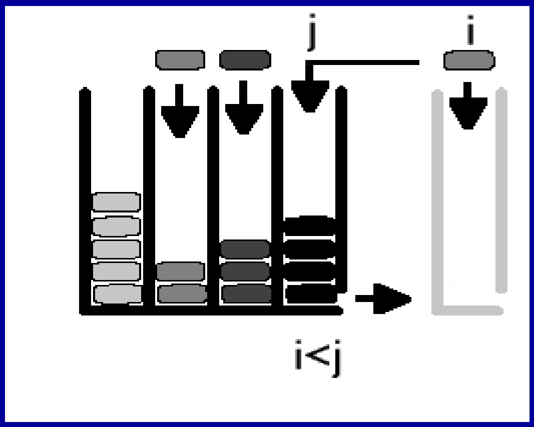
$$W_0 = M, \text{ marker set}$$

$$W_i = SKIZ_{Z_i(f) \cup M}(W_{i-1})$$

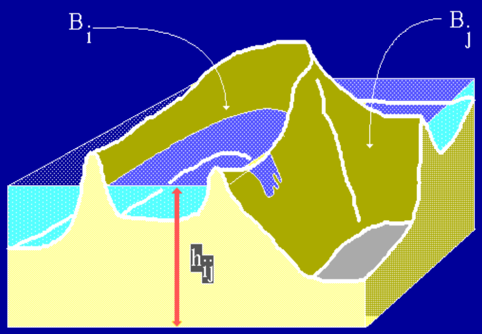
This algorithm is simpler than the classical one: there is no minima detection



- **Hierarchical queues**



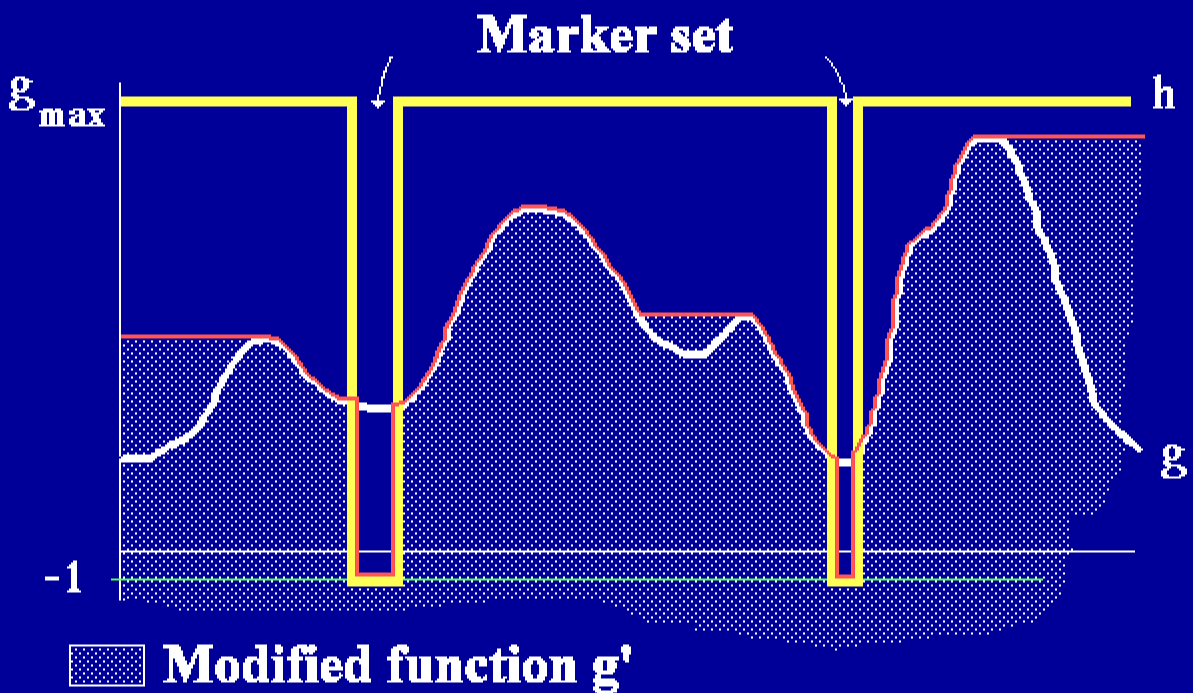
A token at level i<j (the current level) may appear. In this case, it is treated as a token at level j (the i-queue is no longer alive)



With marker-controlled watershed, overflow is the rule and not the exception

SWAMPING (aka HOMOTOPY MODIFICATION)

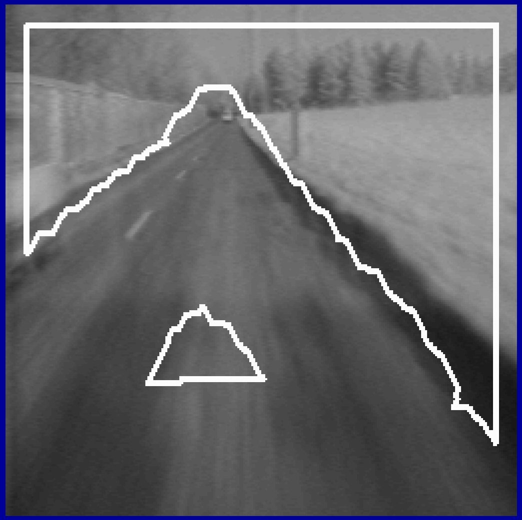
Based on reconstruction, swamping allows to build a new function whose minima correspond to the markers.

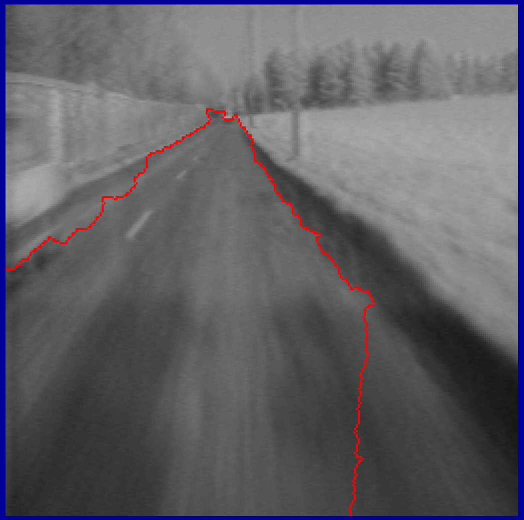


1) a marker function is defined:
$h(x) = -1$ iff $x \in M$
$h(x) = g_{max}$, if not

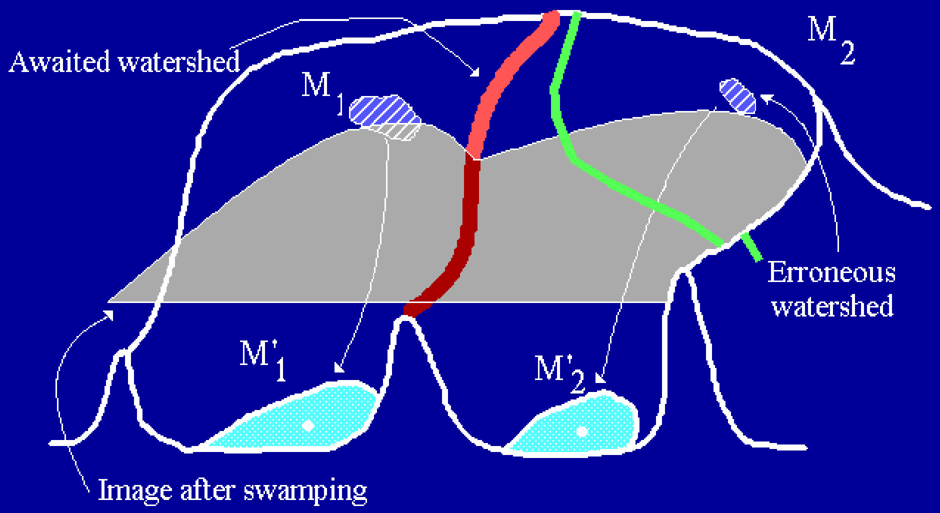
2) The reconstruction of h over $g' = \inf(g,h)$ is made:
$R^*_{g'}(h)$ → swamped function

POSITION OF THE MARKERS



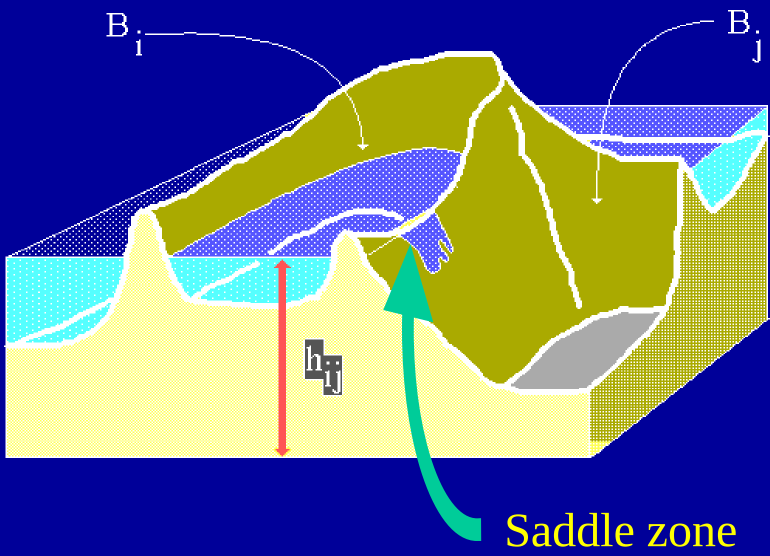


Segmentation obtained (right image) with a marker-controlled watershed of the gradient (markers on the left)

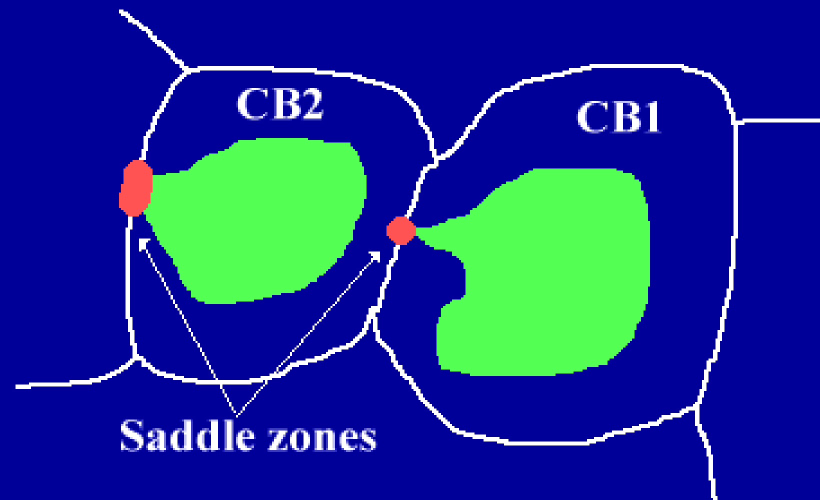


When minima are replaced by markers, it is of outmost importance to control the position of these markers

POSITION OF THE MARKERS (2)



Question:
if we replace the original minima by markers, where to put the markers to insure that the final watershed will be the same?



Notion of lower catchment basin

It's the part of the catchment basin flooded before the first overflow (by the lower saddle zone)

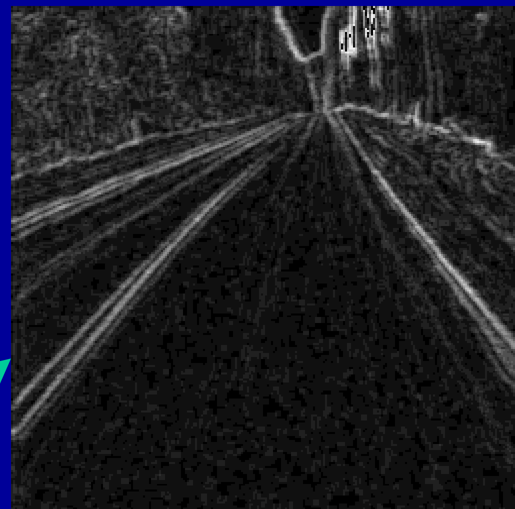
Solution: the markers must be included in the lower catchment basins.

A one-to-one correspondence is not required provided that all the markers included in one lower catchment basin are given the same label.

EXAMPLE OF MARKER-CONTROLLED WATERSHED

Road segmentation

gradient

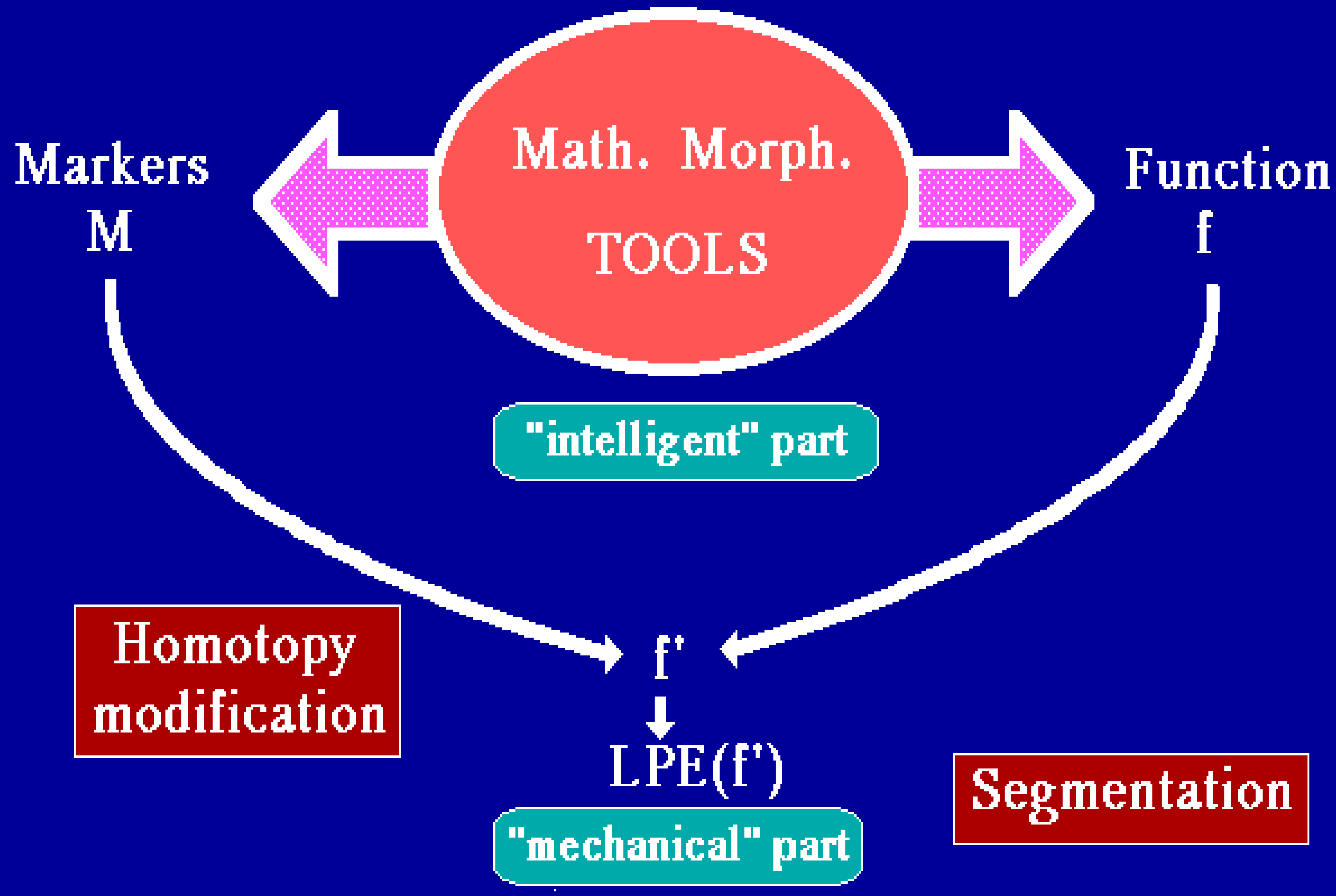


Original image

markers

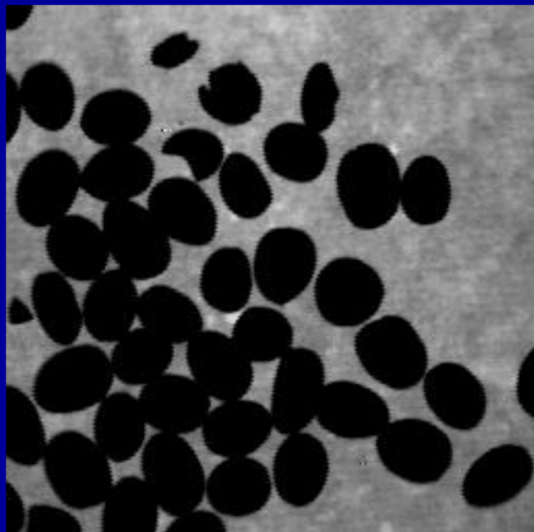
Marker-controlled watershed of the gradient

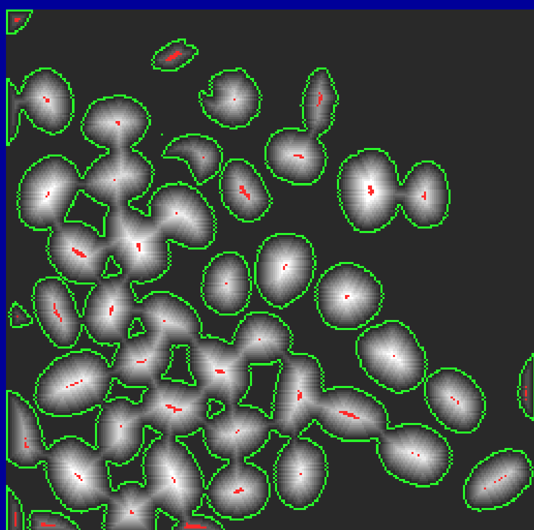
THE SEGMENTATION PARADIGM



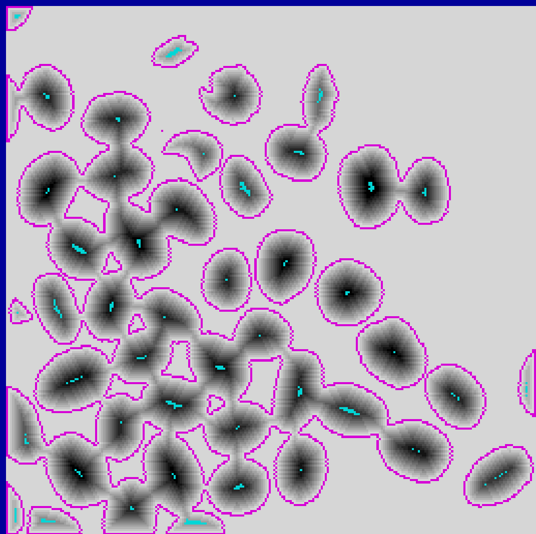
APPLICATIONS

Coffee grains





The distance function of the set is computed. This distance function is inverted and its watershed is performed. The marker set is made of the maxima of the distance function.

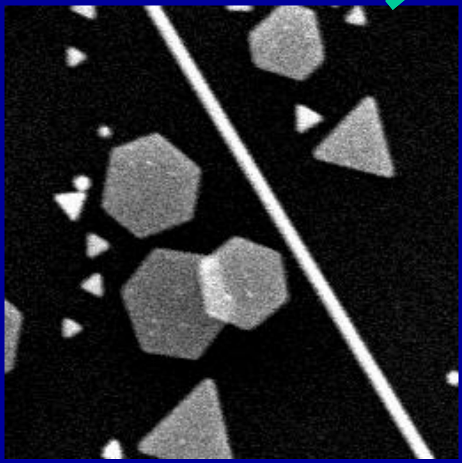


The watershed is performed on the support of the distance function. The maxima are filtered to avoid over-segmentation due to parity problems.

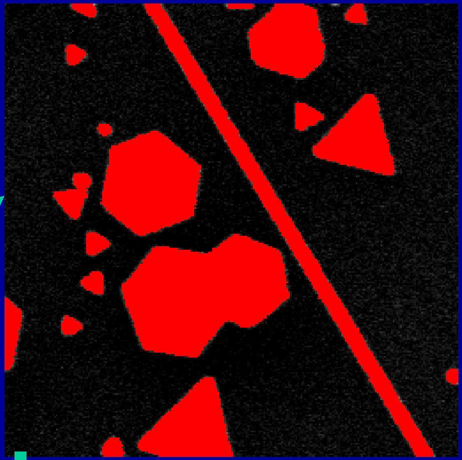
APPLICATIONS (2)

Silver nitrate grains on a film

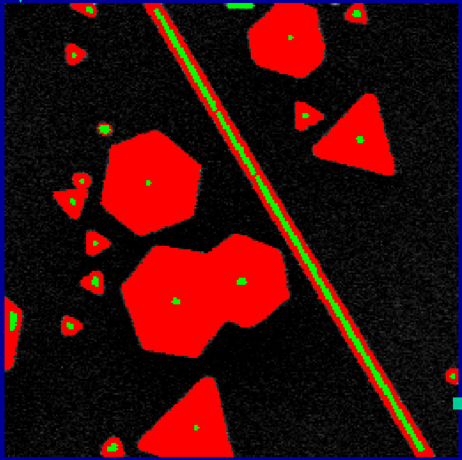
Problem: segmentation of the grains, even when overlapping



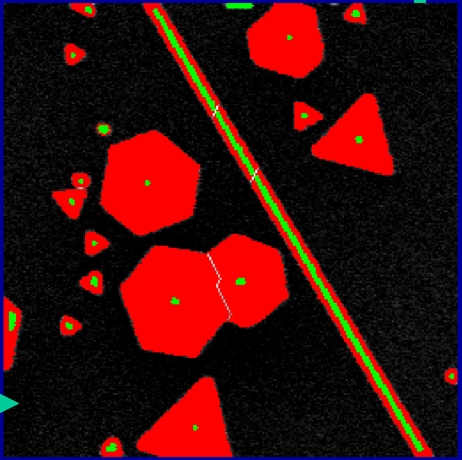
Original image



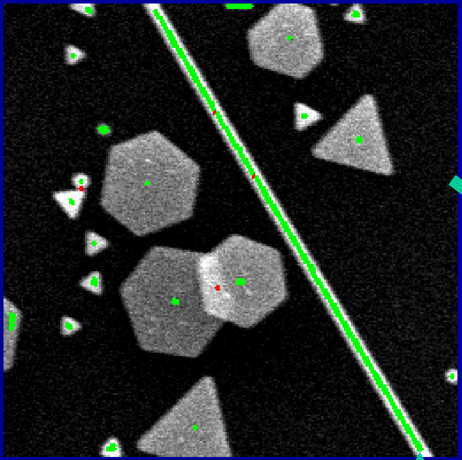
Mask of the grains



1st markers, maxima of distance function



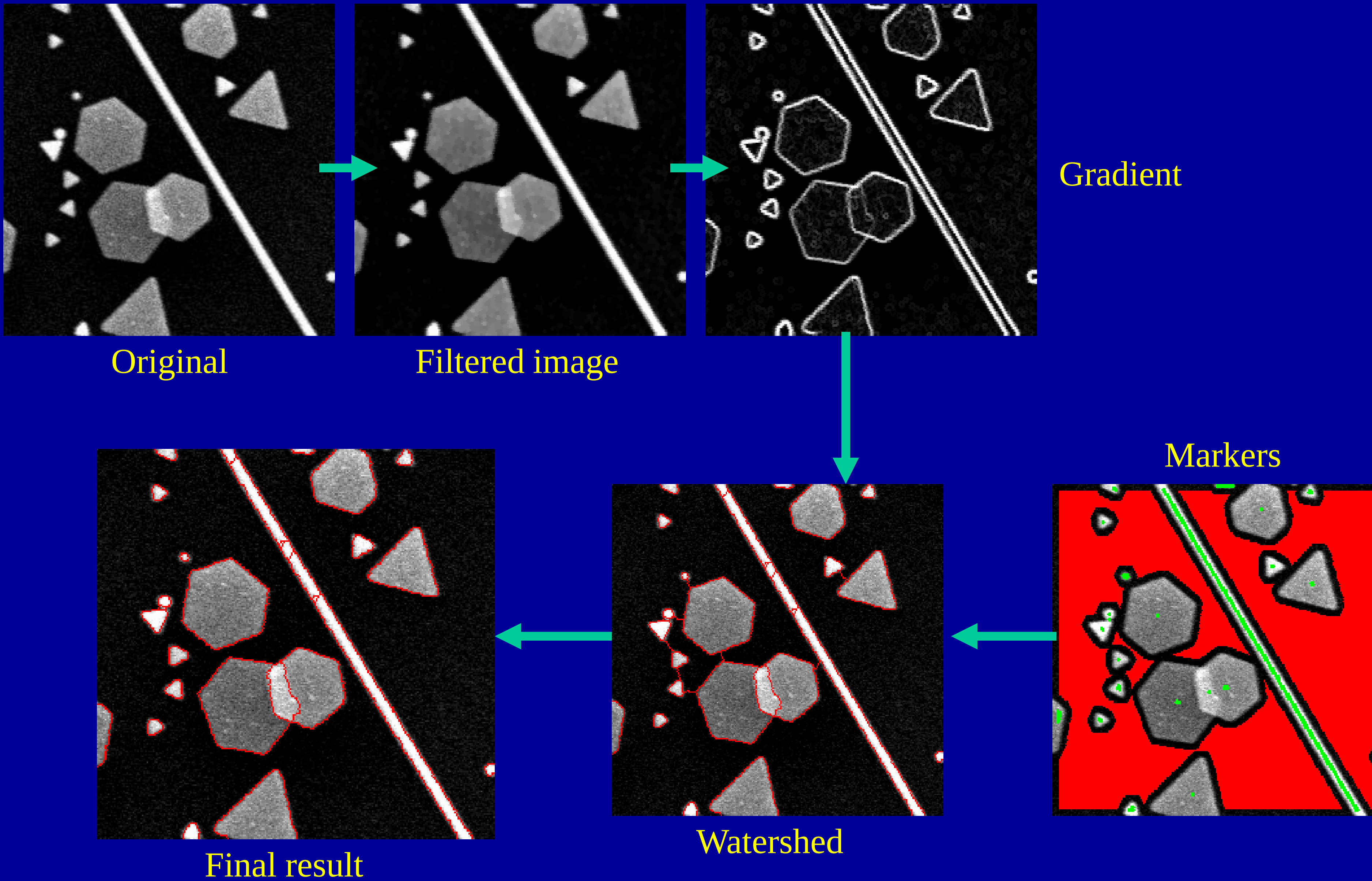
Watershed of the distance function



2nd marker set

The background marker is added. Final marker set

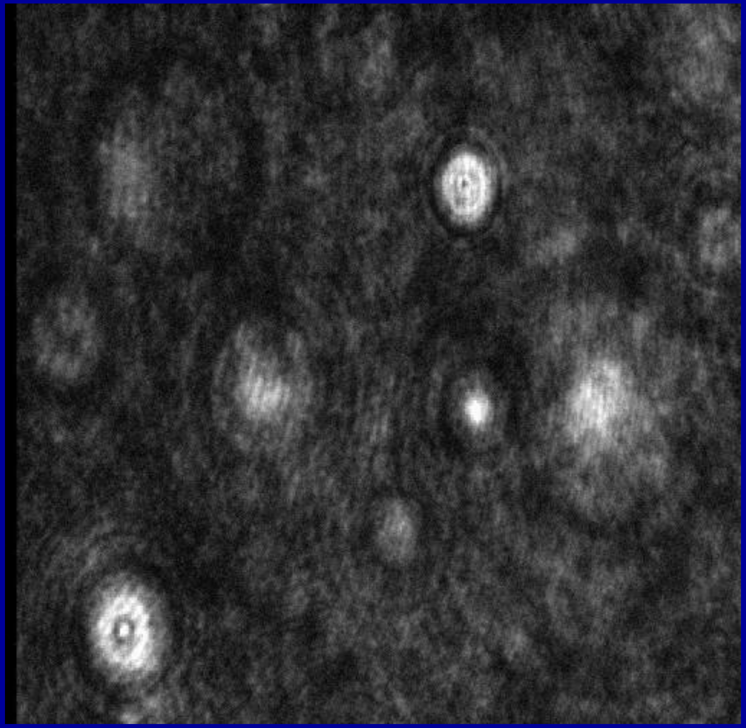
APPLICATIONS (3)



APPLICATIONS (4)

3D restitution of water drops from an hologram

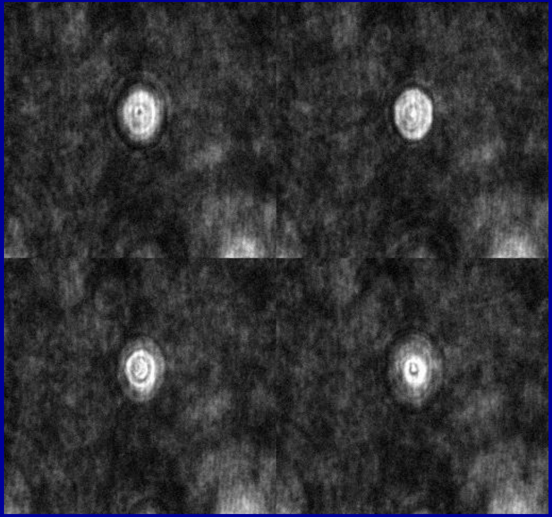
A 3D image of an aerosol (artificial fog) is generated from an hologram. Various sections of the 3D image are taken with a low focus depth camera.

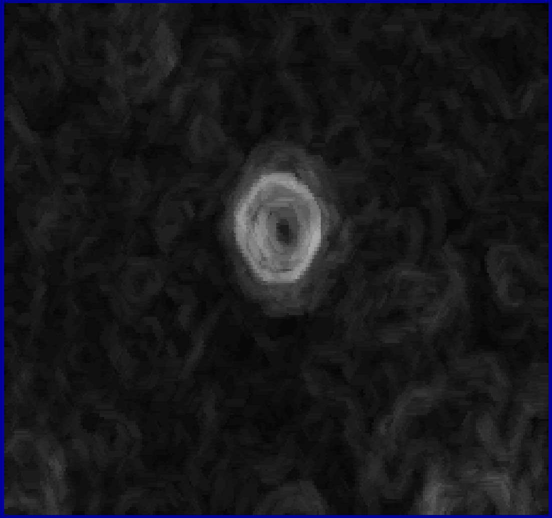


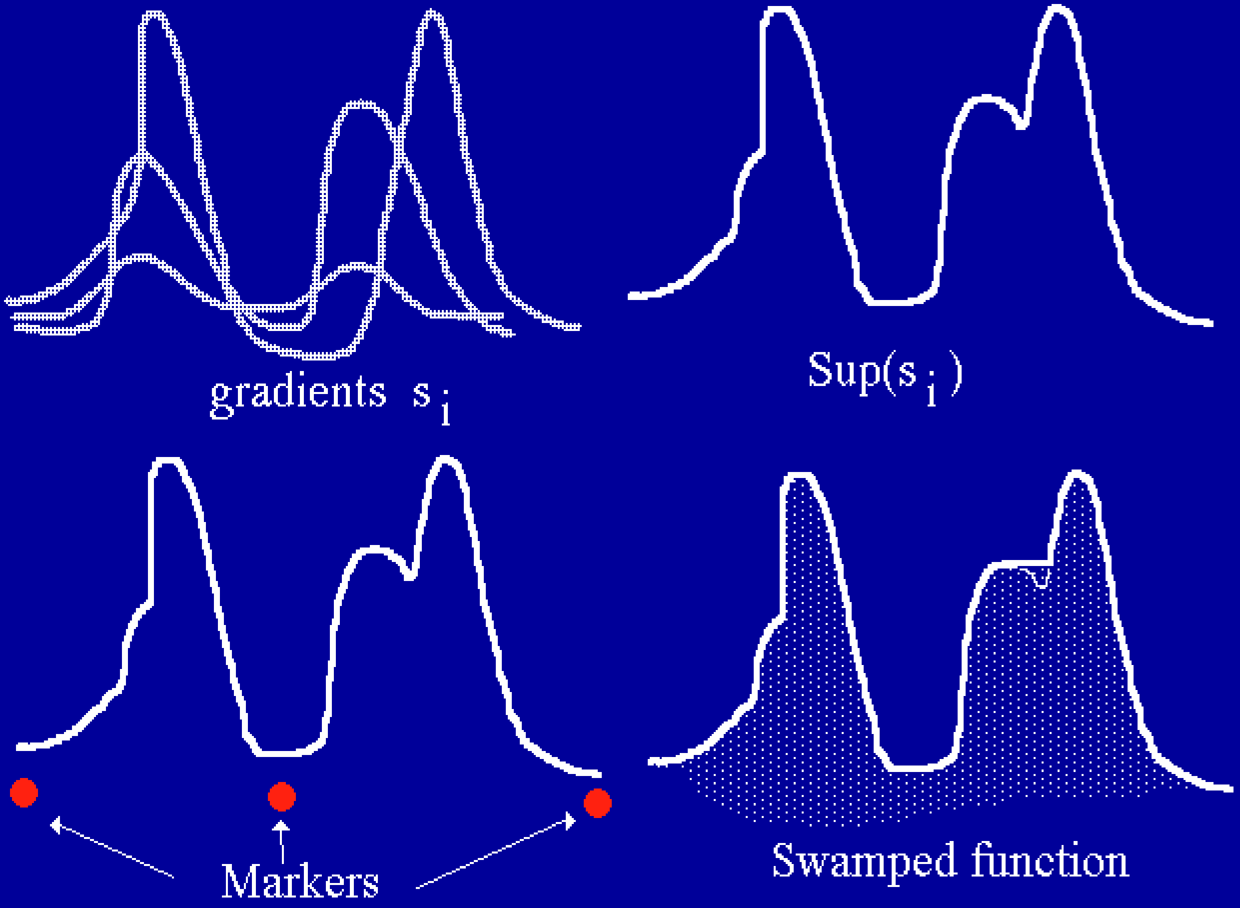
- **n sections s_i**
- **find the best contour**
- **position x, y, z of each drop**
- **volume**

APPLICATIONS (5)

Criterion:
Sup of the gradients



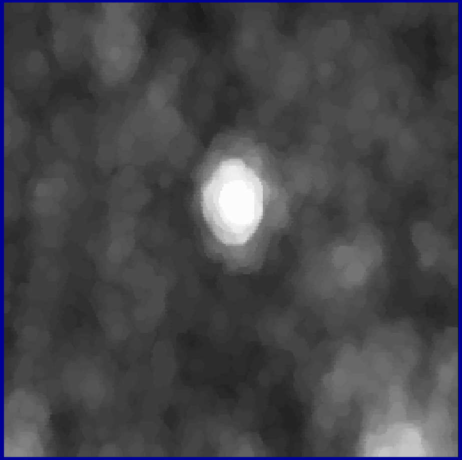


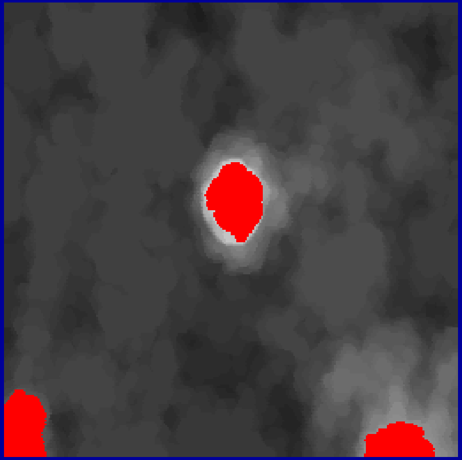


APPLICATIONS (6)

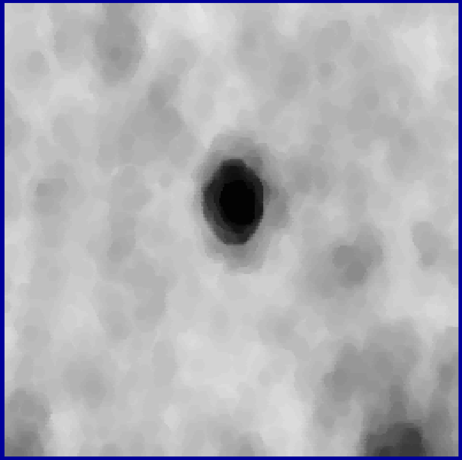
Markers:

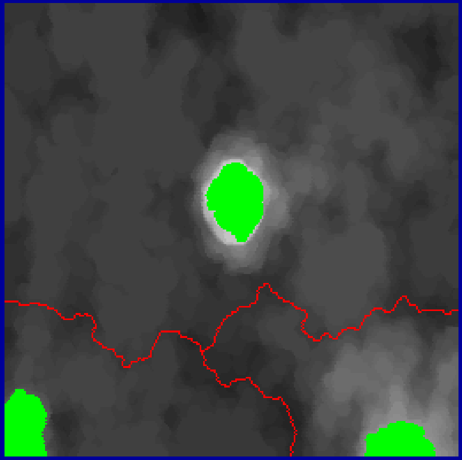
- **Drops → significant maxima of the filtered sup of all the sections**





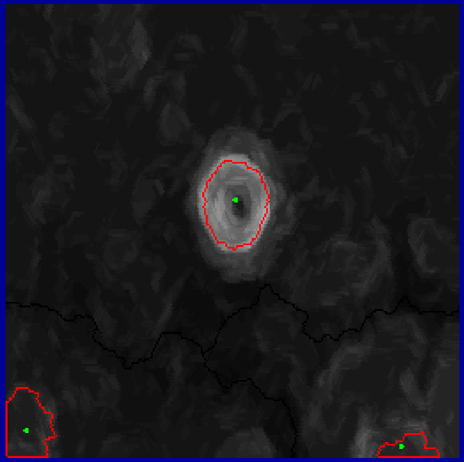
- **Background → watershed of the sup image (inverted)**



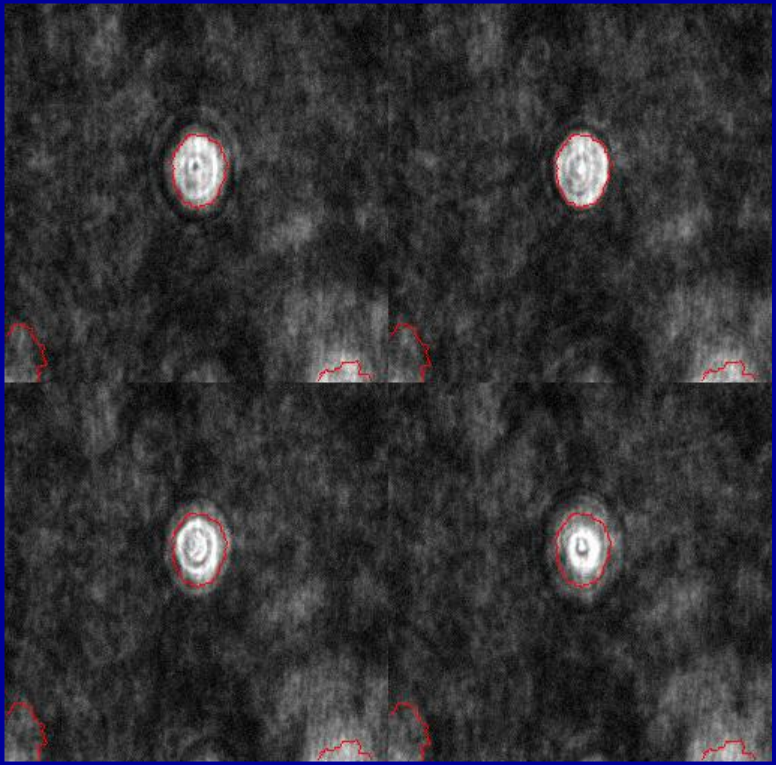


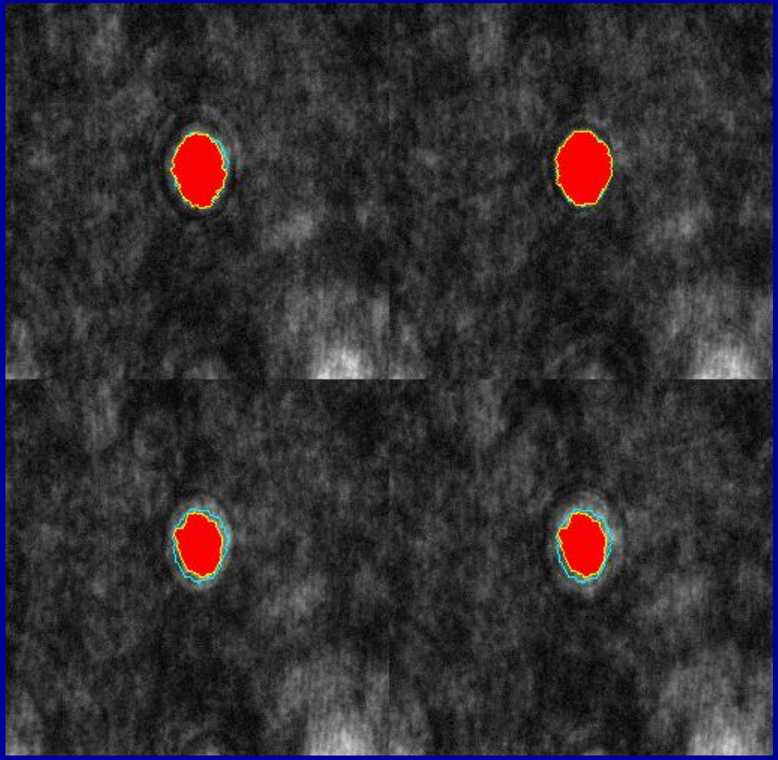
This watershed is a marker-controlled watershed (markers of the watershed are the drop markers)

APPLICATIONS (7)



Final watershed (left). The same watershed image superimposed on the different sections (right).





To find the best section, a marker-controlled gradient watershed is performed on each section with the same set of markers (result in blue) and the best fit with the previous contour is determined. The corresponding section gives the z-position of the drop.

APPLICATIONS (8)

Traffic lanes segmentation

The markers of the lanes are determined by an automatic thresholding. The marker of the background is the complementary set of a dilation.

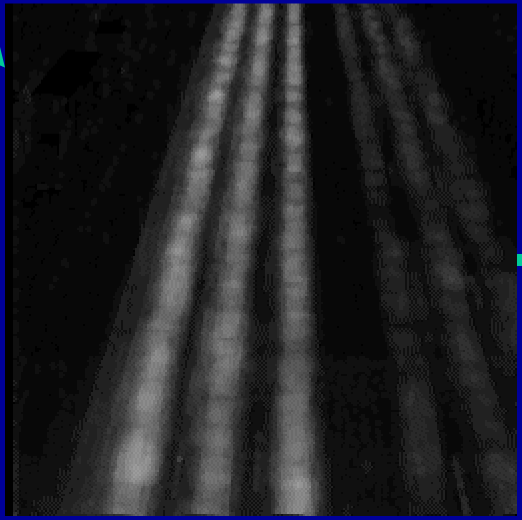
From a sequence of n images f_i, two images are computed:

- The mean,

$$\Sigma f_i /n$$

- The mean of absolute differences,

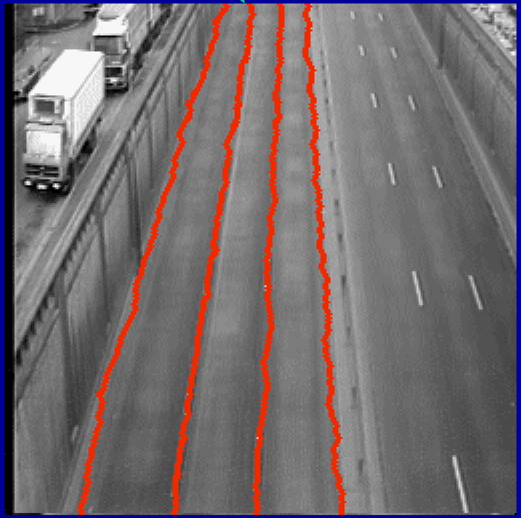
$$\Sigma |f_i - f_j|/n$$



APPLICATIONS (9)



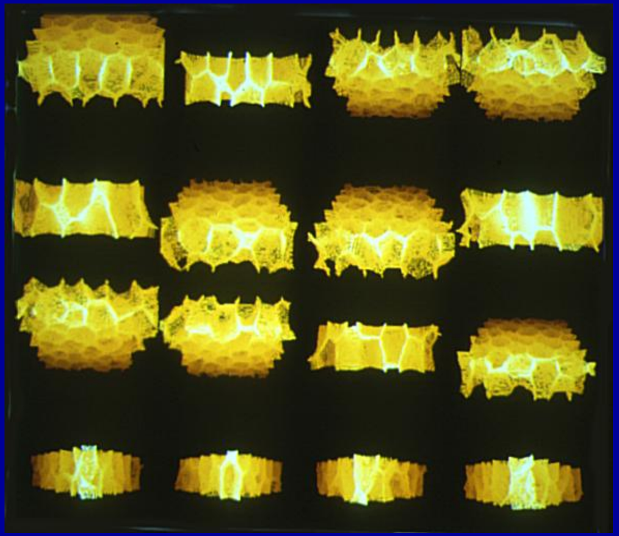




- **Extraction of road marking by a top-hat transform**
- **Calculation of the distance function of the road marking between the markers**
- **watershed of the distance function**

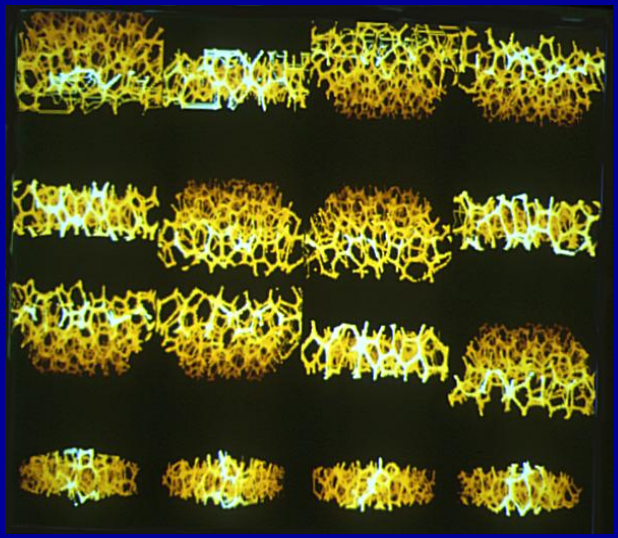
APPLICATIONS (10)

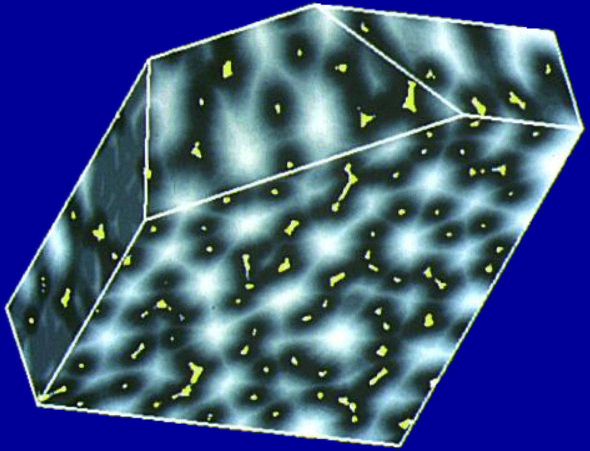
3D segmentations based on distance functions



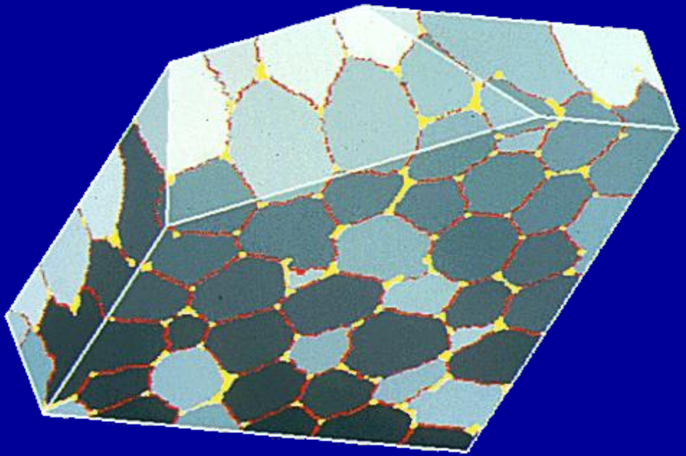
3D watershed

Polyester foam





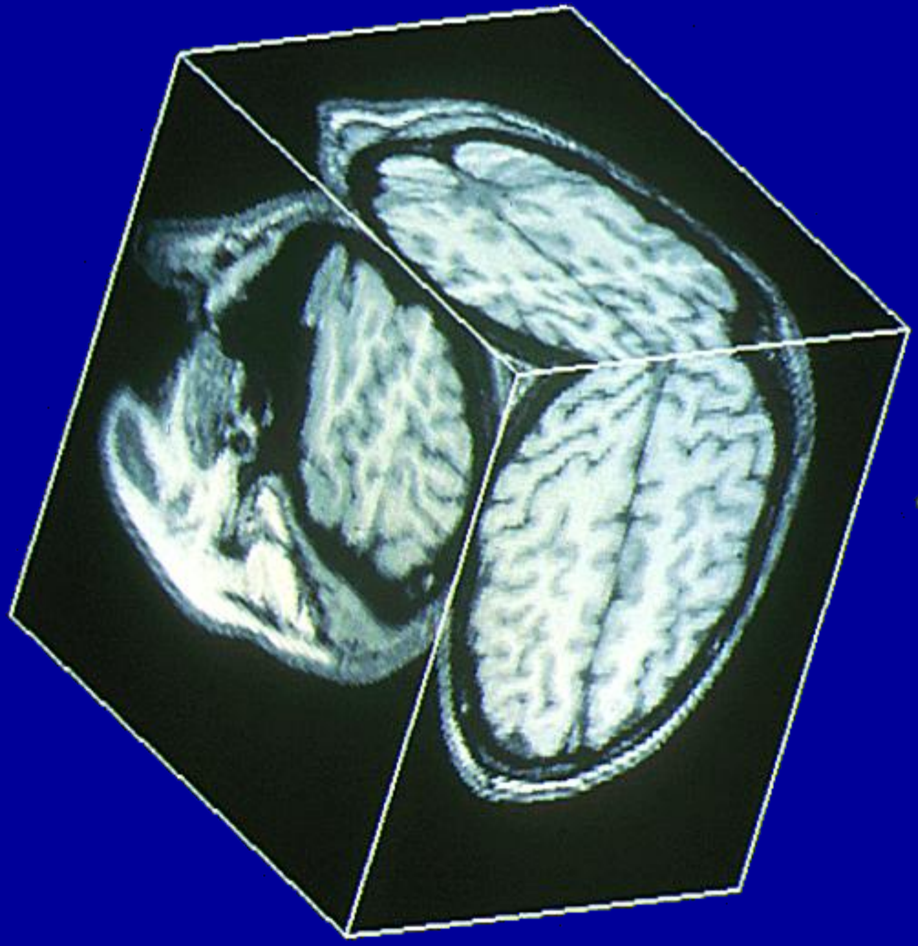
Distance function

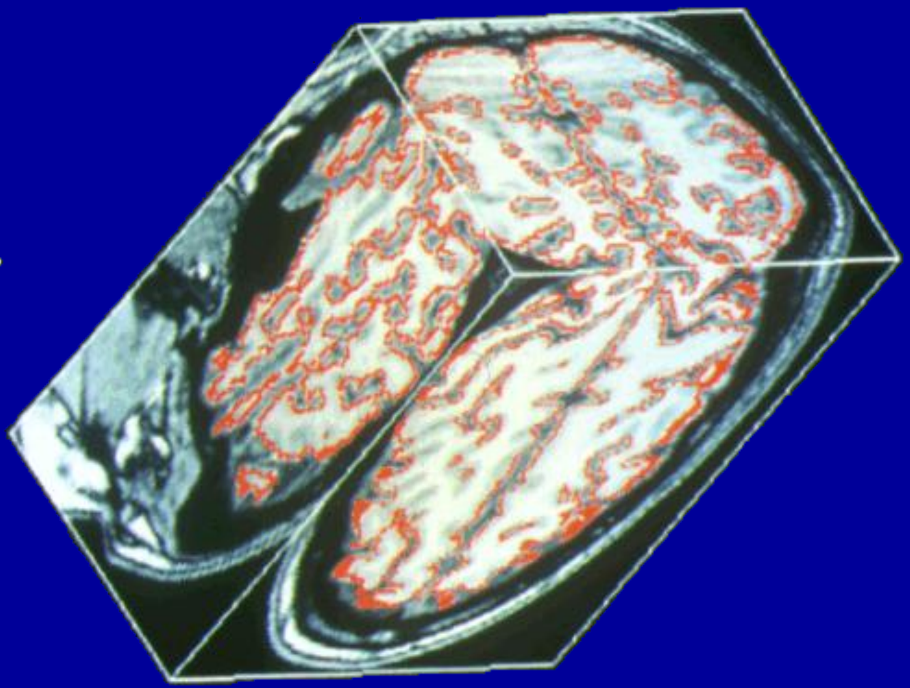


APPLICATIONS (11)

3D segmentations based on gradients

3D brain NMR image





HIERARCHICAL SEGMENTATION, WATERFALLS

Pending problems

It is not always possible to prevent over-segmentation by marker-controlled watershed because it is not always possible to find good markers and/or segmentation criteria.

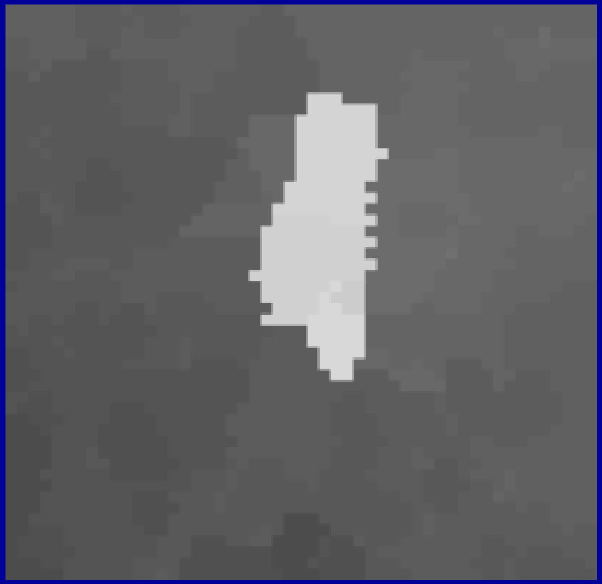
Is it necessary to define markers for the objects **AND** for the background?

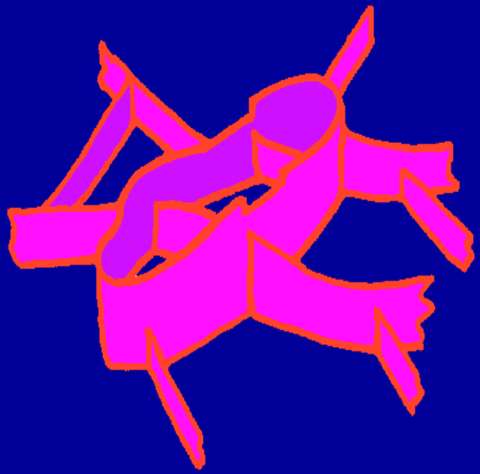
How to be sure that the markers are well-positionned?

Use of multiple criteria and comparison of watersheds.

OVER-SEGMENTATION AND PERCEPTION OF IMAGES

A simple illustration using a mosaic image

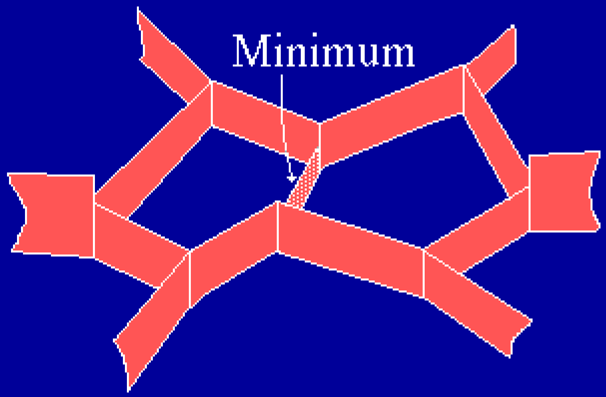


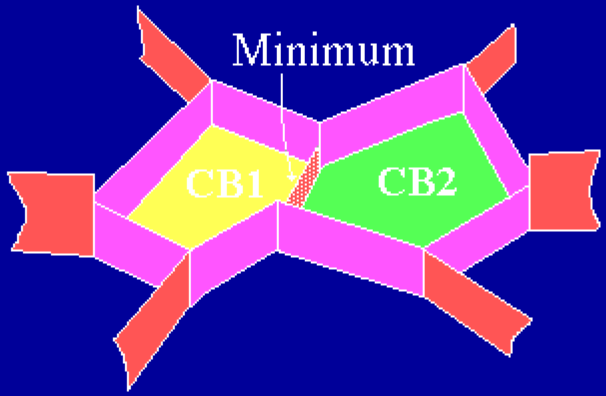


Despite the fact that the image is over-segmented, the white blob can be easily distinguished from the background because, at the same time, the boundaries between the regions inside the blobs and the boundaries inside the background are less contrasted than the boundaries wich separate the blob and the background. Both the blob and the background are marked by boundaries with a minimal contrast.

GRAPH DEFINITION

Arcs of minimal height





In the mosaic image, each arc c_{ij} separates two catchment basins CB_i and CB_j. The valuation v_{ij} of the arc is given by:

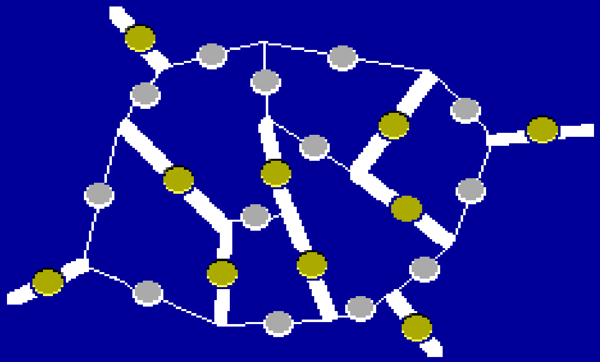
$$v_{ij} = | g_i - g_j |$$

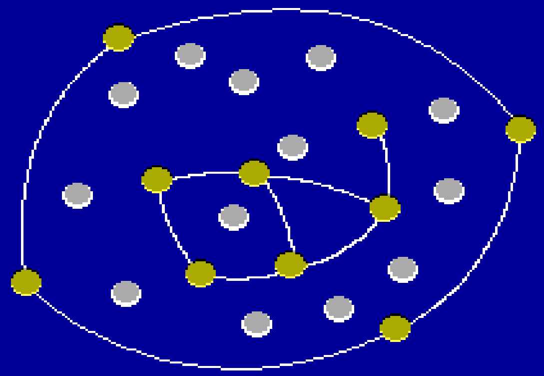
where g_i and g_j are the grey values in the catchment basins.

An arc c_{ij} is said to be minimal if its valuation is lower than those of all the other arcs surrounding CB_i and CB_j

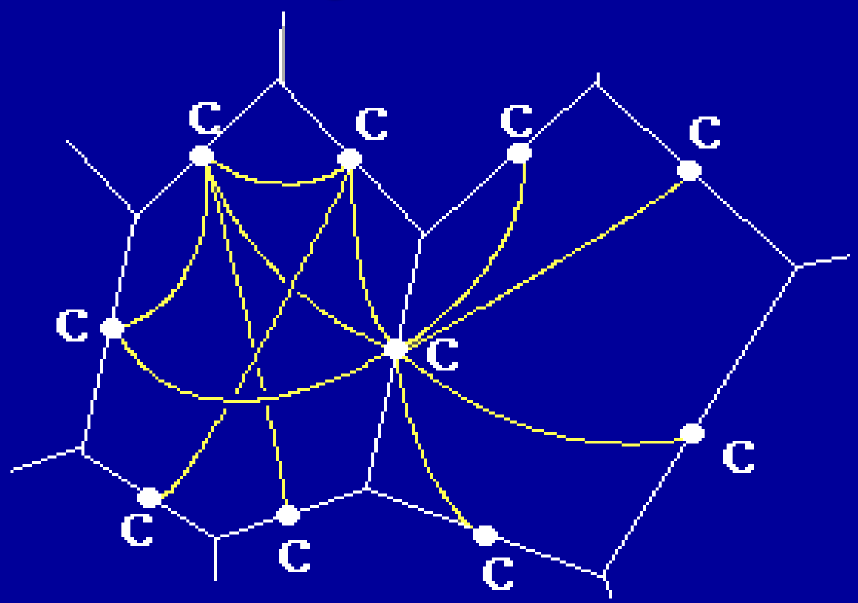
GRAPH DEFINITION AND ASSOCIATED WATERSHED

Definition of a new graph



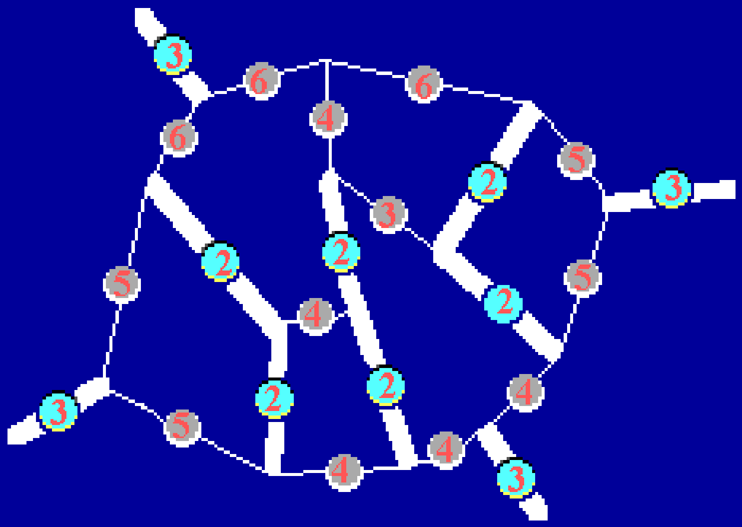


- its vertices correspond to the arcs of the gradient mosaic
- its edges link all arcs surrounding the same catchment basin
- each vertex is valued by the arc valuation as defined in the gradient mosaic

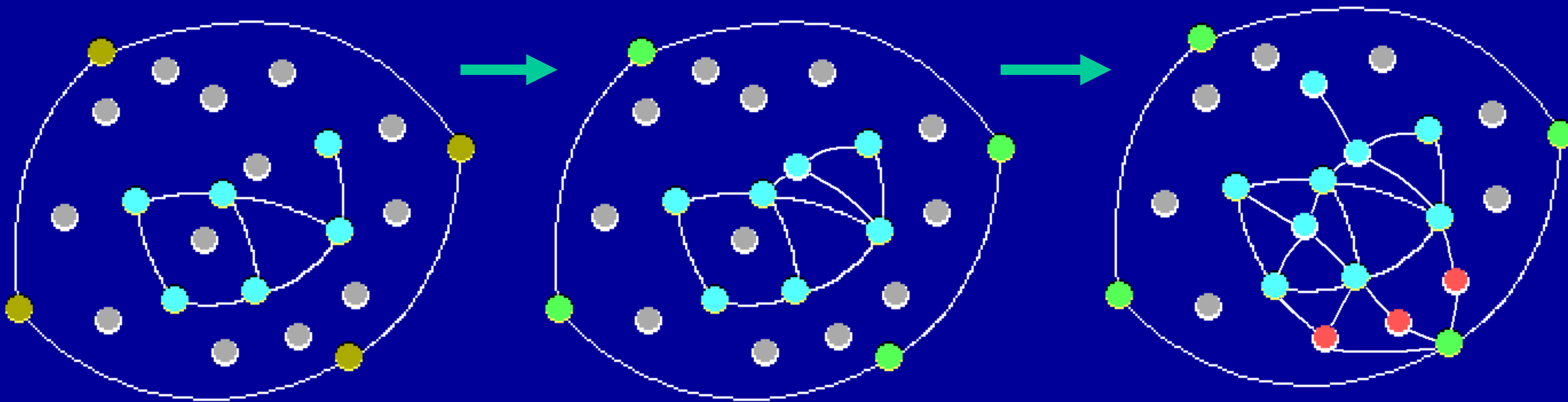


In this representation, the arcs surrounding the same catchment basin are adjacent. Therefore, minimal arcs can be connected although it is not the case in the gradient mosaic, as illustrated above (yellow summits correspond to minimal arcs).

GRAPH DEFINITION AND ASSOCIATED WATERSHED (2)

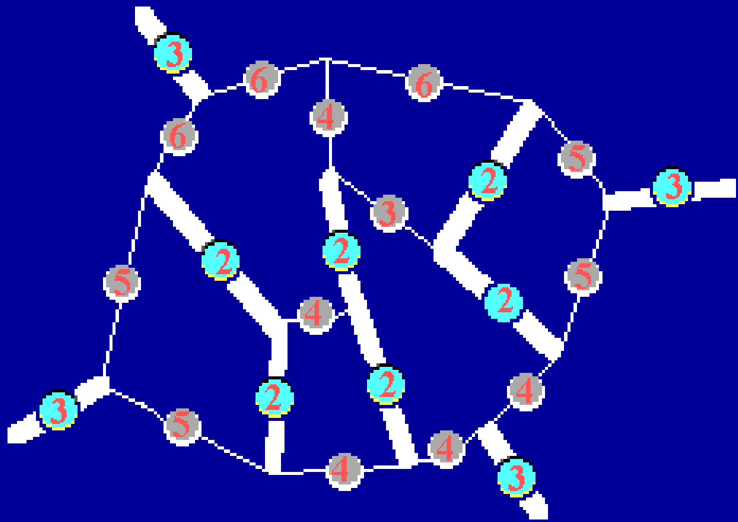


The most significant contours of the mosaic image correspond to those separating regions marked by minimal arcs. They are the watershed lines of the watershed transform defined on the previously defined graph.

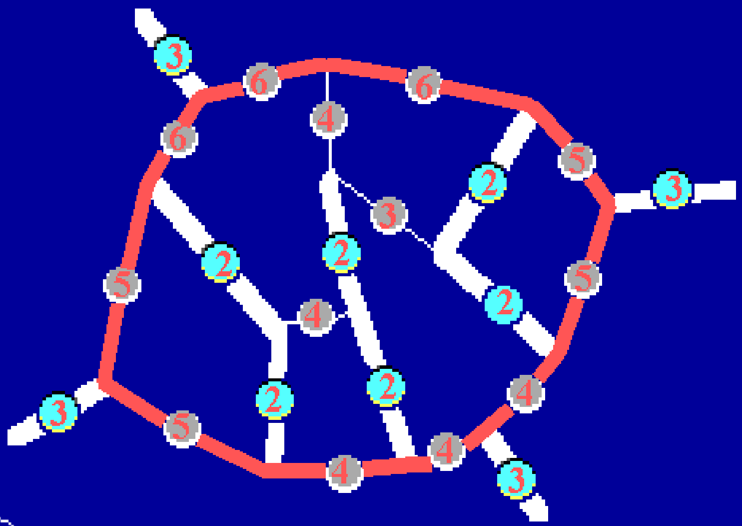


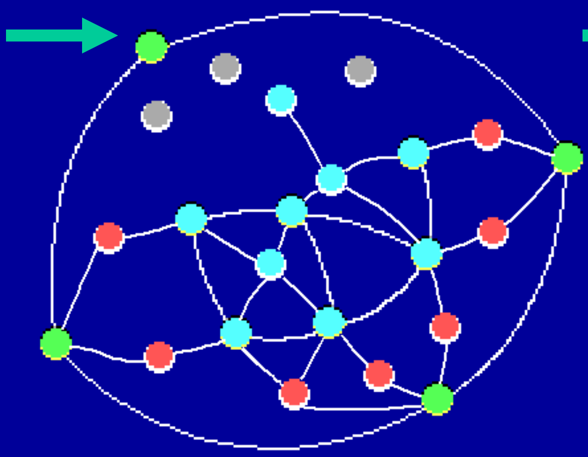
Flooding, step 1 (in blue) Step 2, two CBs, in blue & green Step 3, first dams in red

GRAPH DEFINITION AND ASSOCIATED WATERSHED (3)

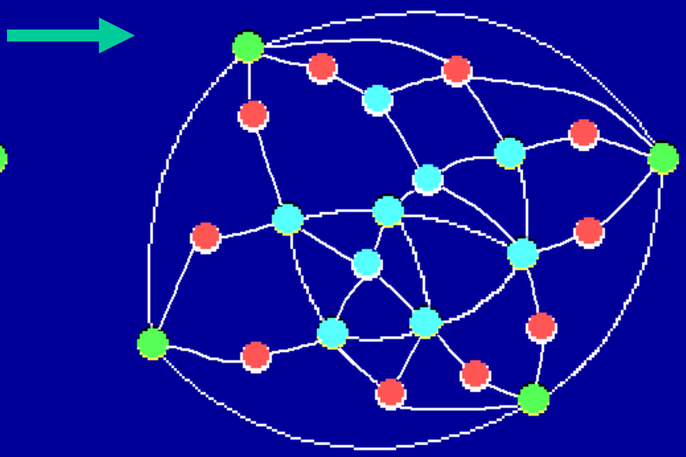


Arcs of the gradient mosaic corresponding to the watershed lines.



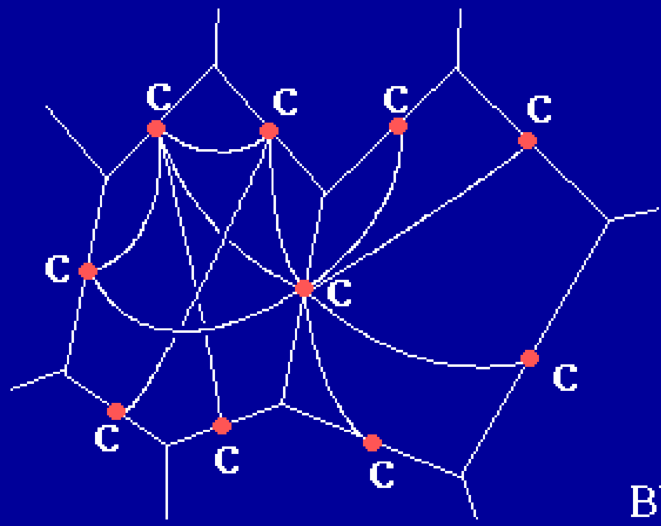


Step 4

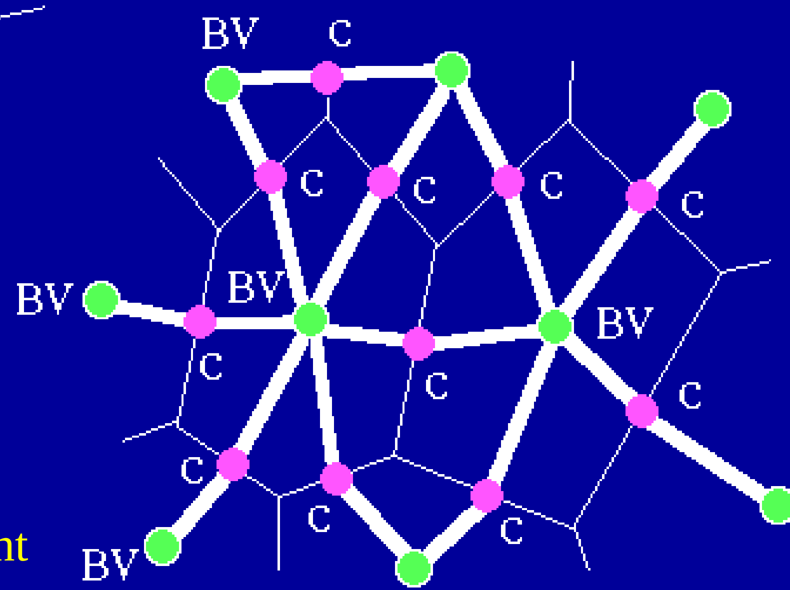


Final watershed

FROM A 3D to A PLANAR GRAPH



The previously defined graph is a 3D valued graph, which is not very handy.



This graph can be transformed into a planar one by the following procedure:

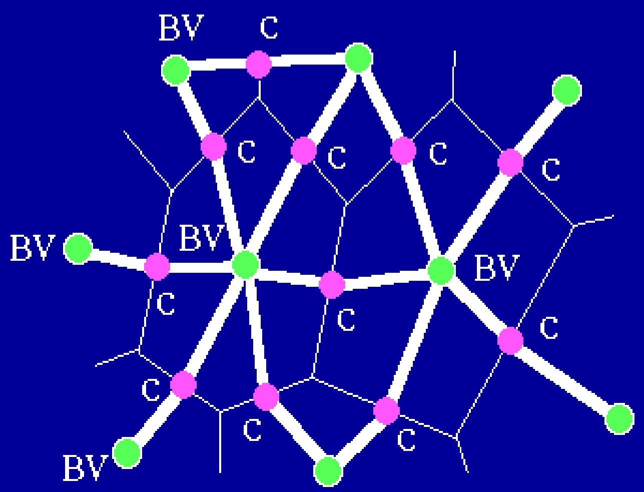
- A new vertex is added in each catchment basin.
- The previous edges are replaced by two successive edges linking the original vertices through the new one.
- The valuation of the new vertex is given by:

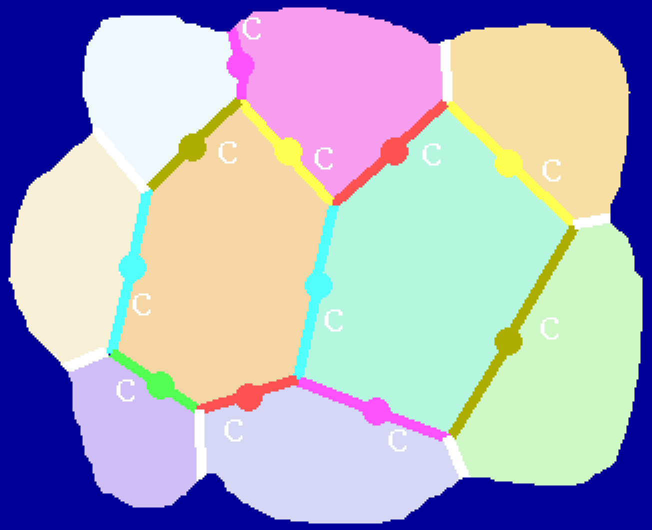
$$\min (v_{ij})$$

where v_{ij} are the valuations of the arcs surrounding the catchment basin.

IMAGE REPRESENTATION

The hierarchical image





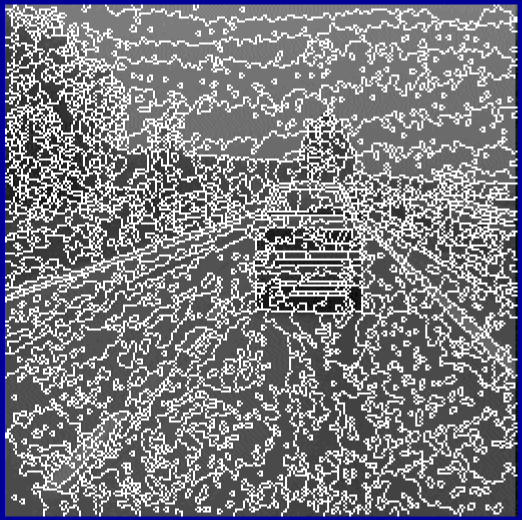

mosaic


gradient mosaic


hierarchical image

An image, named hierarchical image can be build from the planar graph. The catchment basins of the gradient mosaic are filled with grey values corresponding to the valuation of the new added vertices.
The watersheds of this hierarchical image give the higher level of hierarchy (with some restrictions).

HIERARCHICAL SEGMENTATION: EXAMPLE



Initial watershed

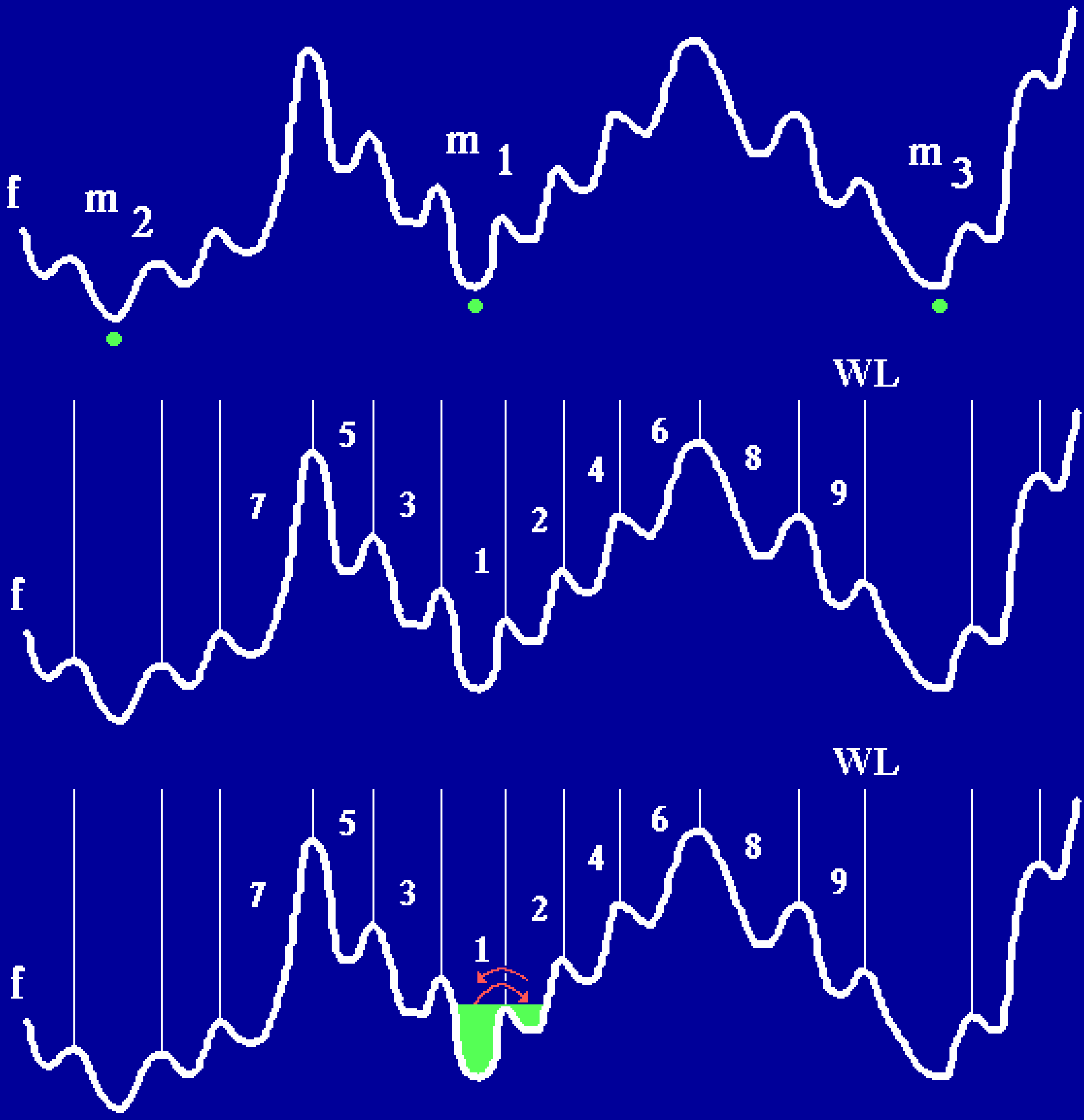
Original image

First level of hierarchy

Mosaic image

WATERFALLS

Introduction



Consider the function f and its watershed. Various catchment basins are numbered from 1 to 9. Consider the flooding from the minimum m_1.

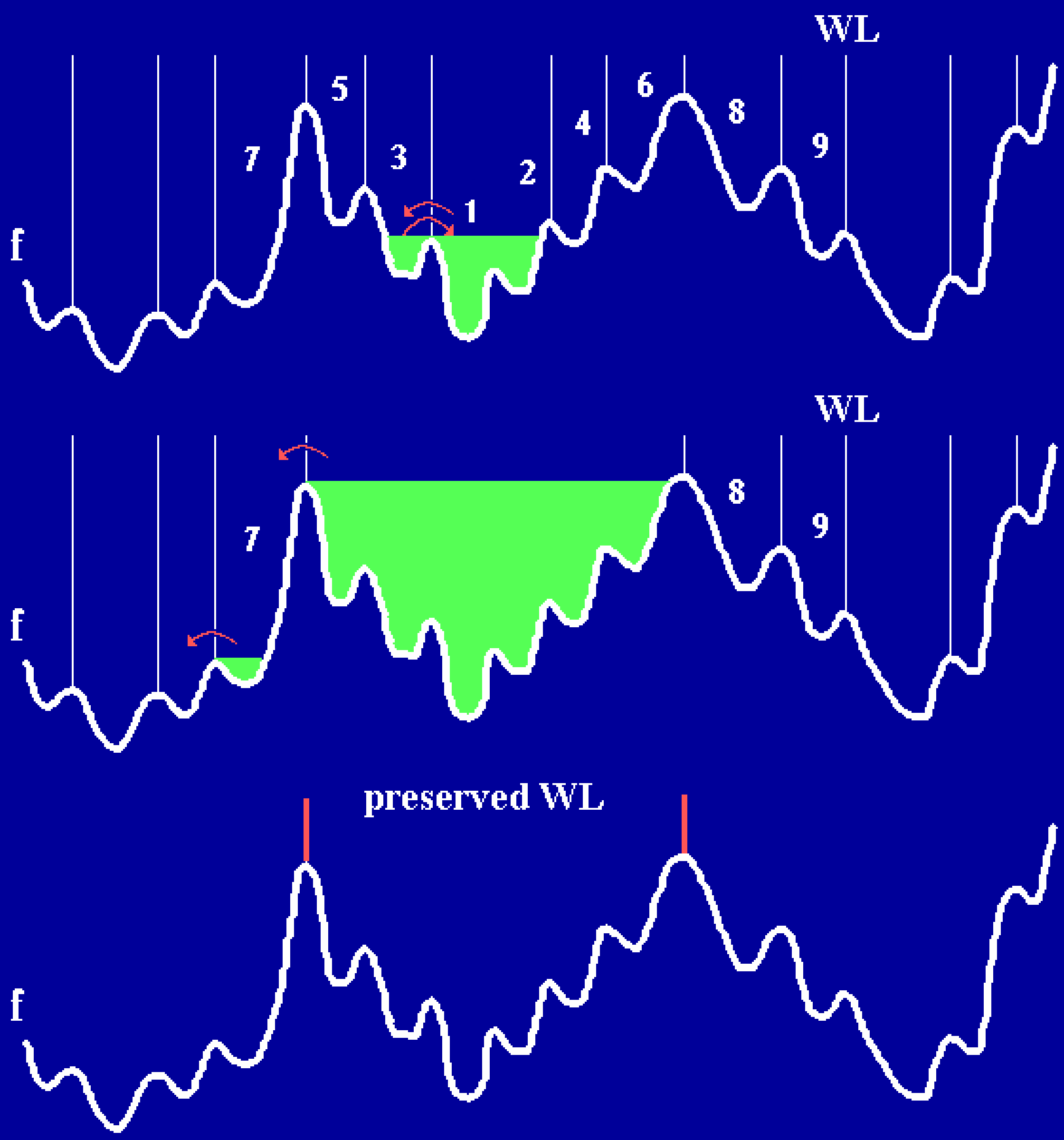
When filling CB1, an overflow occurs towards CB2.

Now, if we fill in CB2, the first overflow occurs towards CB1.

In this case, overflows (waterfalls) are symmetrical.

Therefore, the part of the watershed line separating CB1 from CB2 can be removed and the floods in CB1 and CB2 can be merged.

WATERFALLS (2)

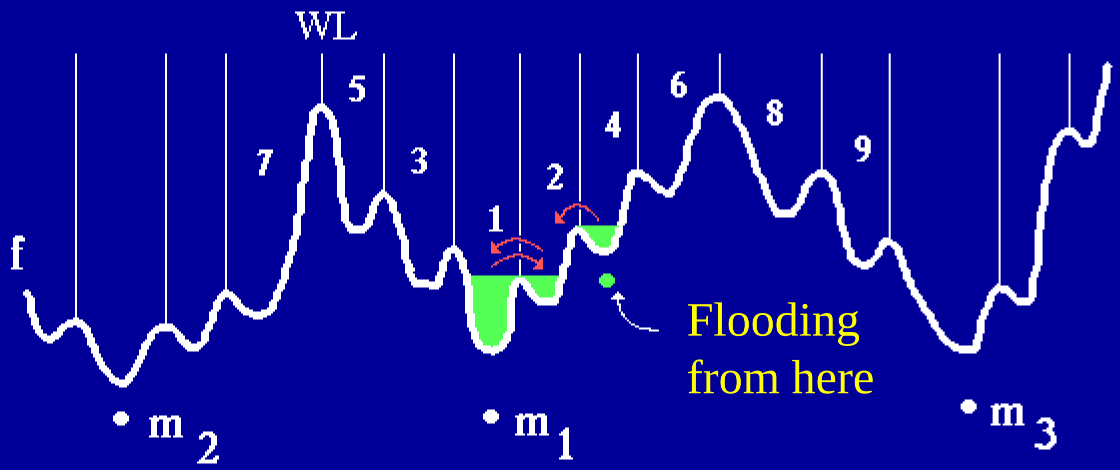


If this flooding process is iterated, the flood invades CB3 which in return, when flooded, pours into the merged basins CB1 and CB2. Here again the waterfalls being symmetrical, CB3 is merged to the flood.

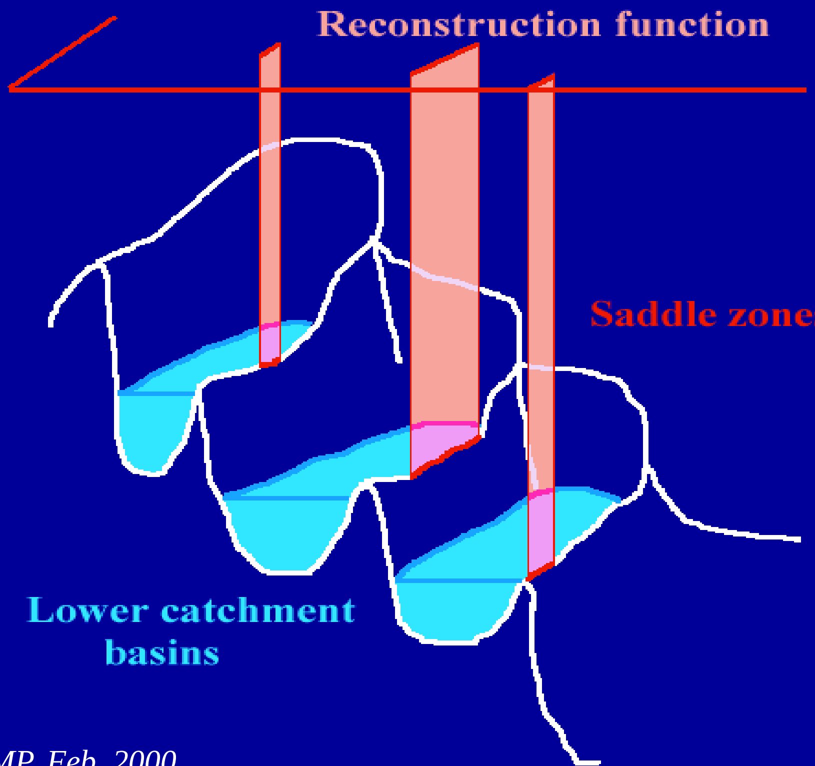
Step by step, because, in each case, waterfalls are symmetrical, all the catchment basins from 1 to 6 are merged.

But, when the flood pours into CB7, the situation changes. Now, if we flood CB7, the waterfall is no longer symmetrical. Therefore, the watershed line between CB7 and the merged basins must be preserved.

SIGNIFICANT CB/ARCS AND RECONSTRUCTION



The previous process does not work if we start from any basin. However, the flooding in the end reaches the significant CBs.

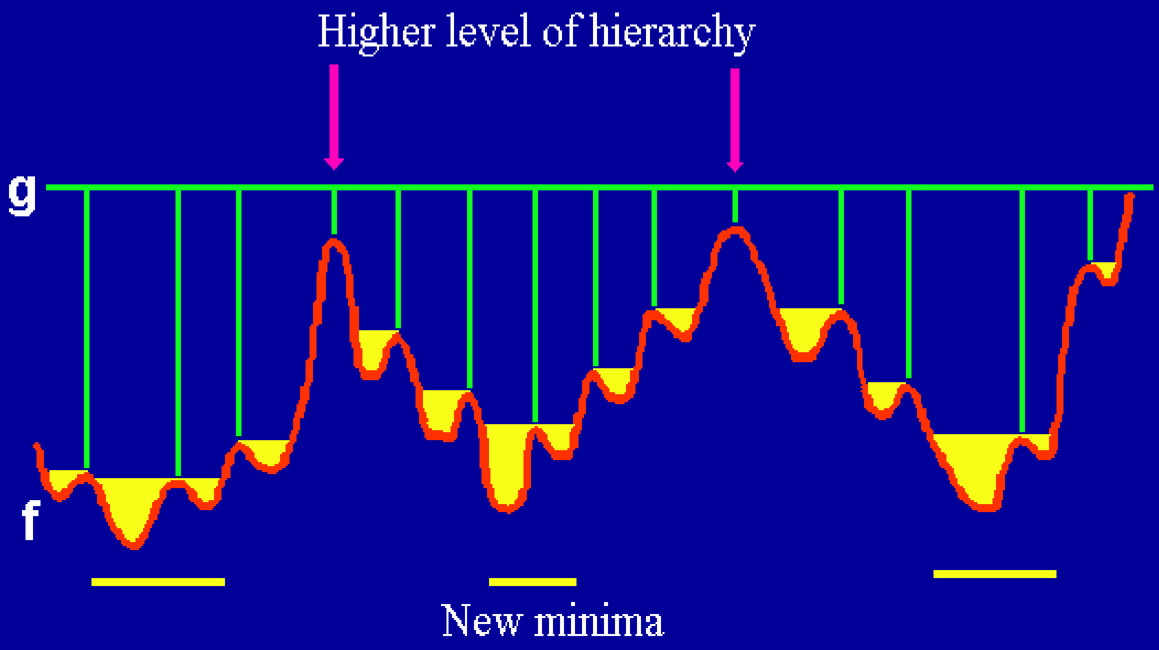


The successive floods generates the lower catchment basins associated with each CB (flood just before the overflow through the lower saddle zone).

This can be achieved directly by a dual reconstruction of the initial function by the lower saddle zones.

SIGNIFICANT CB/ARCS AND RECONSTRUCTION (2)

Instead of using the lower saddle zones (difficult to get them), the entire watershed lines can be used. The result will be identical because the lower saddle zone is the region surrounding the catchment basin at the lowest altitude.



f, initial function

let us define g:

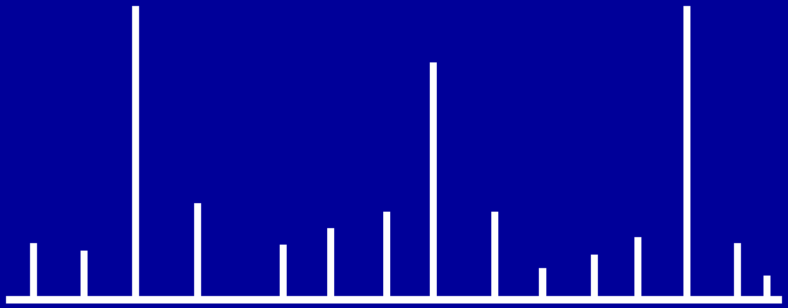
g (x) = f (x) iff x belongs to the watershed lines of f
g (x) = max if not

$$h = R^*_f (g)$$

WT (h) → hierarchy

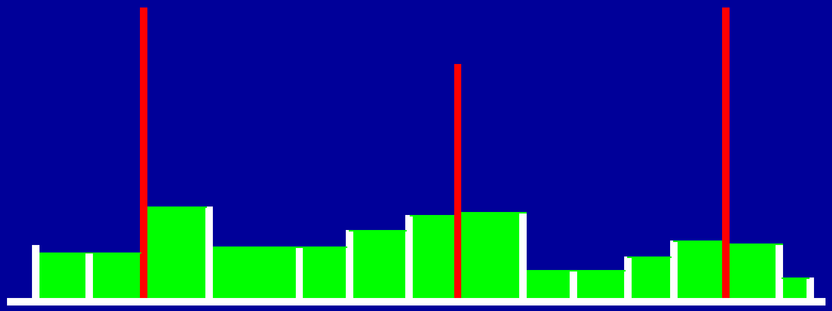
WATERFALLS AND MOSAIC IMAGES

In this case, the hierarchical approach and the waterfall approach are identical. The waterfall transformation is the generalisation for any function of the hierarchical approach.

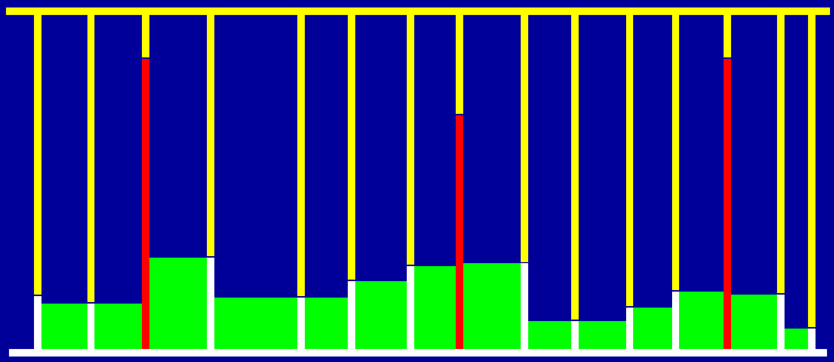


Gradient mosaic image

The minimal valuation of the catchment basin corresponds to the height of the lower saddle zone. This valuation produces the same result as the reconstruction of the gradient mosaic function by the lower saddle zone.



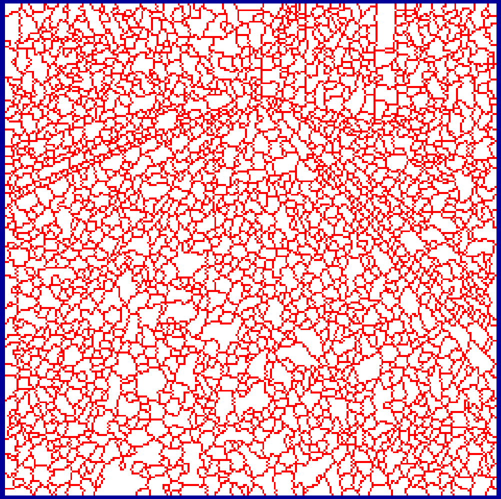
Hierarchical image

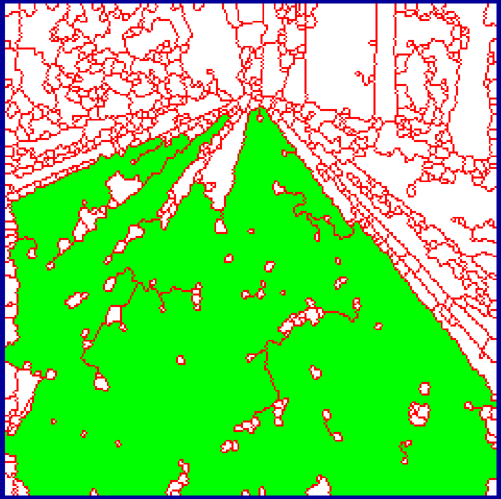


Waterfall image

APPLICATION EXAMPLES

It's just a watershed over the watershed...





The hierarchical segmentation produces a new catchment basin (in green) witch can be used as marker of the road.

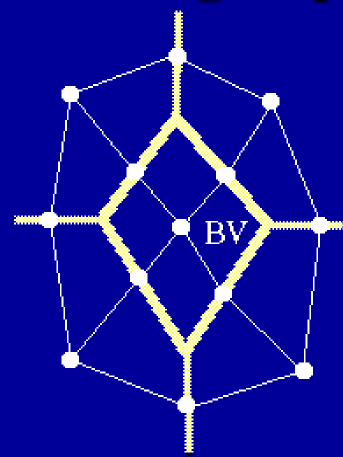
Then, the outside marker can be chosen among the significant catchment basins.

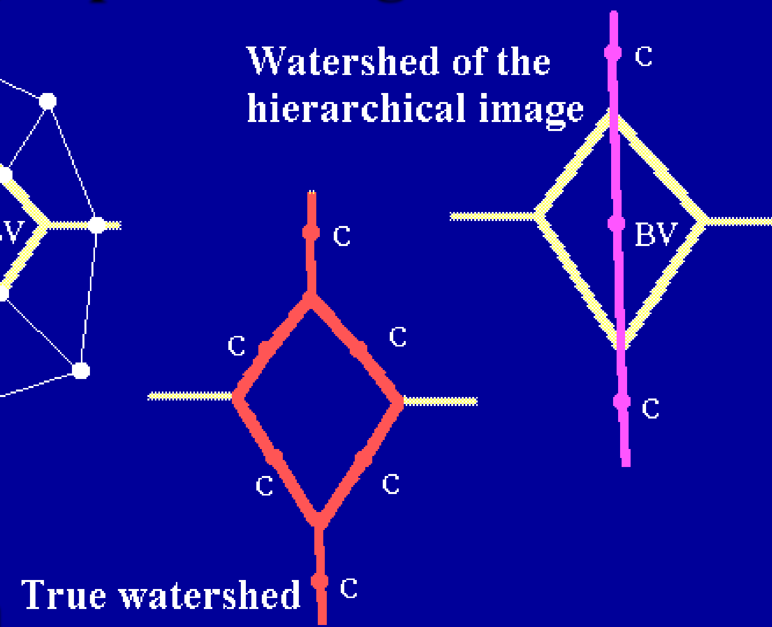
ADVANTAGES AND DRAWBACKS OF WATERFALLS

• It's a non parametric approach

• The waterfall can be iterated, leading to possible higher levels of hierarchy

• Some special features need a special treatment (these structures are equivalent to button-holes)



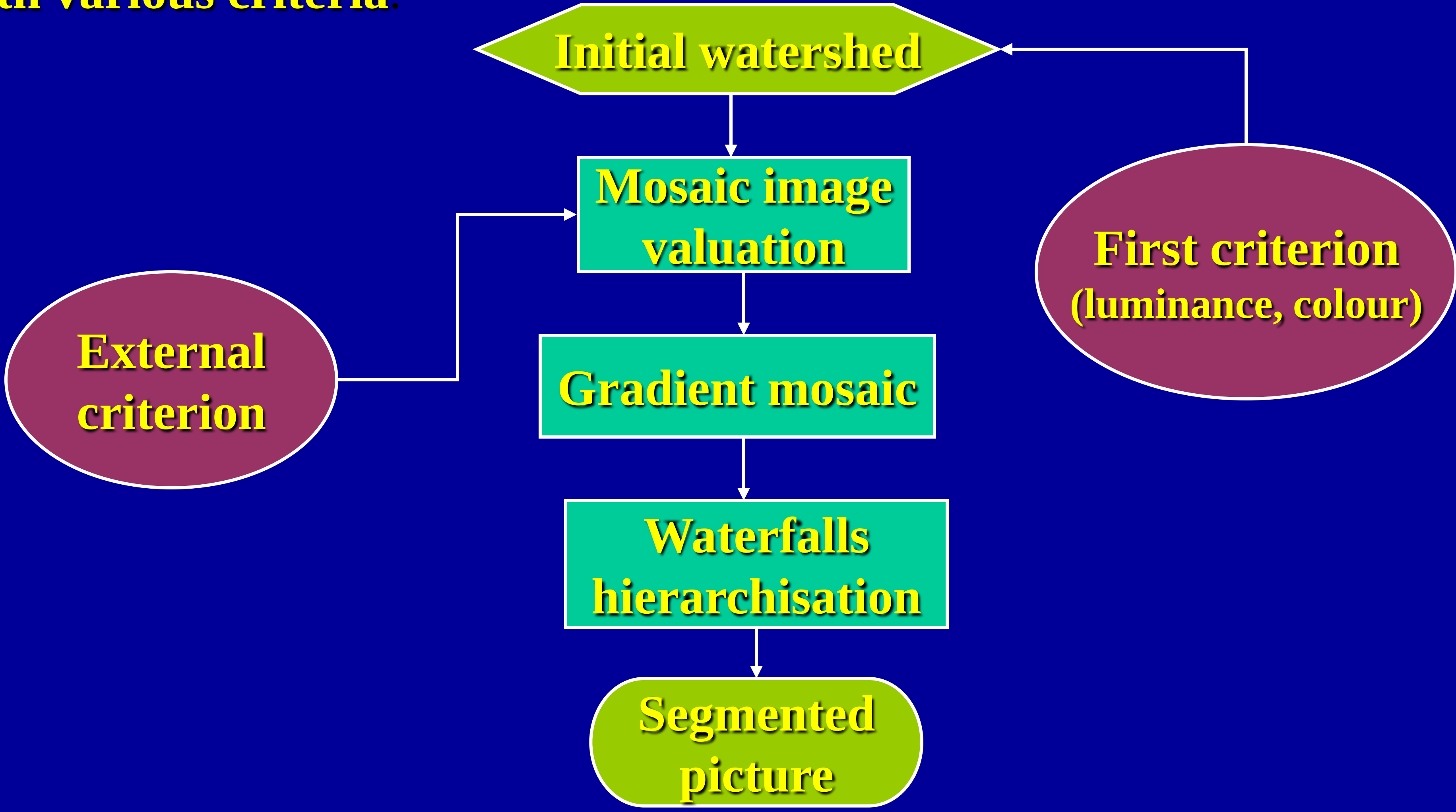


• It is difficult to handle regions with different characteristics (textured/non textured), but a geodesic approach (masking) is possible

• Stop criterion not available

THE WATERFALLS PARADIGM

The waterfall transform is a general technique which can be used with various criteria.

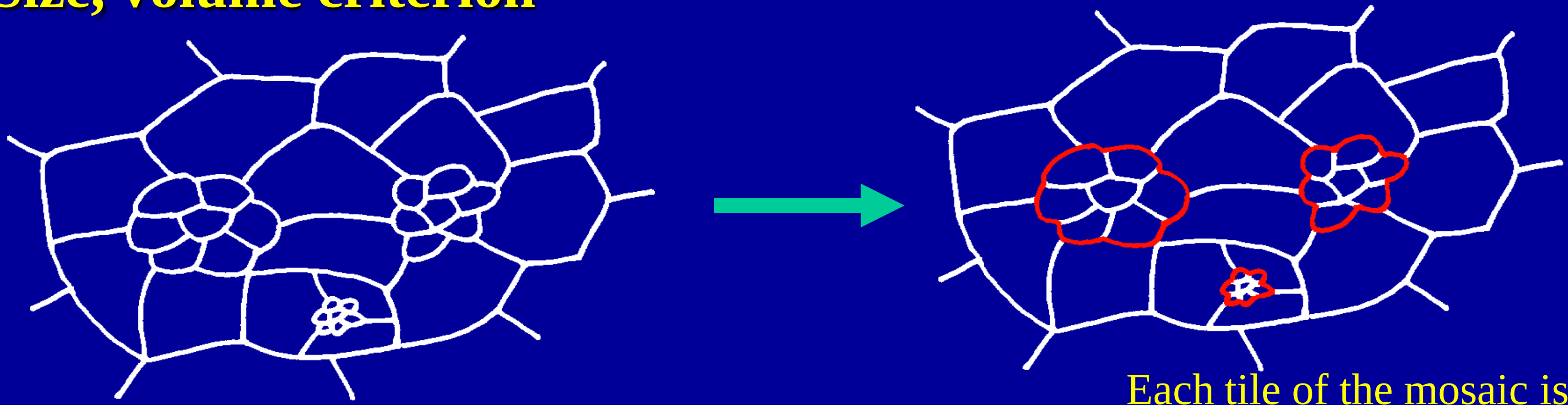


Pending problem: How to use waterfalls in a multi-criteria segmentation?

EXAMPLES

- **Color segmentation**

- **Size, volume criterion**

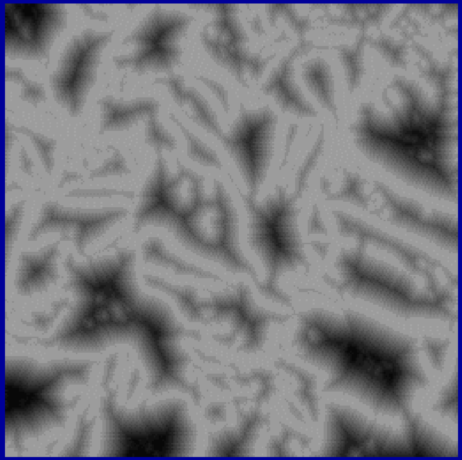


Each tile of the mosaic is valued by its area

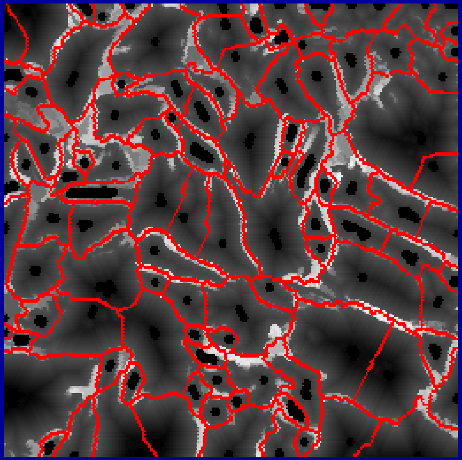
- **Dynamics of minima, of contours, etc..**

DETAILED APPLICATIONS

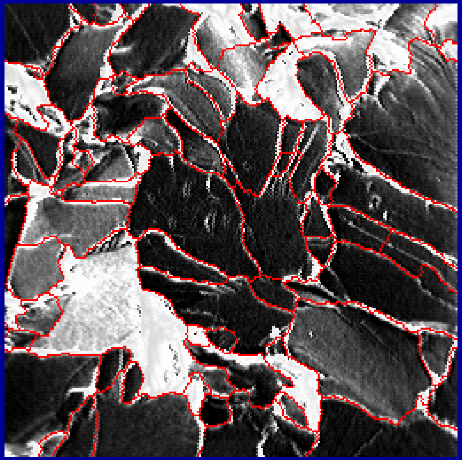
Cleavage fractures in SEM steel images



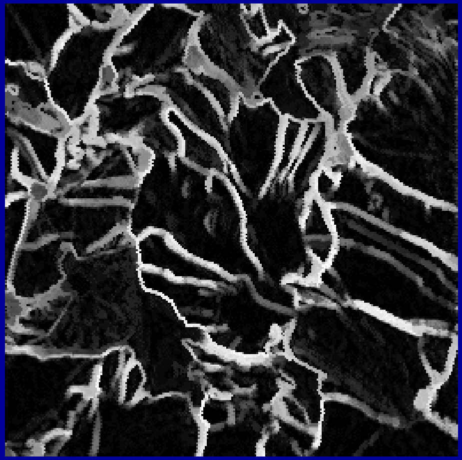
Distance function



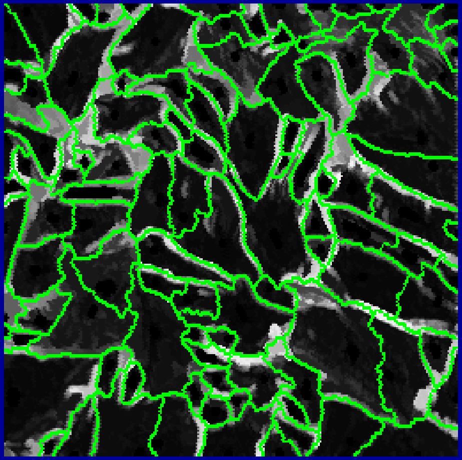
First watershed



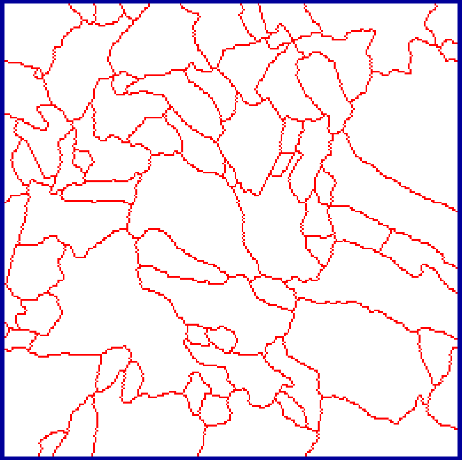
Common dams in both watersheds



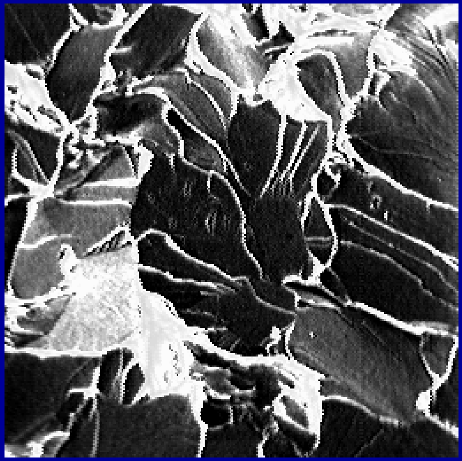
Contrast function

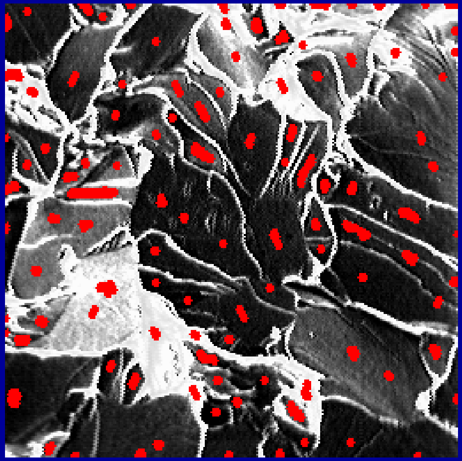


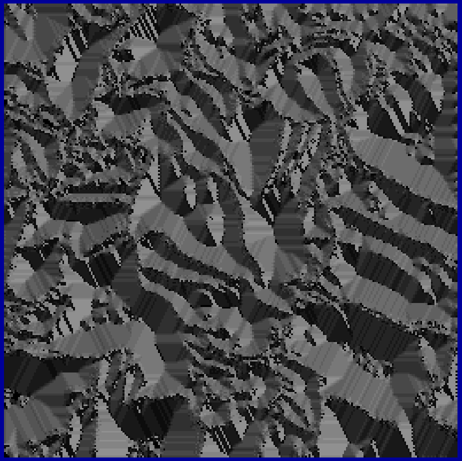
Second watershed



DETAILED APPLICATIONS (2)



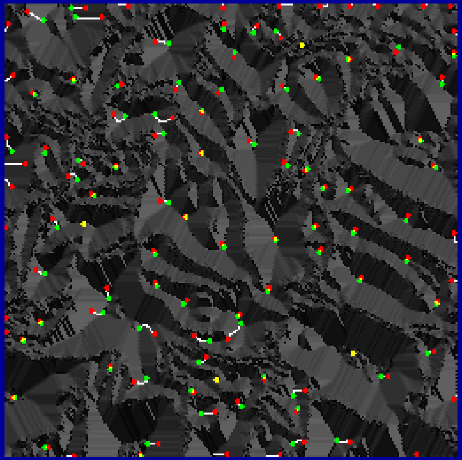


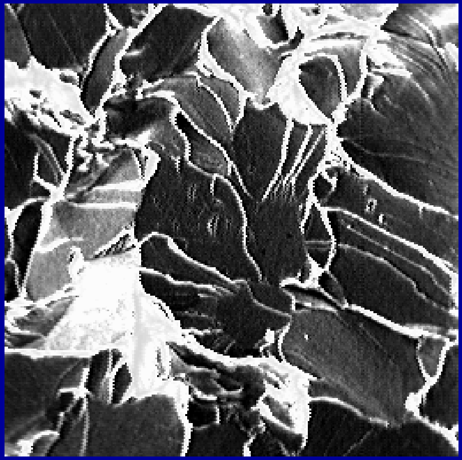


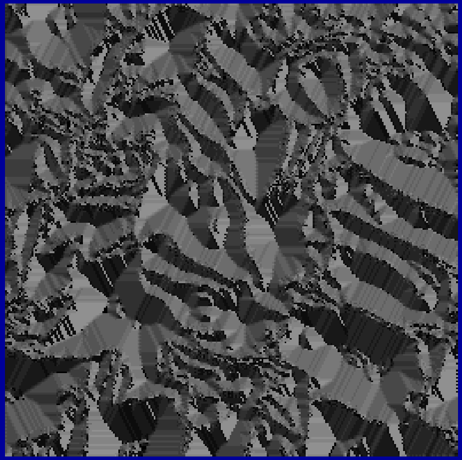
Stereoscopic pair

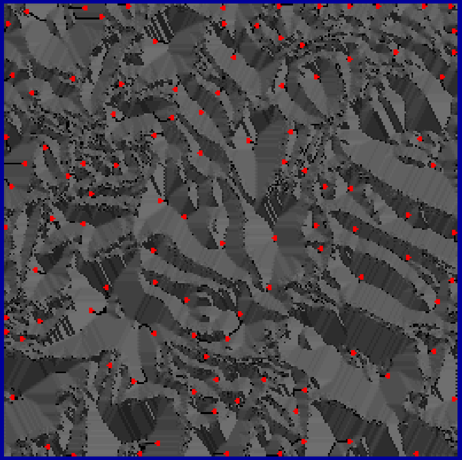
Markers of the first image

Azimuth of distance function





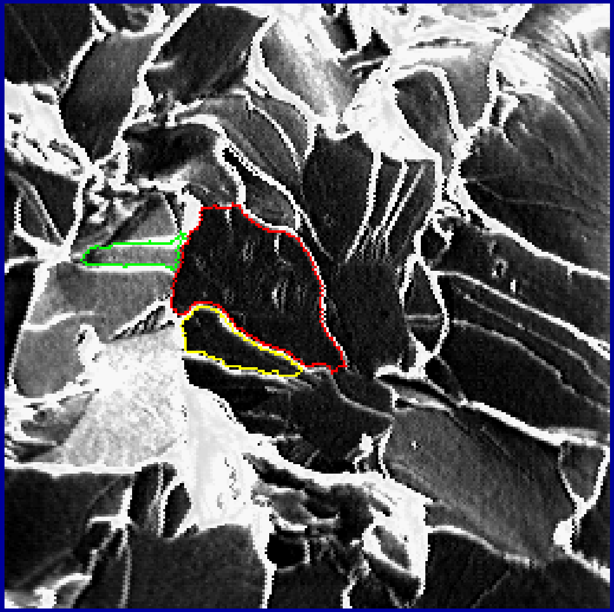




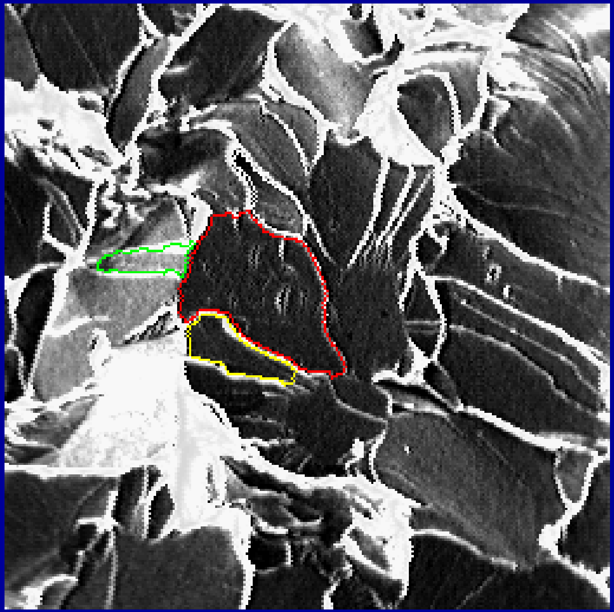
Azimuth (2nd image)

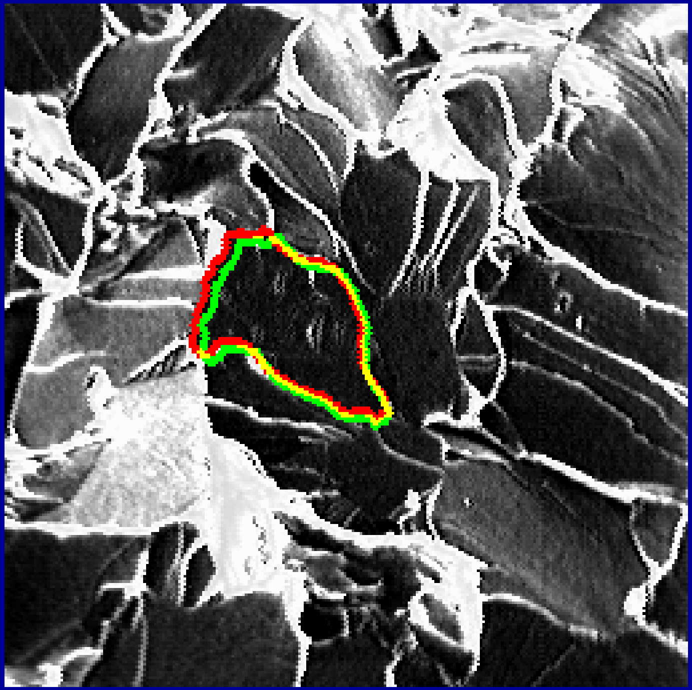
The markers of the first image are thrown on the second one…
and migrate along the steepest slope to give the new markers (in green).

DETAILED APPLICATIONS (3)



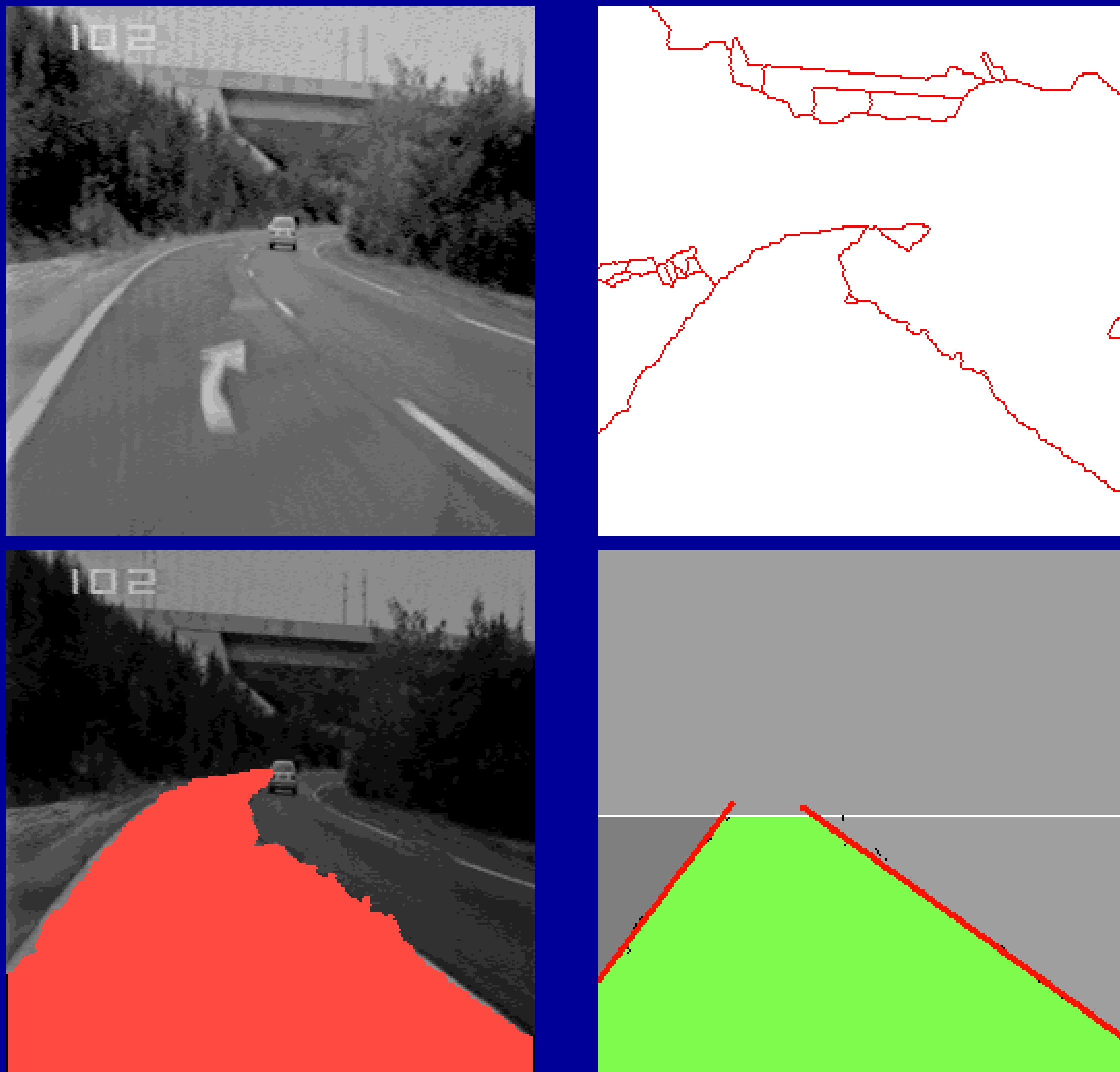
On the right, contour of facets on the first image and the homologous ones in the second image.
Below, displacement of a single facet which can be measured, allowing the computation of its altitude.





DETAILED APPLICATIONS (4)

The PROMETHEUS project



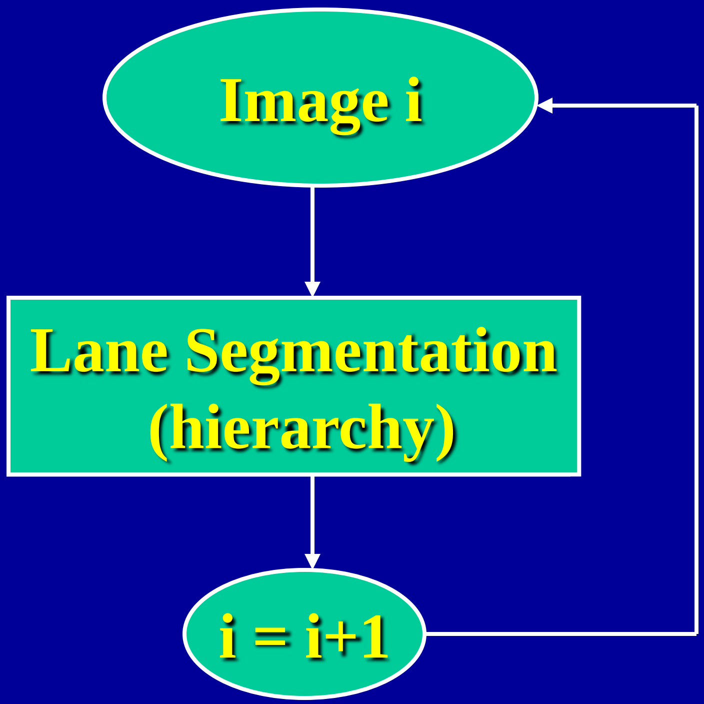
Road segmentation and obstacle detection

Two phases:

• primary road or lane segmentation (hierarchical watershed). No information is shared between pictures in the sequence
• Definition of a road/lane model (sometimes very basic) and use of this model to build the markers which will be used for the segmentation of the next picture.

DETAILED APPLICATIONS (5)

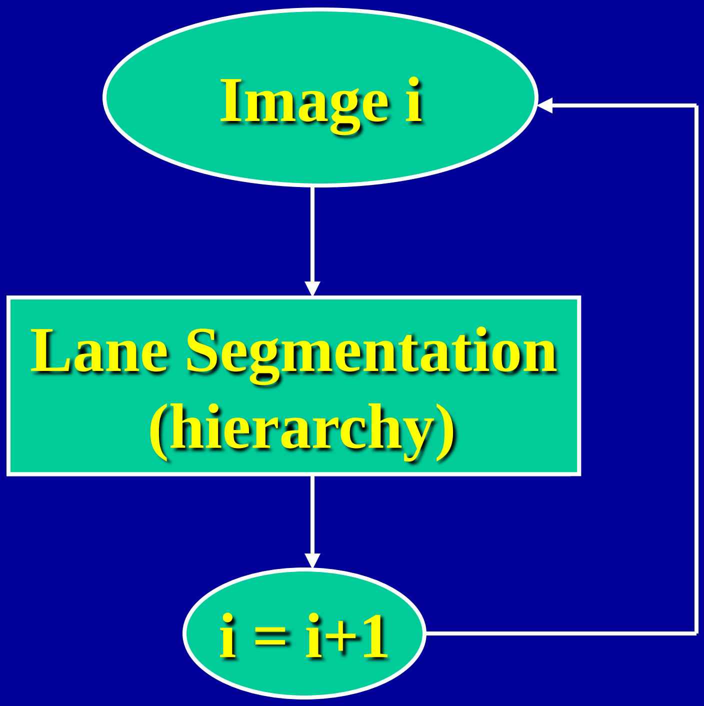
Phase I

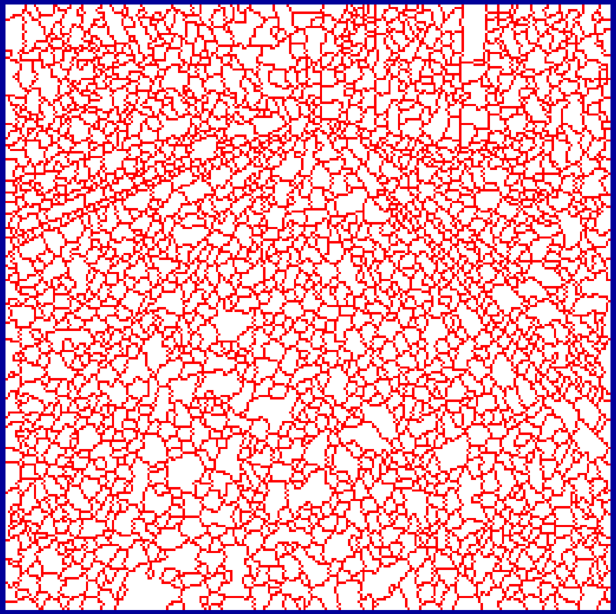


Initial image

DETAILED APPLICATIONS (5)

Phase I

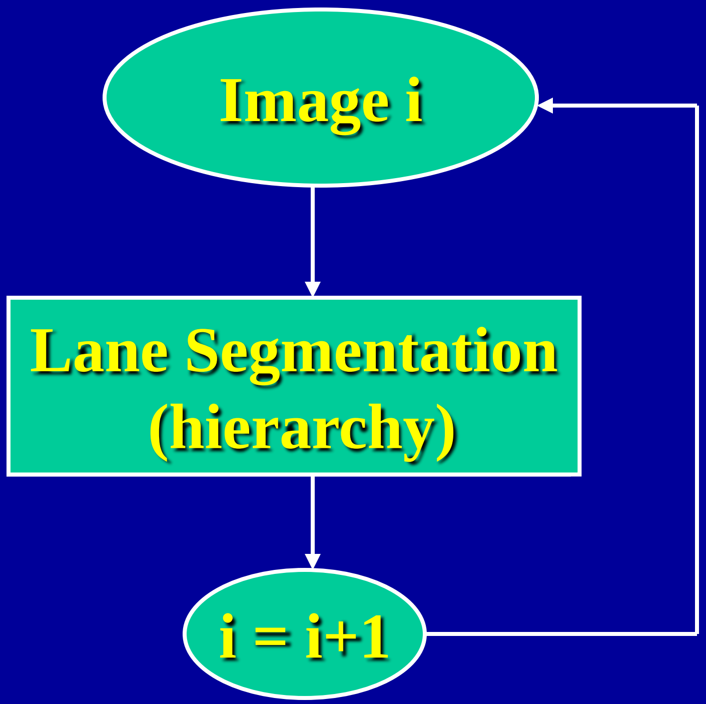


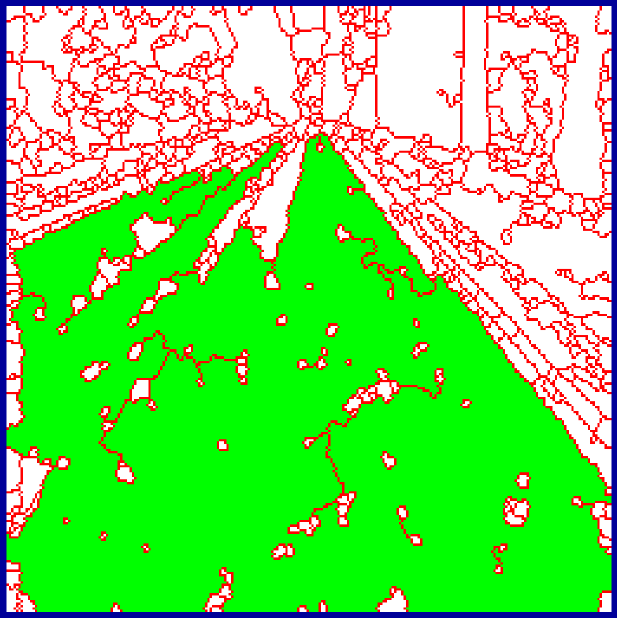


First segmentation

DETAILED APPLICATIONS (5)

Phase I

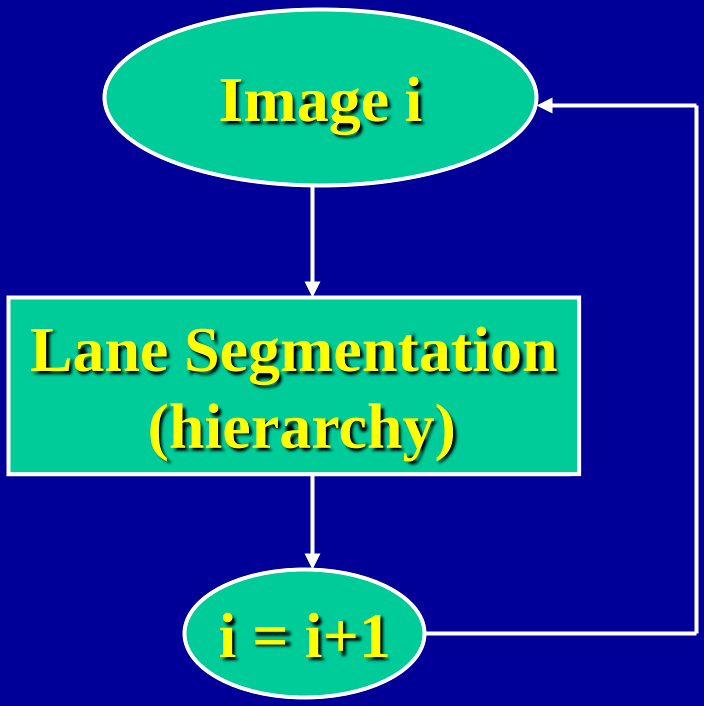




Second level of hierarchy and marker extraction

DETAILED APPLICATIONS (5)

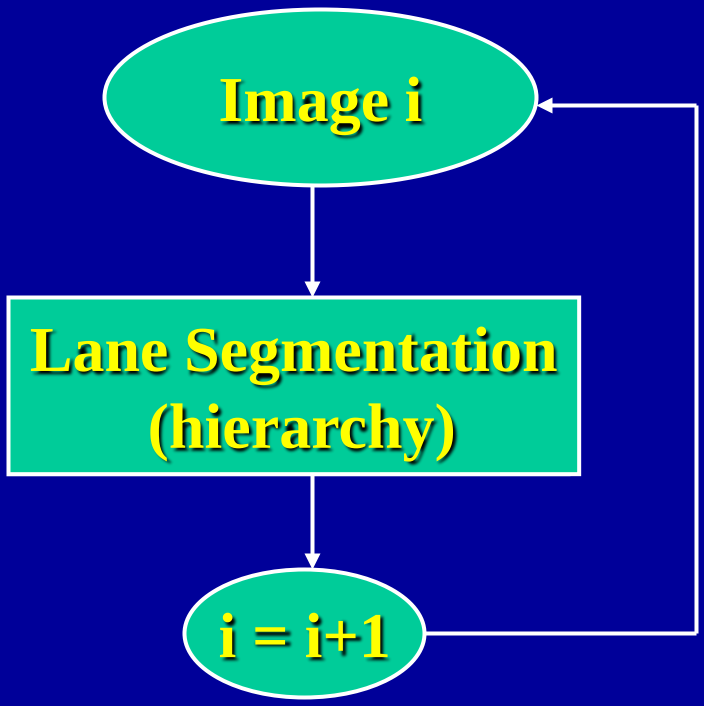
Phase I



Final segmentation

DETAILED APPLICATIONS (5)

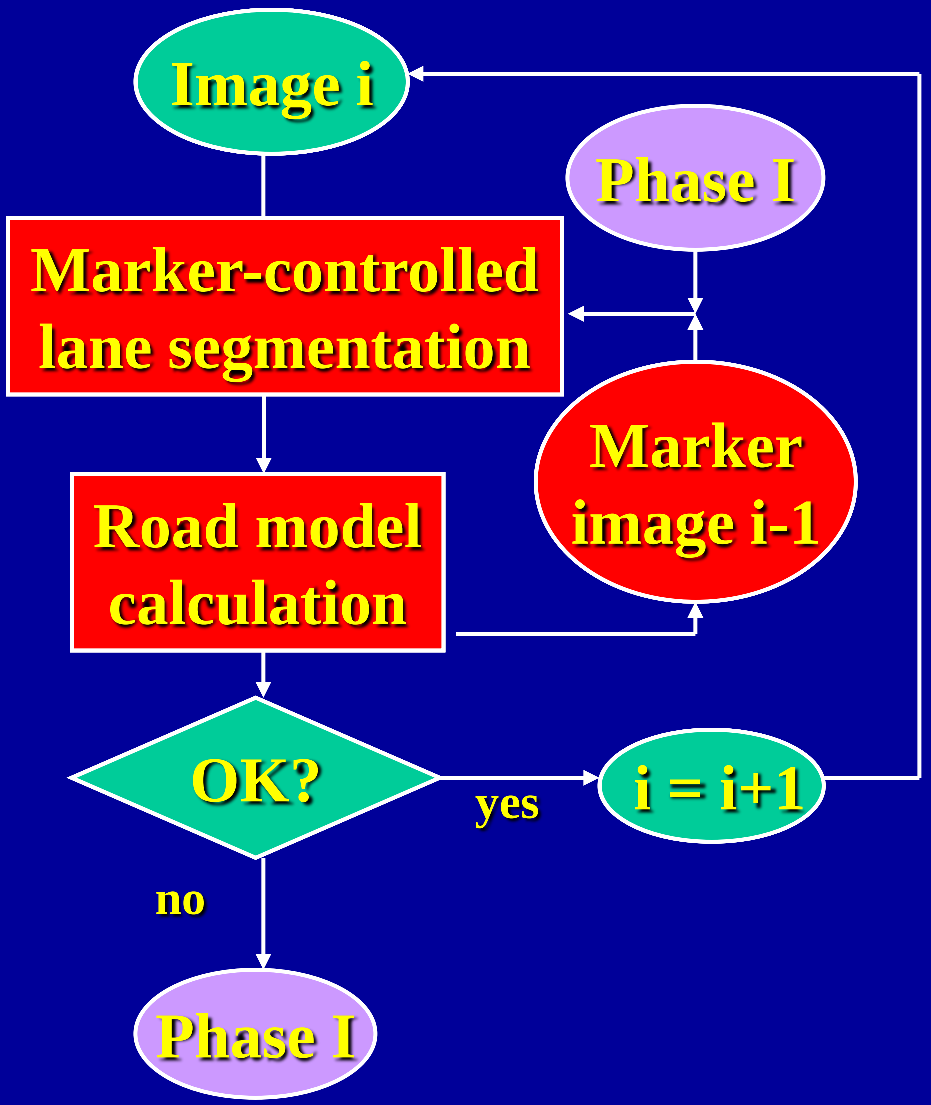
Phase I



Example of detection on a complete sequence

DETAILED APPLICATIONS (6)

Phase II

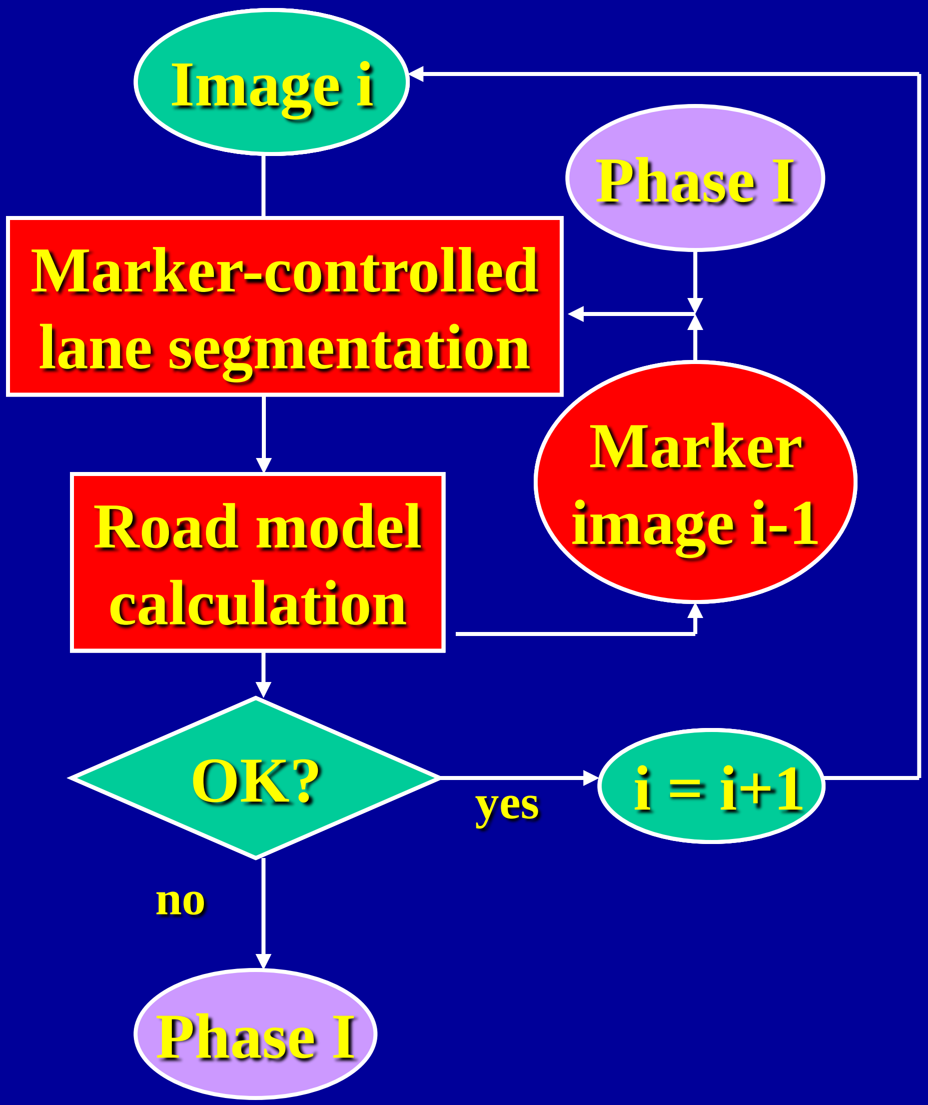


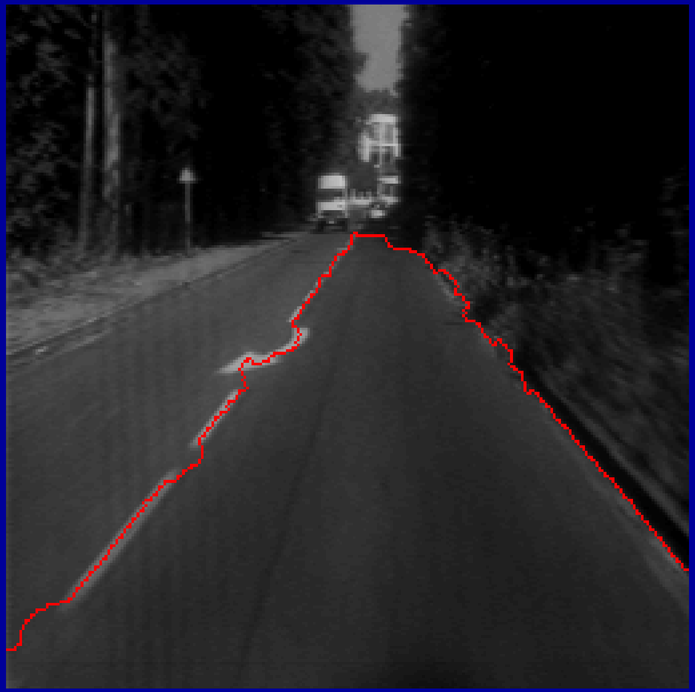
Sequential image at time i

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



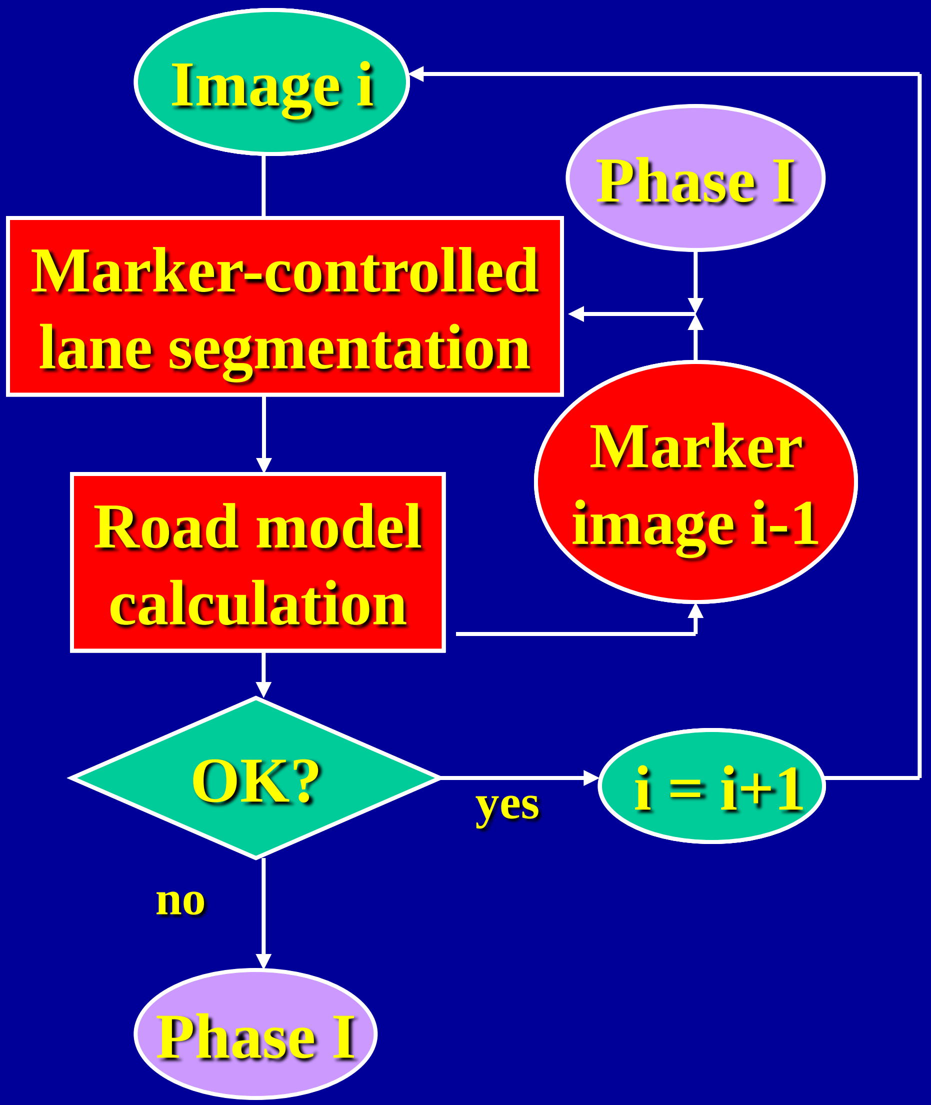


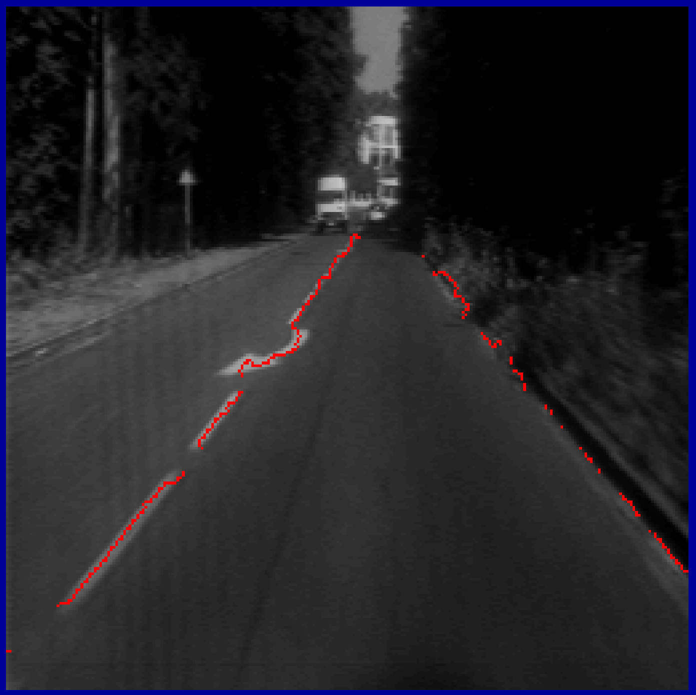
Marker-controlled segmentation of the lane (marker generated by the previous image)

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



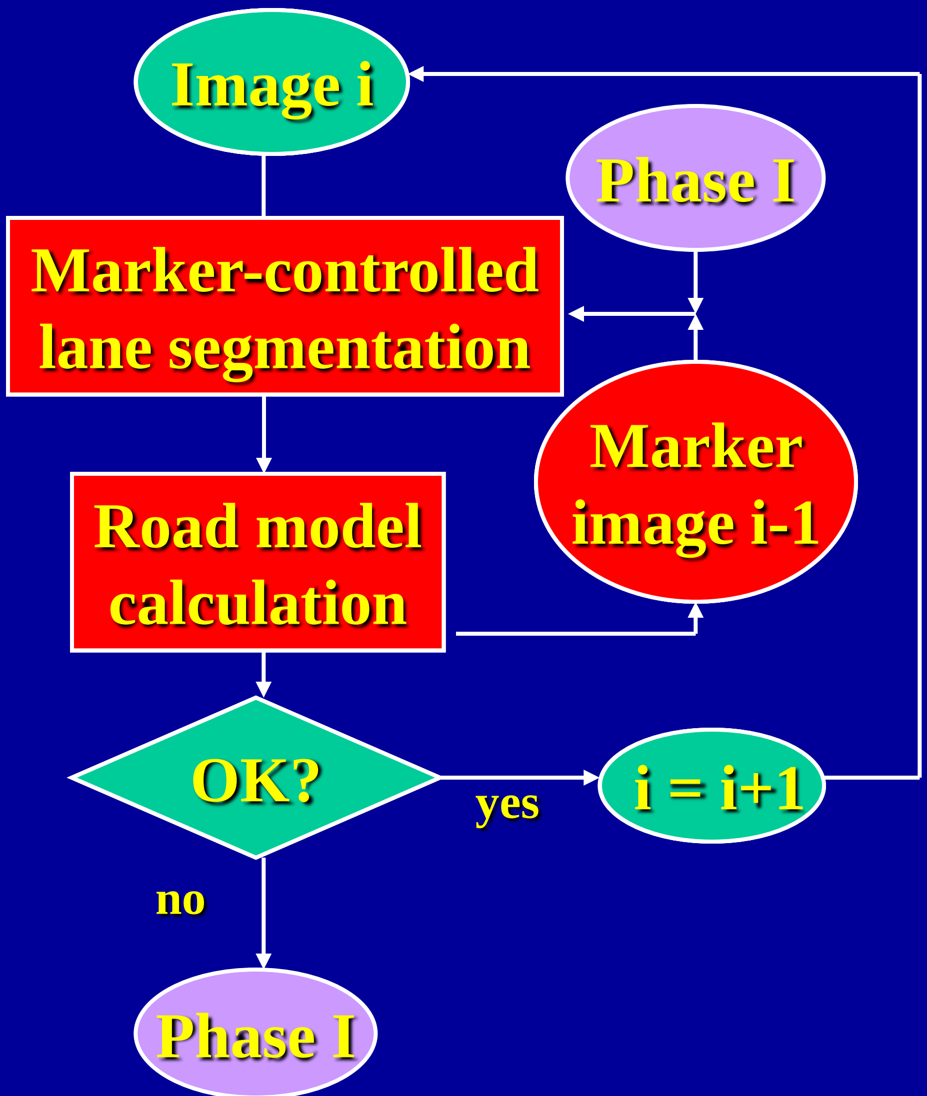


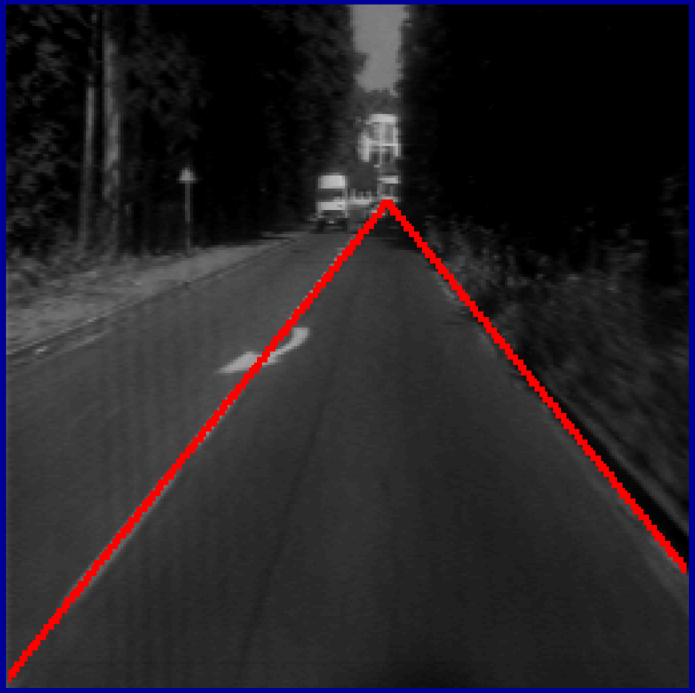
Those pixels belonging to the contours of the lane are selected...

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



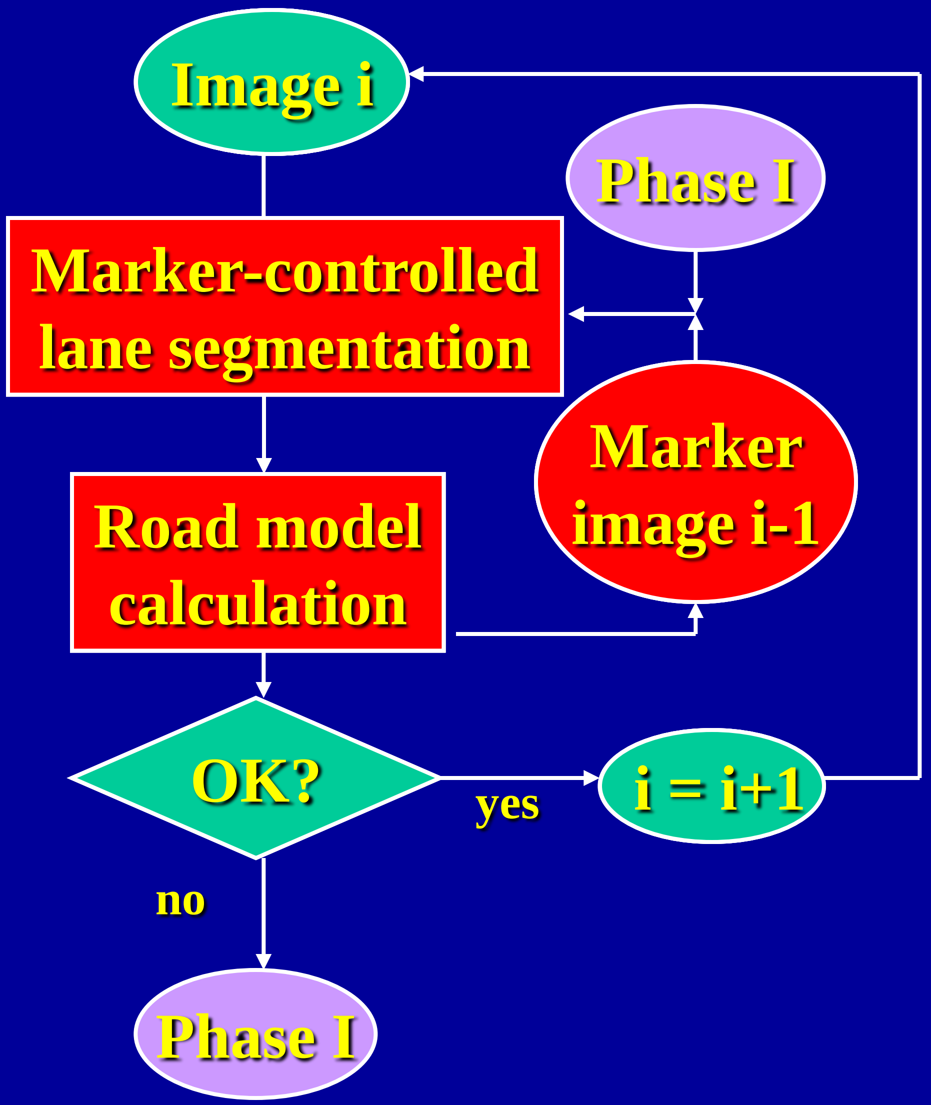


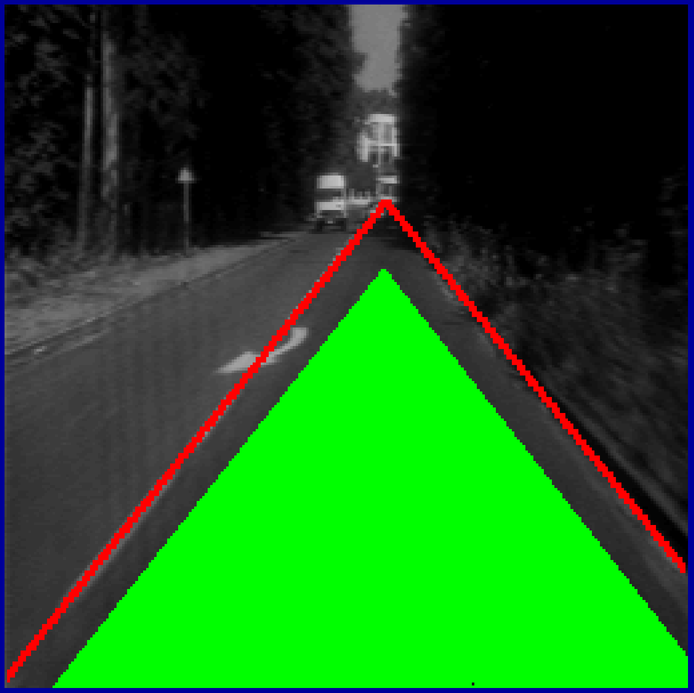
…and used to adjust a lane/road model

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



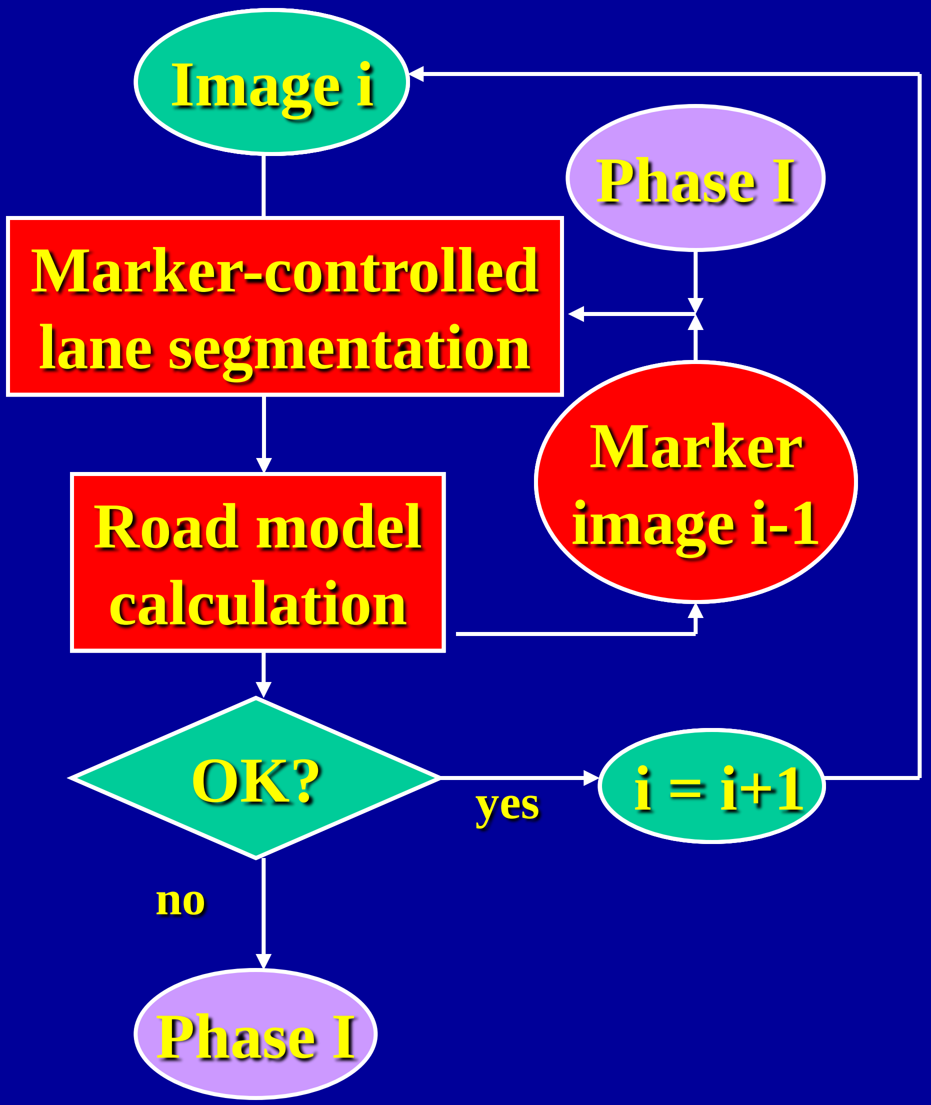


The road/lane model leads to the generation of a new marker

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II

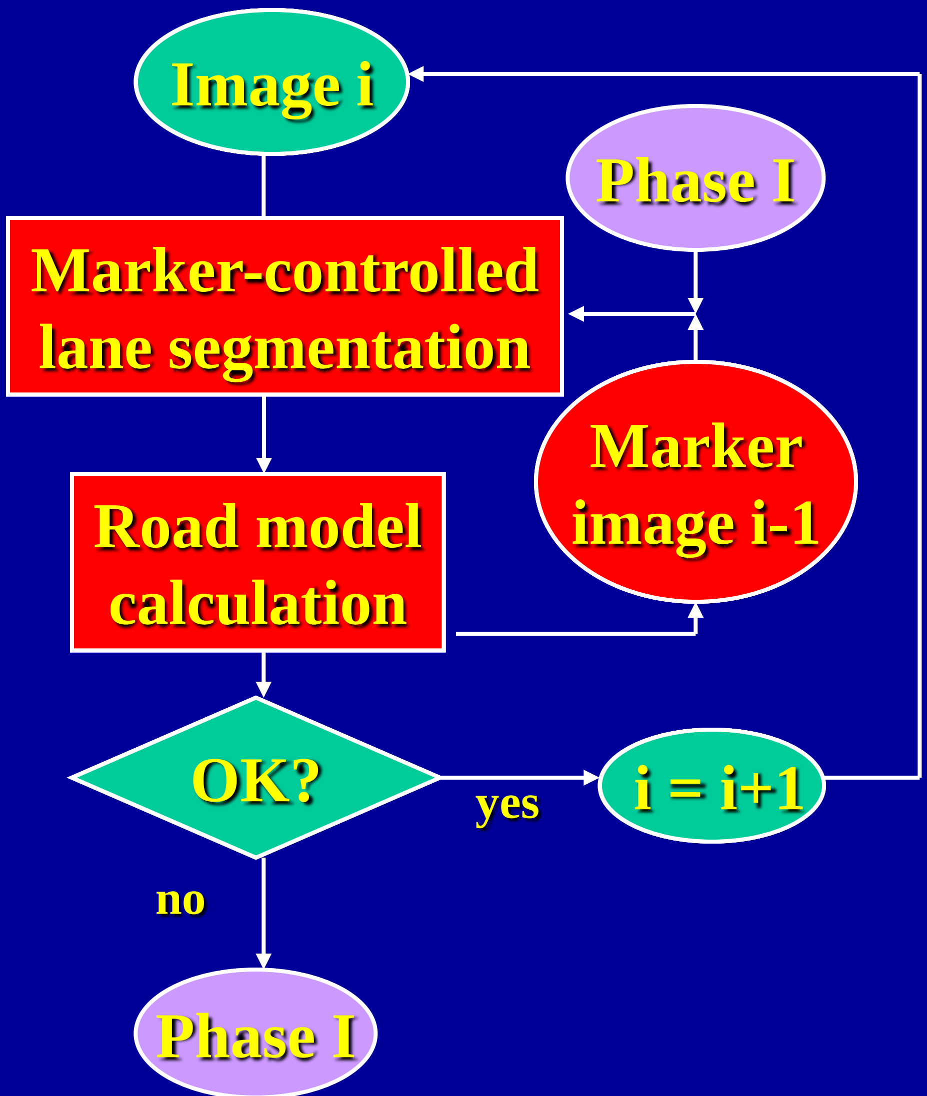


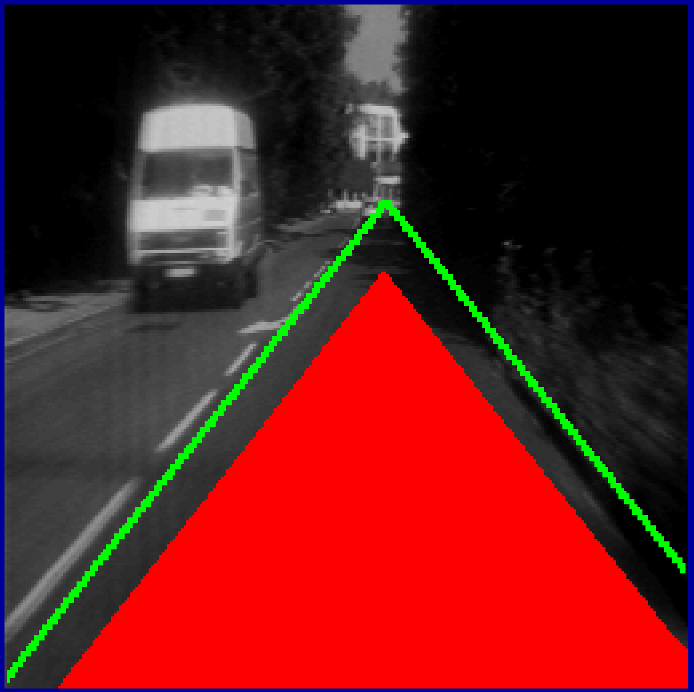
If no error occurs, the next image is processed

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



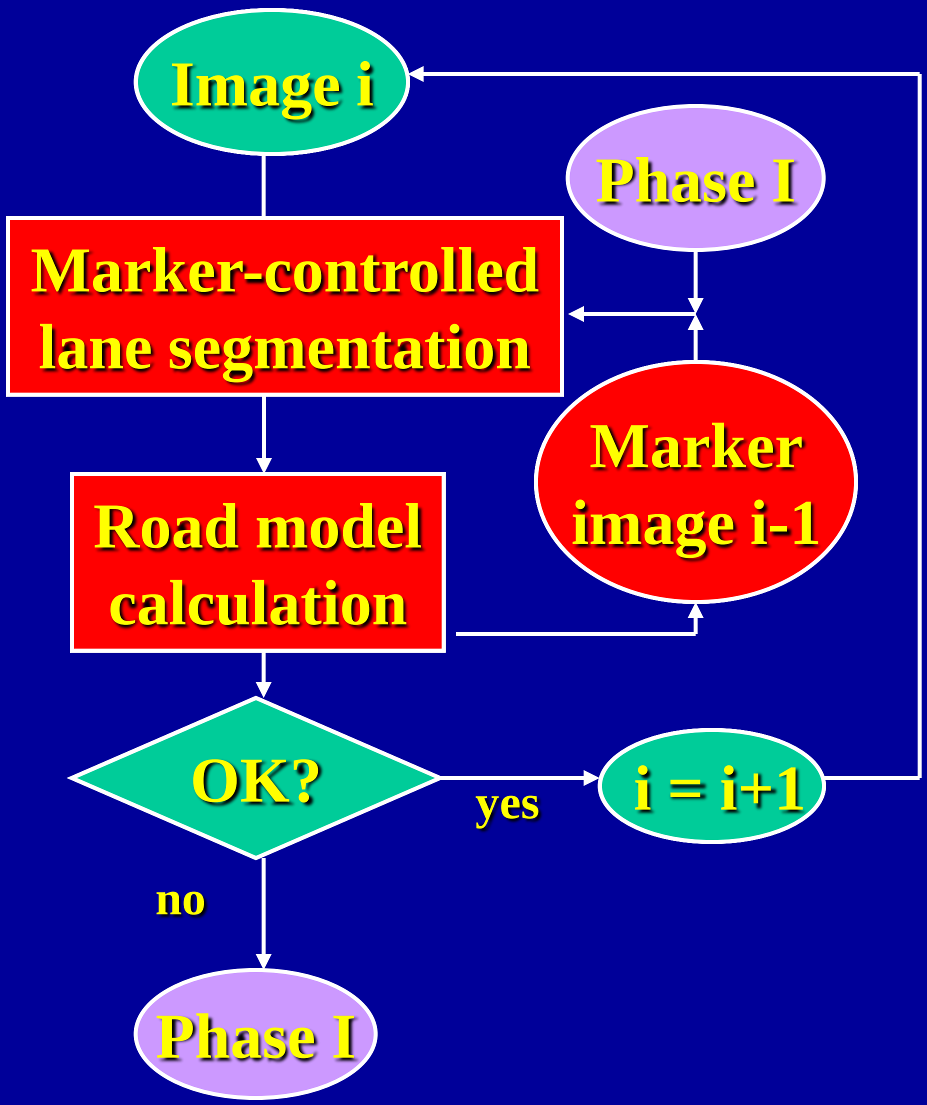


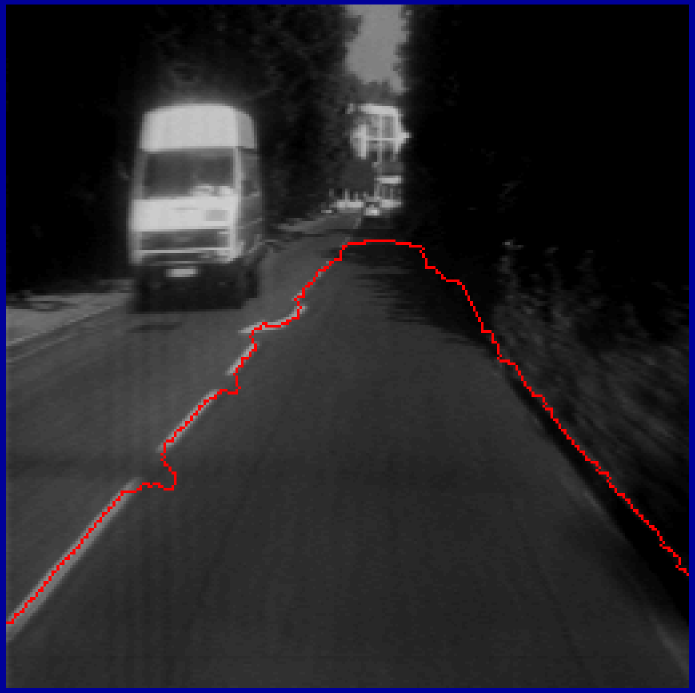
The previous marker is used to segment the current image

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



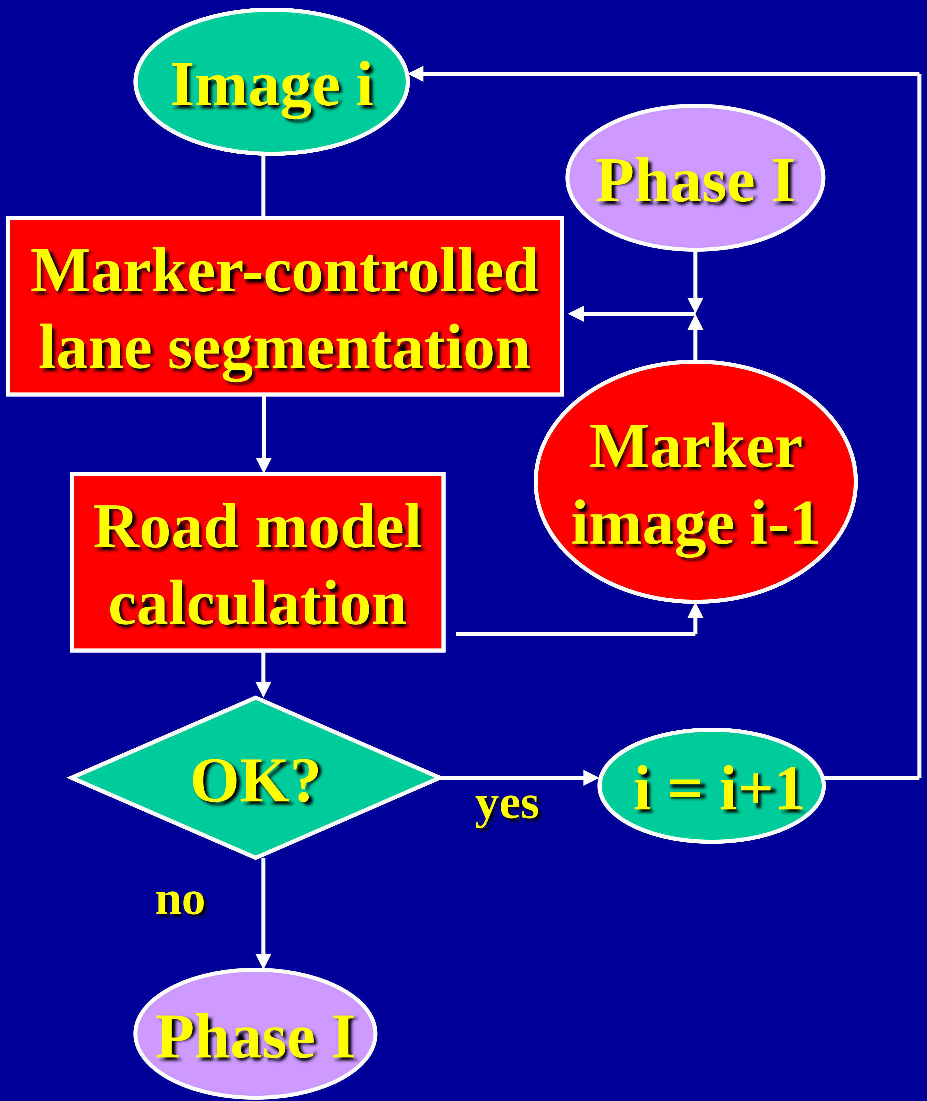


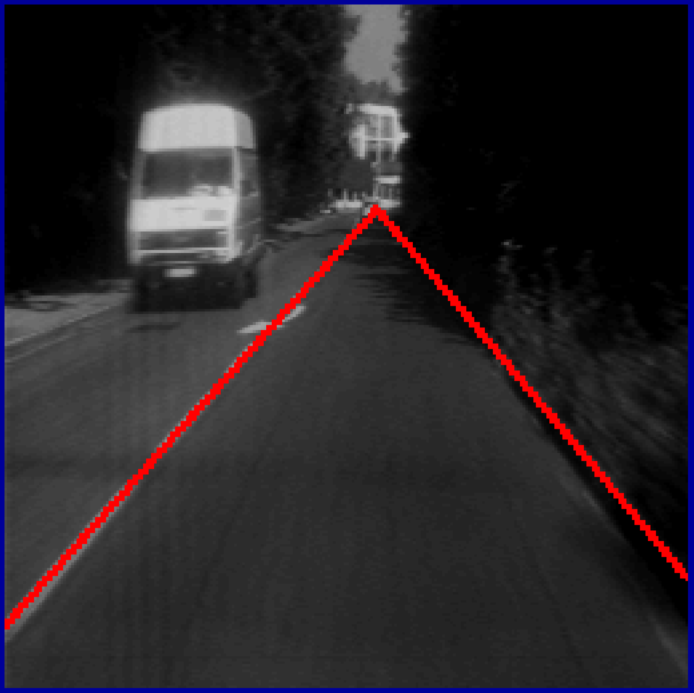
The previous marker is used to segment the current image

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II



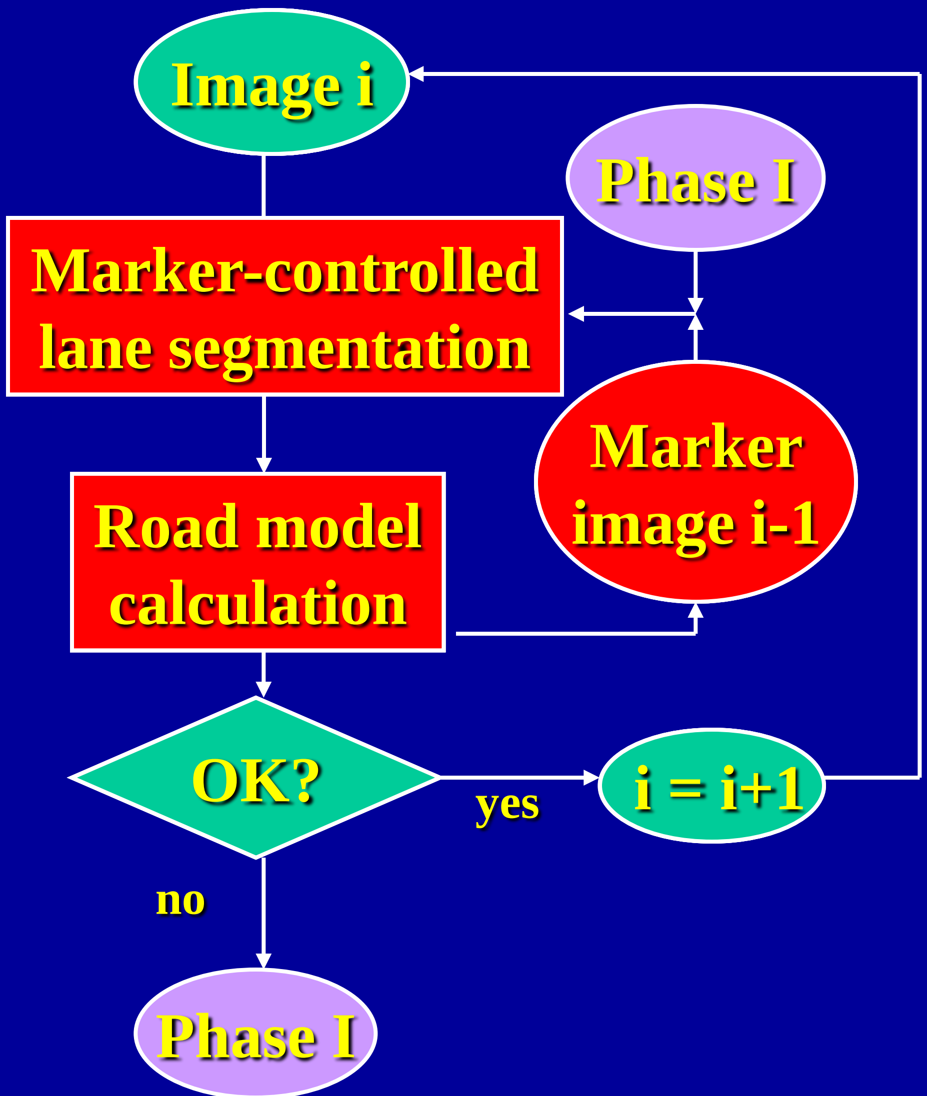


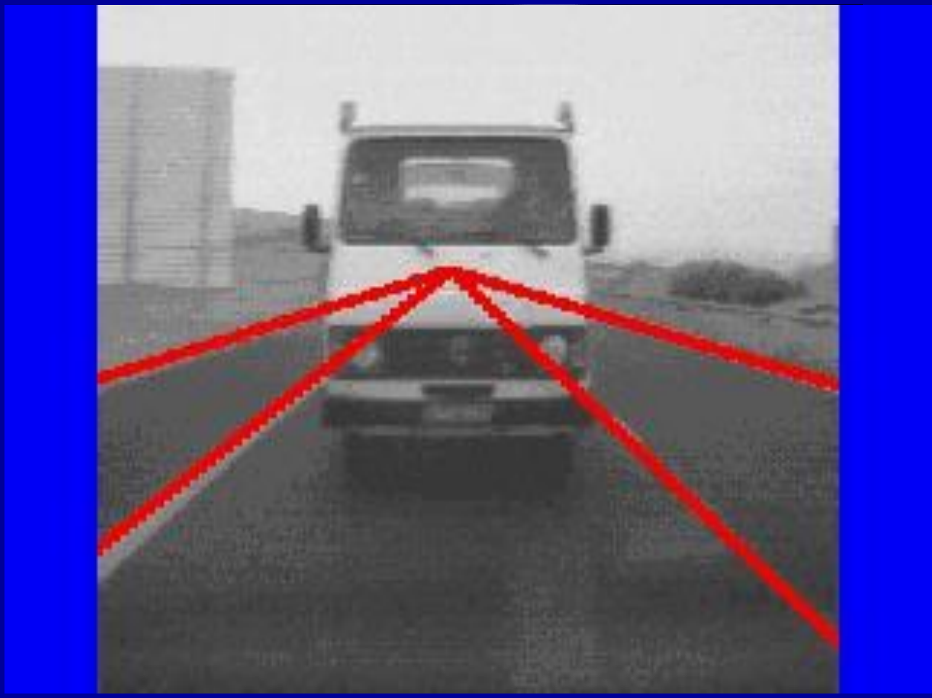
And a new adjustment of the road/lane model is performed

Note that, despite its apparent complexity, this phase is faster than phase I (no hierarchical segmentation).

DETAILED APPLICATIONS (6)

Phase II





Demonstration of the process on a complete sequence (three lanes road model)

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http://cmm.ensmp.fr/~beucher/publi.html