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EXTREMA OF GREY-TONE FUNCTIONS AND MATHEMATICAL MORPHOLOGY

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Abstract

Some applications of the use of extrema of grey-tone functions are given. After a brief presentation of the basic transformations of the Mathematical Morphology involved in this process, the notion of watersheds is introduced. The computation of watersheds is explained in terms of propagation and in terms of thinnings. Finally, two examples of use are presented on real-world images.

People involved in image analysis, when working with sets, are mainly faced with the following question : does the object I am looking for exist or not ? Mathematical Morphology tries to answer this question using the fundamental concept of Hit or Miss transformation and the correlated transforms which are erosion, dilation and so on. The same people, when working with functions (That is with most of the real-world pictures) are faced with the same problems as they were with sets. Unfortunately, in that case, the answer is a bit more complicated, due mainly to the fact that there is no longer that simple binary situation. Nevertheless, when looking at a picture, one can easily detect objects because they are lighter or darker than the background, for instance. So, at a first level of recognition, the most important information is not given by the various grey levels of the image, but on the contrary, by some neighbourhood relationships between the different elements of the picture. These relationships are often controlled by some extremal features. A zone in the picture lighter than the background can be for example the marker of an object. In this paper, we shall present some morphological transformations using extrema of a function, and their application to picture segmentation.

Extrema of a function. Definition and computation

We shall assume that our function of interest f fullfills "good" properties of continuity. We can define a minimum of that function in the following way : suppose that we walk along the graph of this function considered as a topographic surface. Starting from one point and using a never ascending path, as long as we can reach a lower points of the graph, we can say that the former point does not belong to a minimum. On the contrary, if there exists no possible descending path starting from a point of the topographic surface, this point belongs to a minimum. A good representation of a minimum is given for instance by the bottom of a crater of a volcano.

More precisely, we can define a minimum using the various thresholds of f. That is :

Let X_λ be a threshold at level λ of f

$$X_\lambda = \{x \in R^2 : f(x) \leq \lambda\}$$

X_λ may be composed of n connected components X_λ^j

$$X_\lambda = \bigcup_j X_\lambda^j$$

One connected component X_λ^j is said to be a minimum at level λ, iff :

$$X_\lambda^j \cap X_\mu = \emptyset, \quad \forall \mu < \lambda$$

This definition provides a way to compute the minima using the thresholds of the function. More refined methods can be used ([1],[2]).

Watersheds of a function

Another very important notion is the notion of watersheds. The watersheds of a function can be considered as the zone of influence of its minima. An intuitive approach of this notion can be he following (Figure 1).

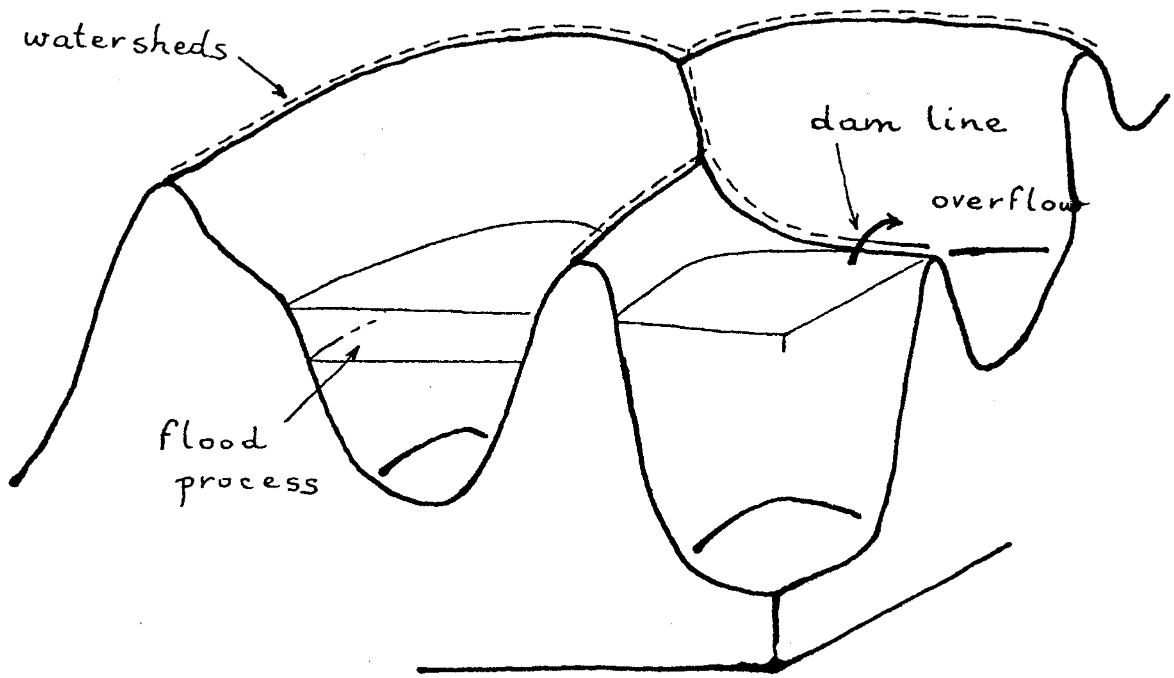


Figure 1 : Watersheds, intuitive definition

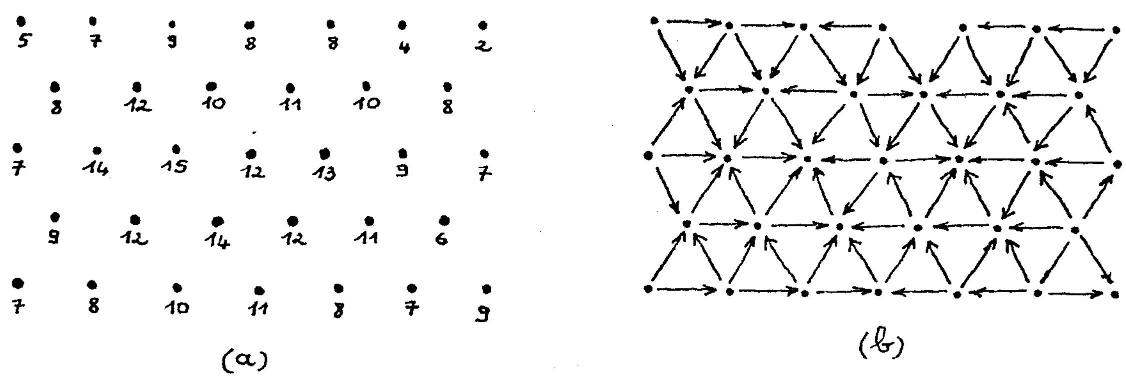
Every minimum is embedded in a basin. One way to determine the extension of that basin is to flood it. Everytime an overflow occurs, a dam can be built on the topographic surface to prevent the overflow. When the dam has drawn a closed contour, we get the boundary of the basin associated to the minimum. The set of all the boundaries for all the minima of the function is called the watersheds of that function ([3]).

Very often, the computation of these watersheds is rather complex. It uses geodesic reconstructions of the various thresholds of the function ([3]). Fortunately, there exists another method for building these watersheds, based on the idea of propagation and, on the other hand, on a transformation called thinning.

Thinning and propagation

Propagation

In order to introduce the notion of propagation, let us consider the digitization of the function f on an hexagonal grid. Every point has six neighbours. Suppose that we are propagating a flow on the graph of the function. Assuming now that the flow has reached the center point, we can easily find which points of the neighbourhood will be flooded. They are those points higher than the center point. If we express these relationships (that is, which points of the neighbourhood will be flooded) using direction signs (Figure 2), we get an oriented graph.



<u>Figure 2</u> : Propagation graph a) original function
b) associated graph

The detection of the points of the function belonging to the watersheds is performed as follows :

- First, we can determine, on the propagation graph, those points which belong to the watersheds. They are the center points receiving the propagation flow from at least two connected zones of their neighbourhood.
- Then, the direction signs starting from these points are deleted (these points being unflooded cannot propagate a flow to their neighbours). By doing so, we define a new propagation graph, on which the entire process is re-run again until we do not gain any new watersheds points.

In practice, this procedure is rather complex, especially as a number of points have not been explained in detail here. However, this process can be performed in a simpler way by using some morphological transforms called thinnings.

Thinnings

Let $T = (T_1, T_2)$ be a two-phases structuring element. The thinning of f by T is a transformation which provides a new function g defined as follows :

iff $\sup_{y \in T_2} [f(y)] < f(x) \leq \inf_{z \in T_1} [f(z)]$, then :

$$g(x) = \sup_{y \in T_2} [f(y)]$$

else,

$$g(x) = f(x)$$

This transformation can be used to determine the watersheds of a function. In fact, it is possible to show that this operation and the process of propagation described above are equivalent if the right class of structuring elements T is used. This class is composed of those structuring elements which preserve the homotopy as described in [5].

Some examples

These transformations based on the extrema of a function are very useful in picture segmentation and contour detection. Two examples will be given as an illustration.

Electrophoresis gels

Bidimensionnal electrophoresis is a powerfull technique for proteins identification. Proteins can migrate on a gel according to their molecular weight and their iso-electrical point (Figure 3).

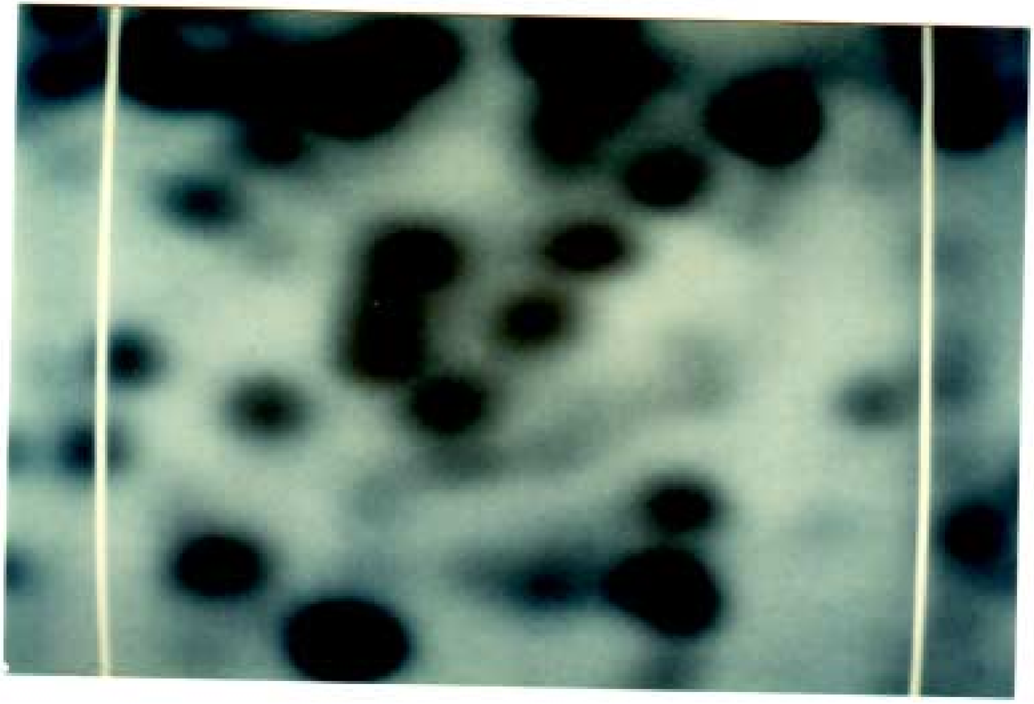


Figure 3 : Electrophoresis of proteīns

When studying that picture, many elements are of prime importance. First, the position of the proteīns blobs because this position is the signature of the proteīn ; next, the density of the blob which is closely related to the activity of the corresponding proteīn. Then, the neighbourhood relationships between the various blobs is interesting when we compare two or more gels of the same mixture.

Figures 4 to 8 explain the entire process. The minima of the grey-tone function are detected. They correspond to the blobs (Figure 4).

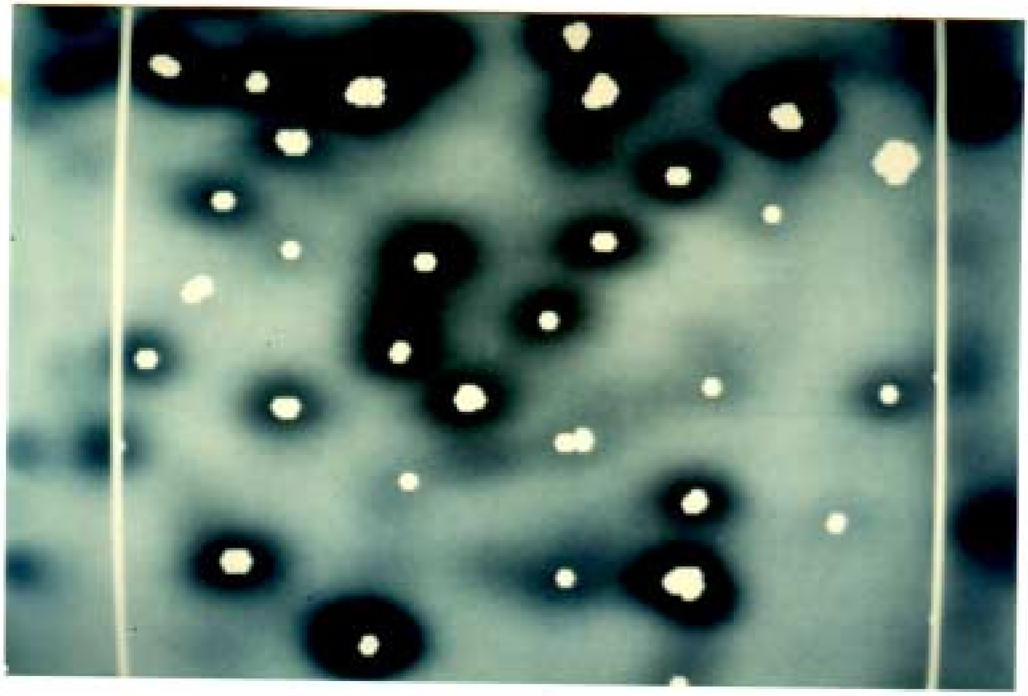


Figure 4 : Minima of the grey-tone function

Then, the centers of these blobs are computed (Figure 5).

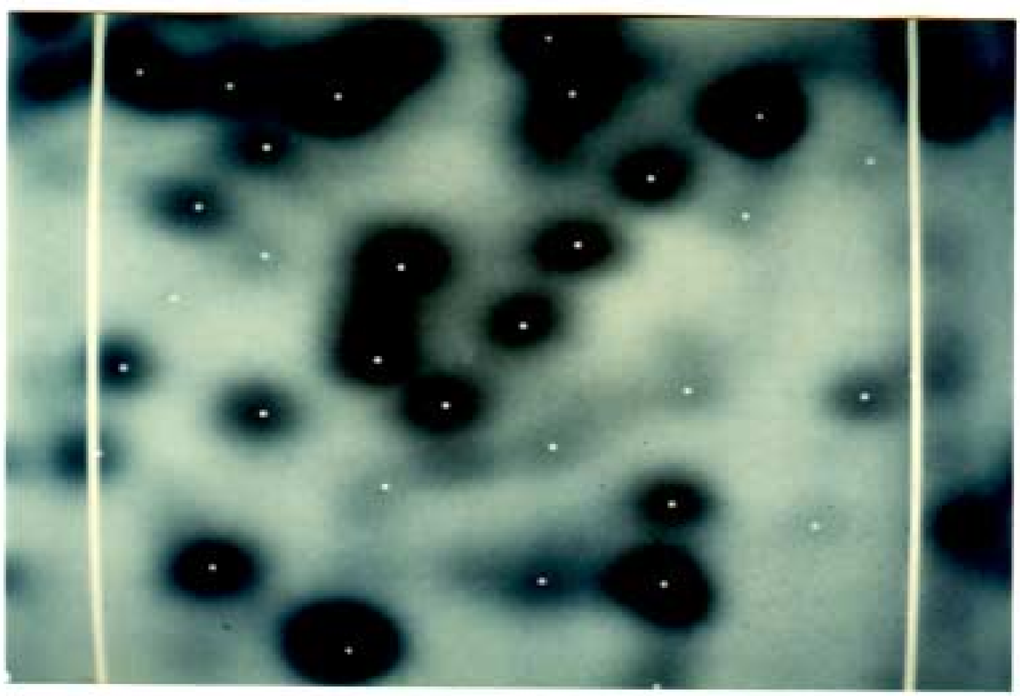


Figure 5 : Centers of the blobs

In order to compute the contours of the blobs, a rather complex process is performed : first, we get the watersheds of the grey-tone image (Figure 6). This transformation provides a good description of the neighbourhood relationships between the blobs. But it also gives us some very useful information : we know that the contour of each blob is somewhere between the watersheds of the grey-tone image and the minima (Figure 7).

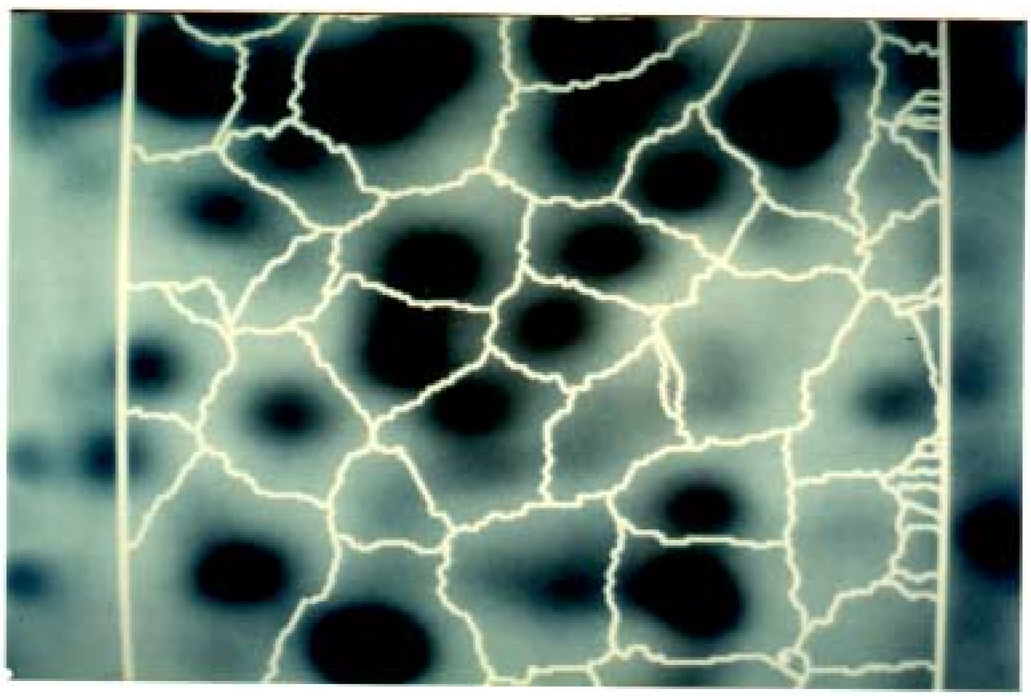


Figure 6 : Watersheds of the grey-tone image.

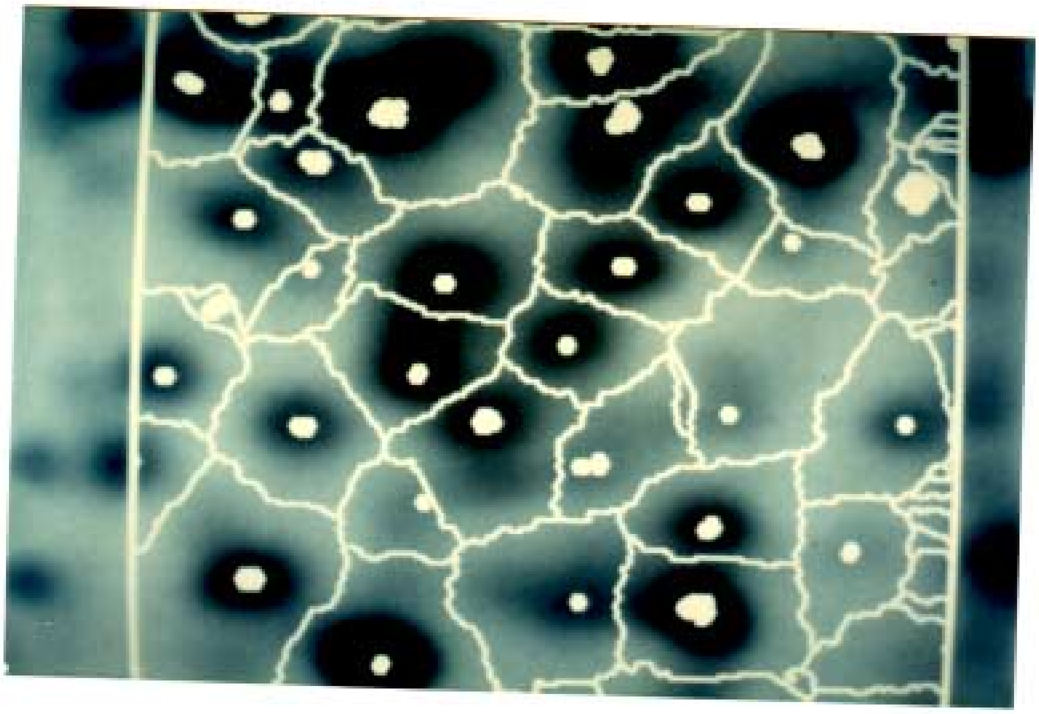


Figure 7 : Partition of the picture. The contours are between the watersheds and the minima.

So, we can now detect the contours. They are the watersheds of the gradient function. The trick is here that the construction of these watersheds is controlled by the topological information available from the watersheds of the grey-tone function. The final result is given on Figure 8.

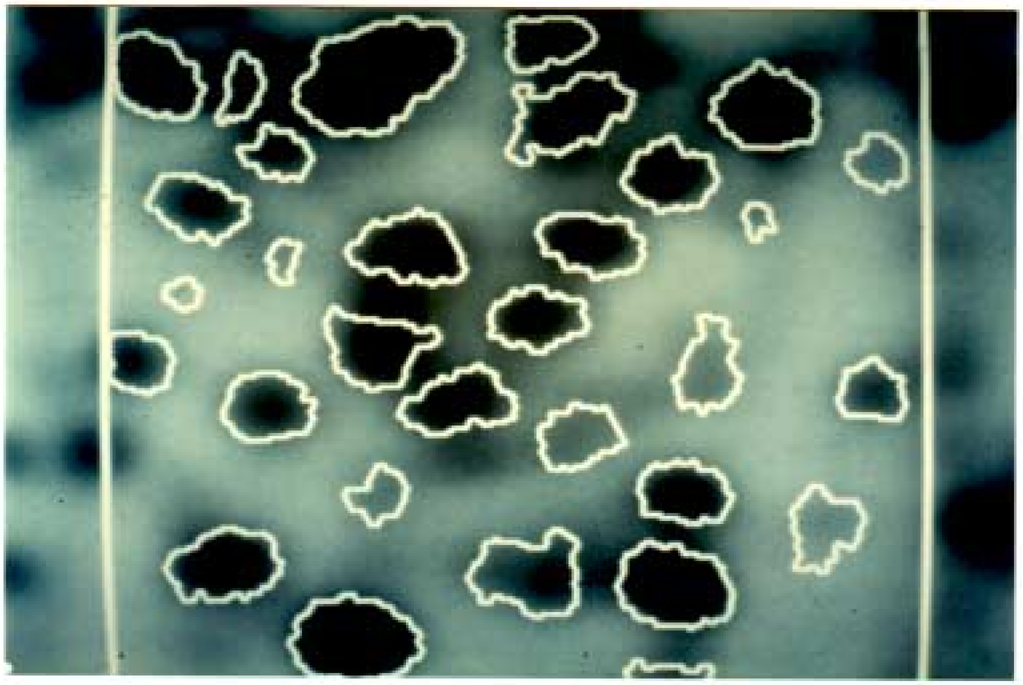


Figure 8 : Contours of blobs

We remark that the number of contour lines is equal to the number of detected blobs. Furthermore, each contour is closed and has a rather regular shape.

Contours of fractures

Using watersheds of the gradient function, we can obtain contours of objects in complex pictures. Figure 9 and 10 give an example of such a procedure, in order to detect the contours of facets in a S.E.M. image of fractures in steel.

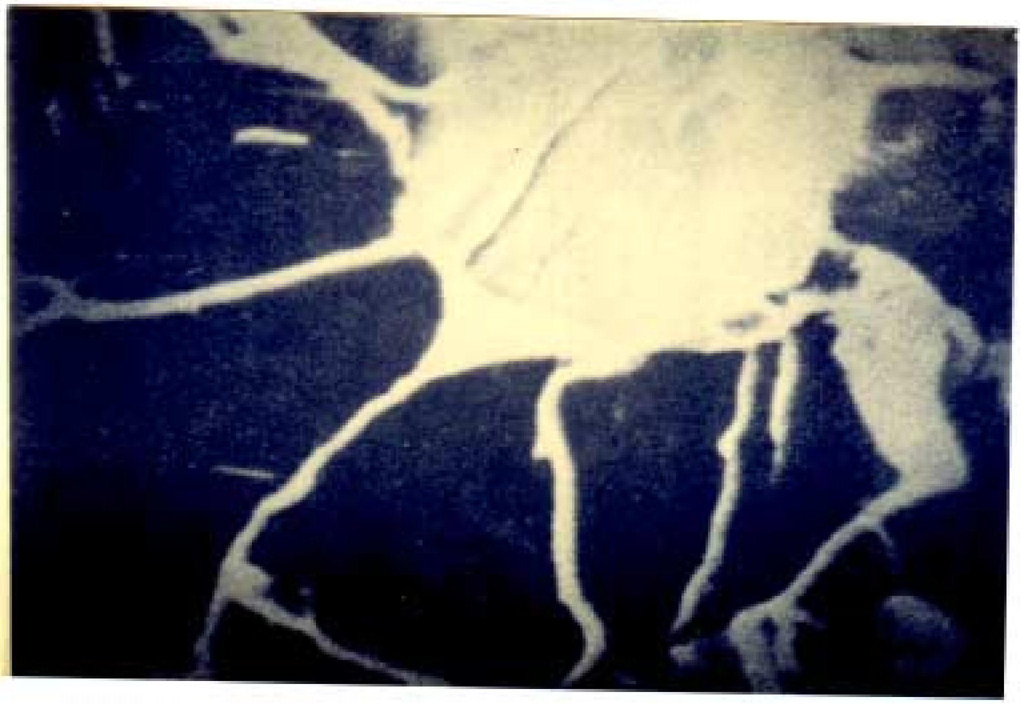


Figure 9 : S.E.M. image of steel cleavage fracture.

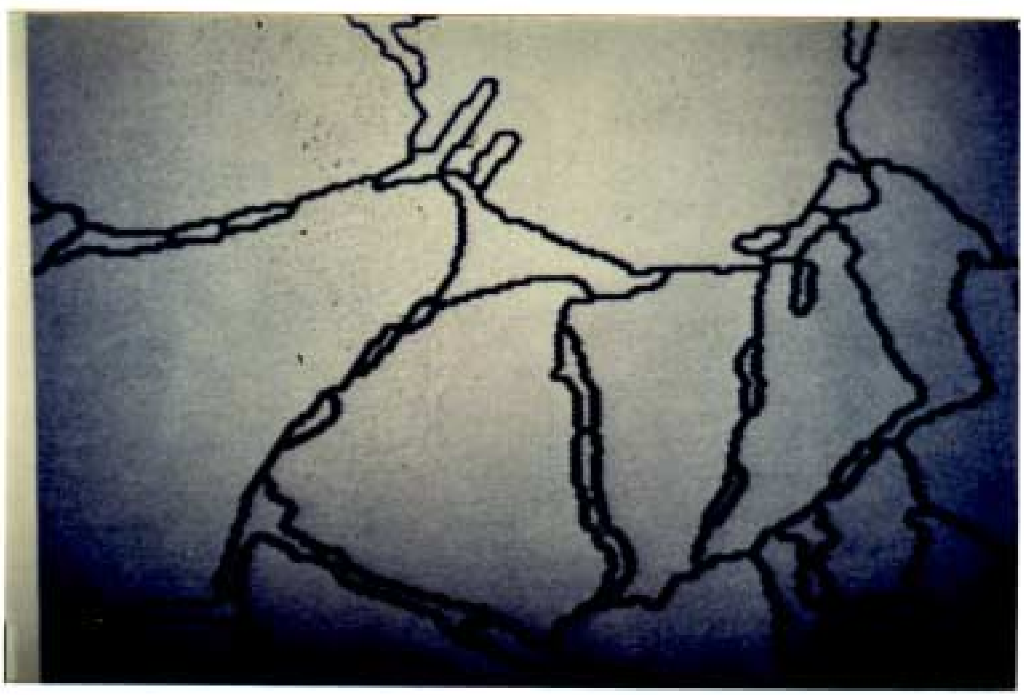


Figure 10 : Contour lines of the facets.

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