GEOVISION 99

Mathematical Morphology
&
Geology

When image analysis uses the vocabulary of earth sciences

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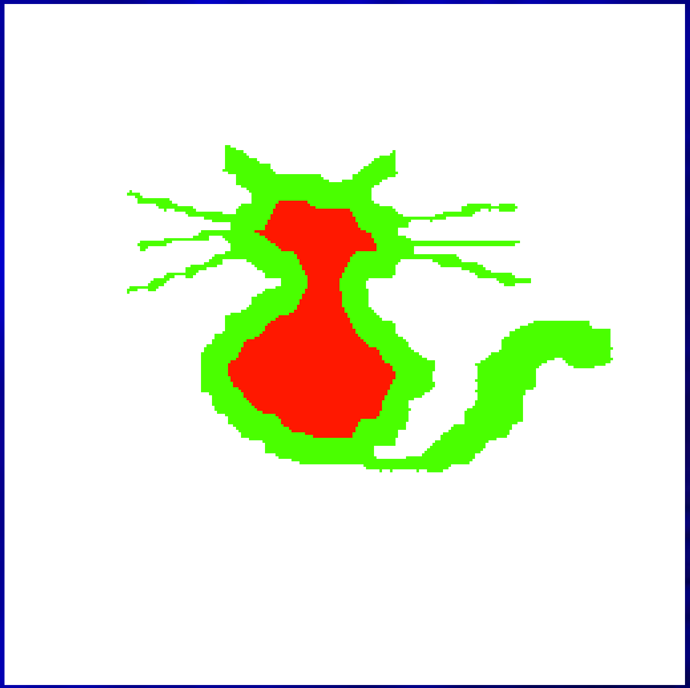
The Basics of Mathematical Morphology

Erosion and Dilation are the basic bricks of MM

$$X \ominus B = \{x : B_x \subset X\}$$

$$X \oplus B = \{x : B_x \cap X \neq \varnothing\}$$



These operators act on sets:

$$Y = \Psi(X)$$

More complex transformations can be defined by concatenation

$$Y = \Psi_1(\Psi_2(\ldots\Psi_n(X)))$$

Morphology & Grey-tone Images

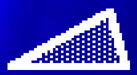
Most of the transformations defined on sets can be extended to functions.

$$f \ominus B(x) = \mathrm{Inf}\left[f(y) : y \in B_x\right]$$

$$f \oplus B(x) = \mathrm{Sup}\left[f(y) : y \in B_x\right]$$

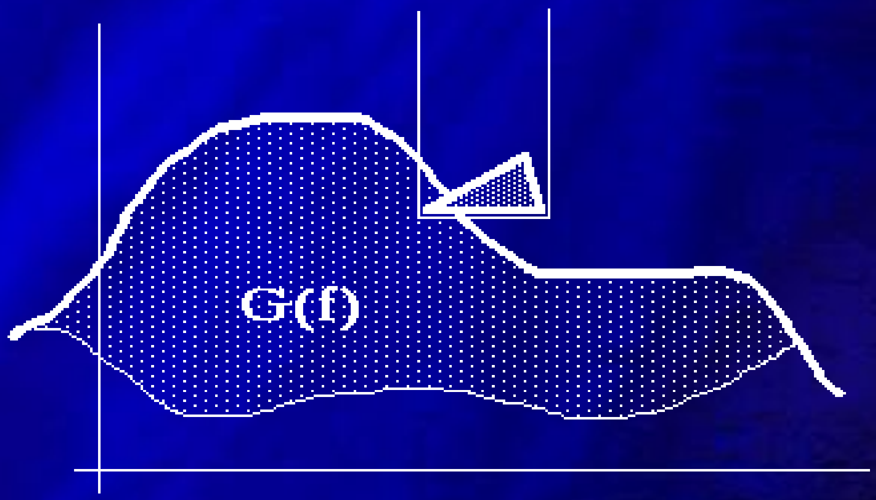
Set under the graph and topographic surfaces

$$G(f) = \{m(x, y) : y \leq f(x)\}$$



The set under the graph of the eroded function is the erosion of the set under the graph of f:

$$G(f \ominus B) = G(f) \ominus B$$

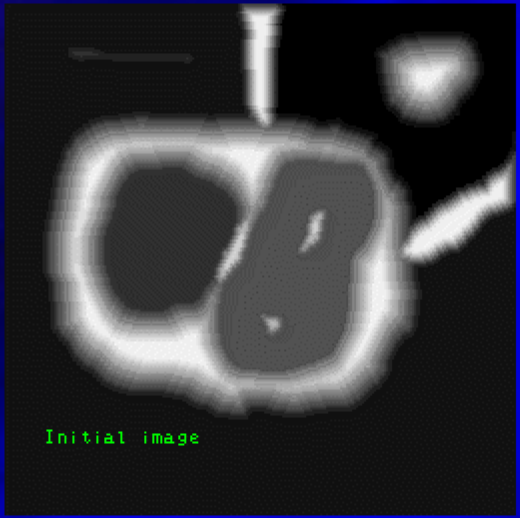


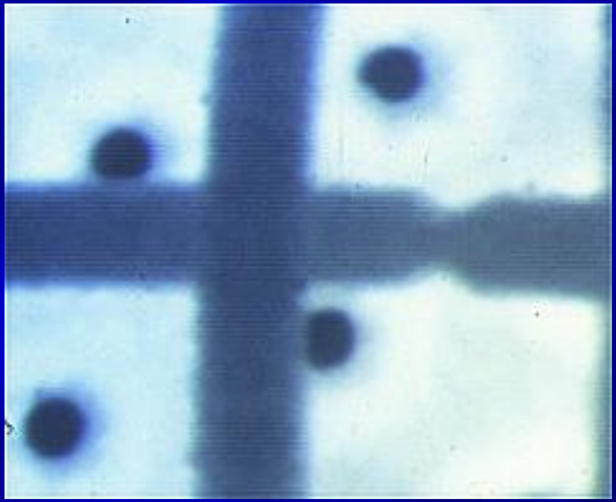
In the same way, the set under the graph of the dilated function is the dilation of the set under the graph of f:

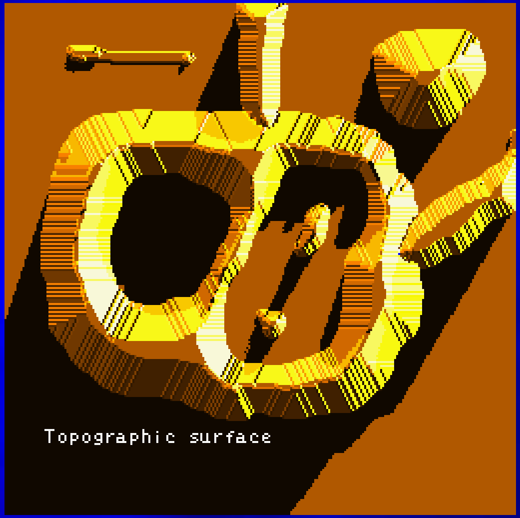
$$G(f \oplus B) = G(f) \oplus B$$

Functions and topographic surfaces

A function in MM is seen as a topographic set.



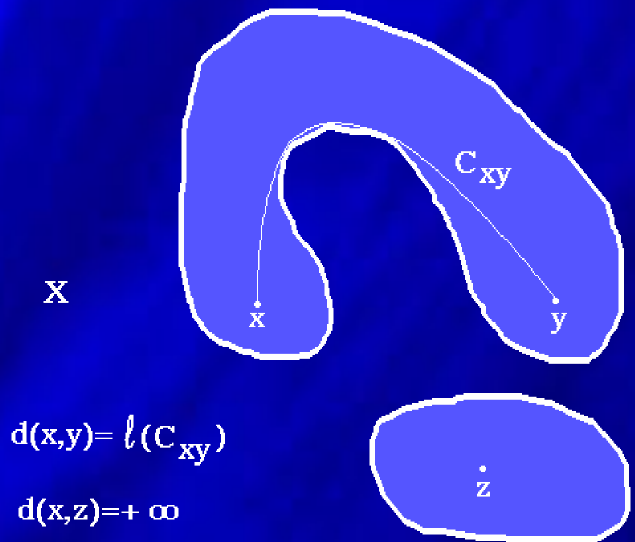


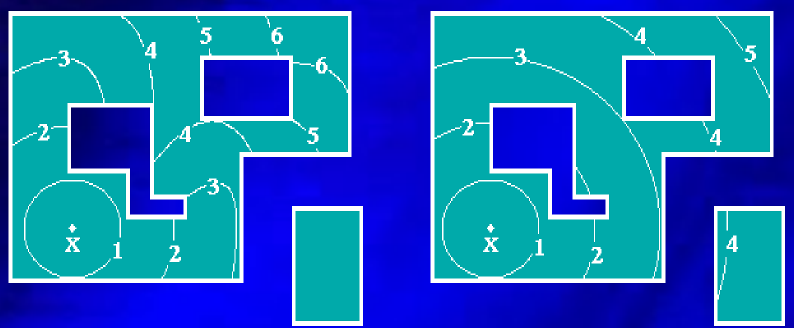


Geodesy

X, geodesic space

**Distance defined in X:
length of the minimal path
included in X between x and y**





(a) (b)

B, geodesic disk of radius r

$$E_X(Y;r)=\{y\in X: d_X(Y^c,y)>r\}$$

$$D_X(Y;r)=\{y\in X: d_X(Y,y)\le r\}$$

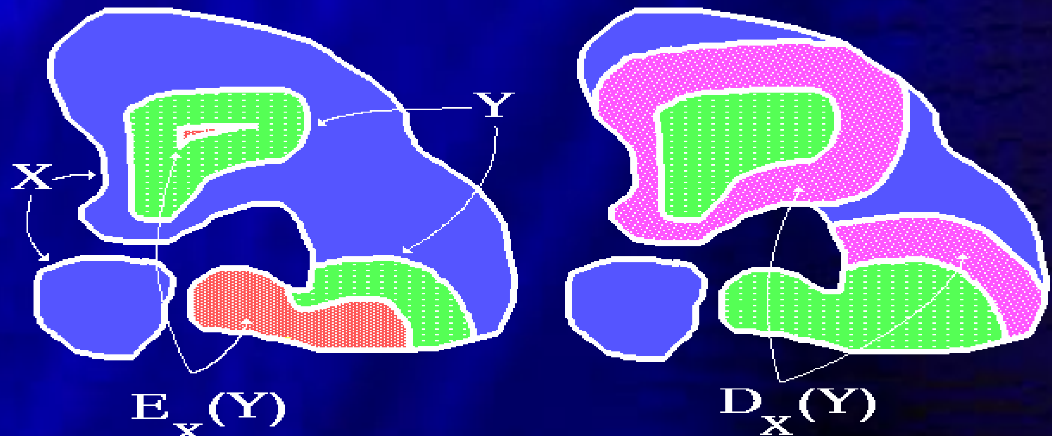


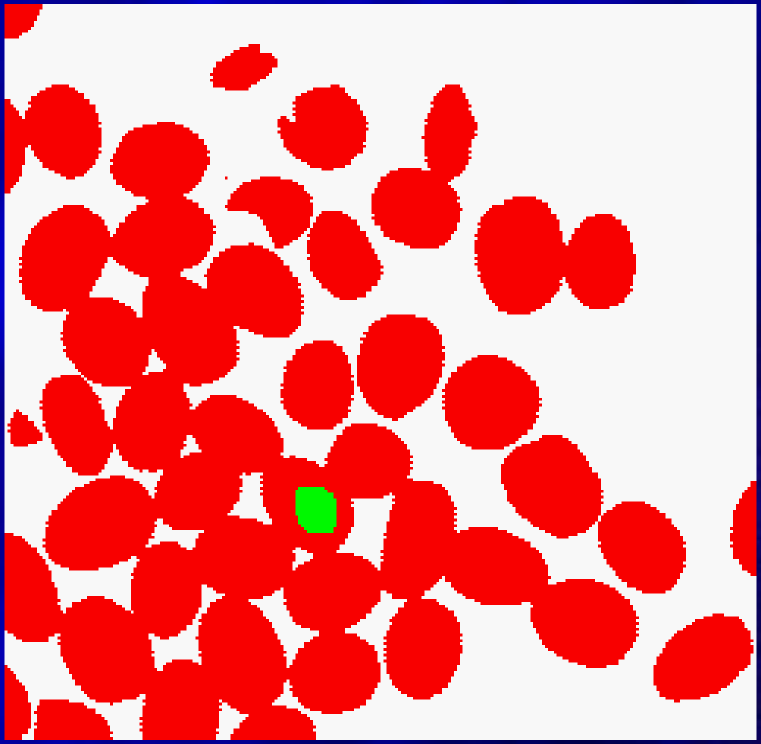
Image reconstruction & filtering

The geodesic dilation until idempotence leads to the definition of the reconstruction

$$R_X(Y) = [D_X(Y)]^{\infty}$$

Y is the marker set

X is the set to reconstruct.



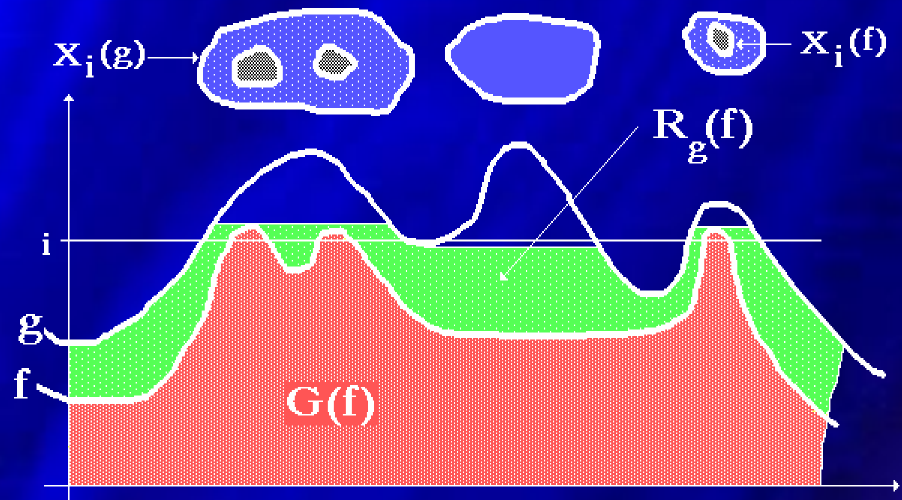
The reconstruction is a powerful way to take into account connectivity through a “simple” dilation

The reconstruction is a morphological filter:
- increasing transform
- idempotent transform

Geodesy & Reconstruction of functions

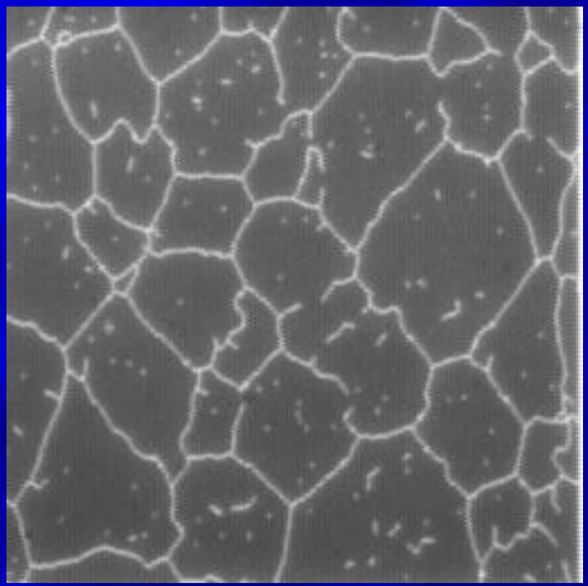
The geodesic transformations for functions use flat structuring elements.

$$D_g(f) = \mathrm{Inf}\left[(f \oplus B), g\right]$$



The reconstruction can also be defined for functions:

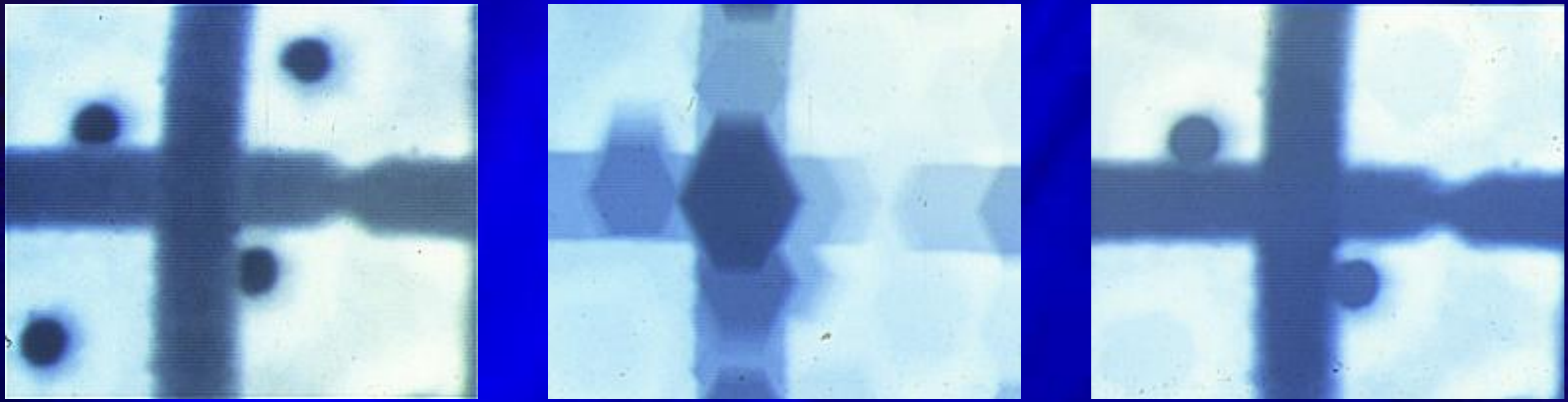
$$R_g(f) = \left[D_g(f)\right]^{\infty}$$



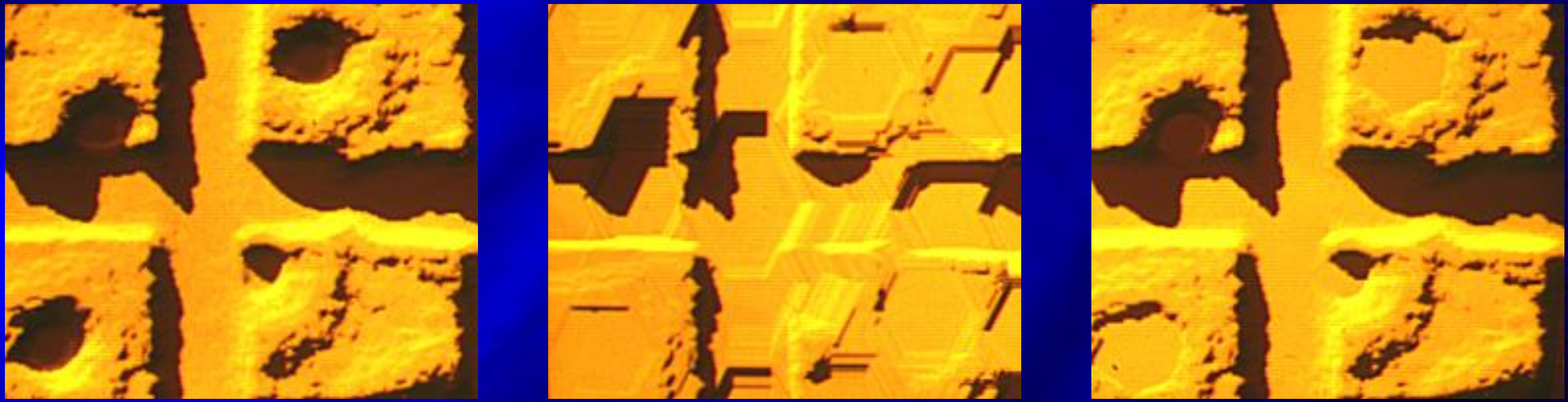
The reconstruction is a powerful filtering tool.

Filters...

Comparison between a classical morphological filter (closing) and a reconstruction filter.



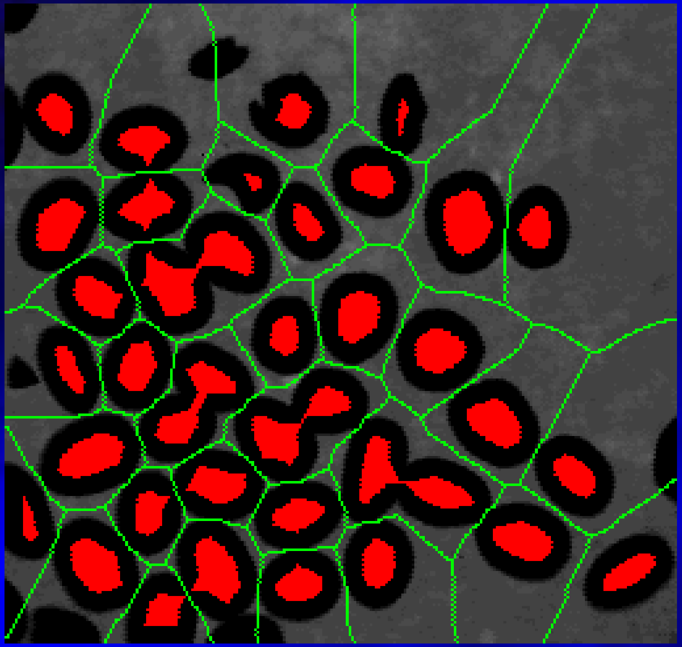
Initial *Closing* *Reconstruction*

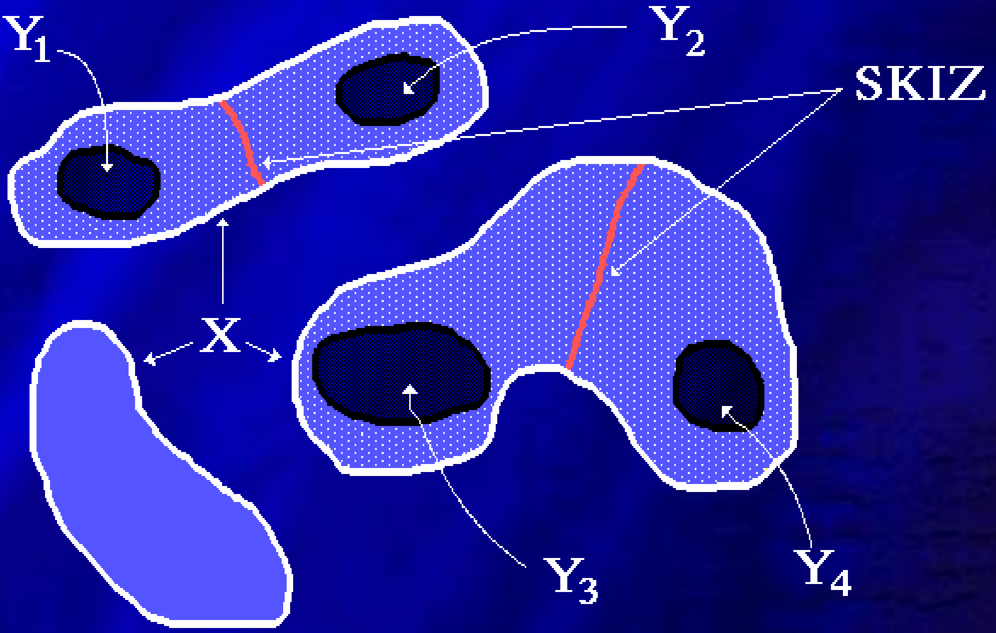


Connectivity and geodesy

SKIZ and geodesic SKIZ

$$IZ(Y_i) = \{x \in E : dist(x, Y_i) < dist(x, Y_j), i \neq j\}$$



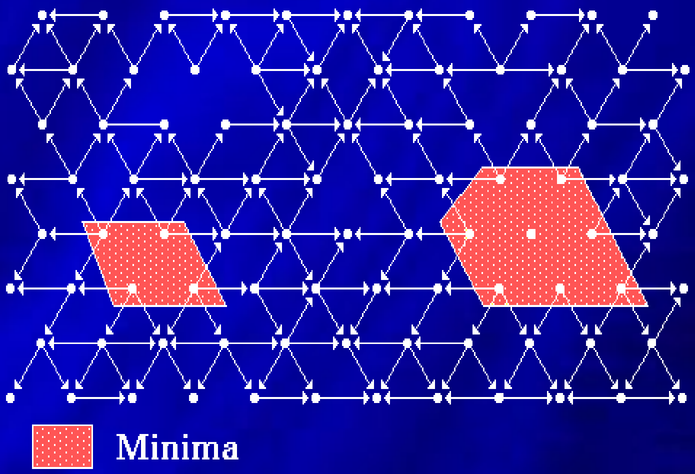


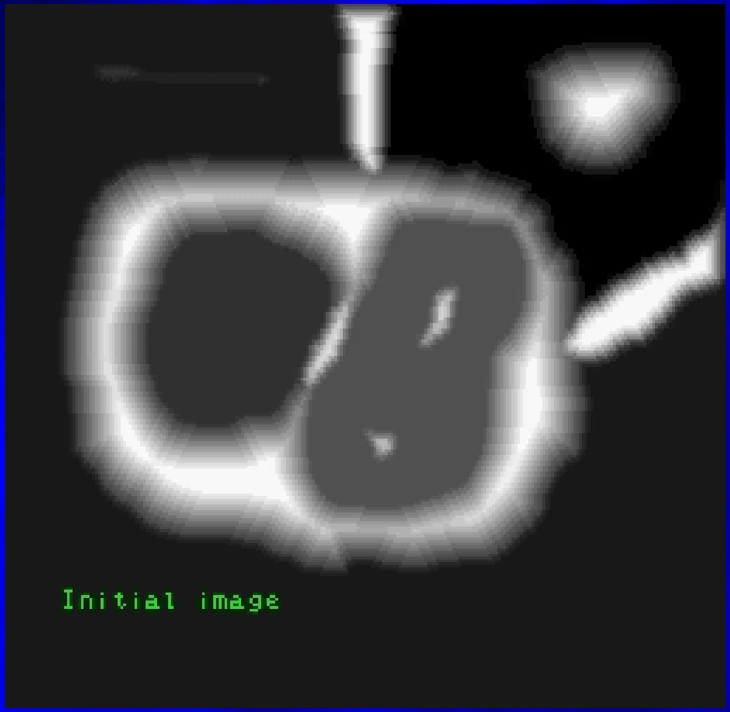
- Dist: euclidean distance
- E: whole space

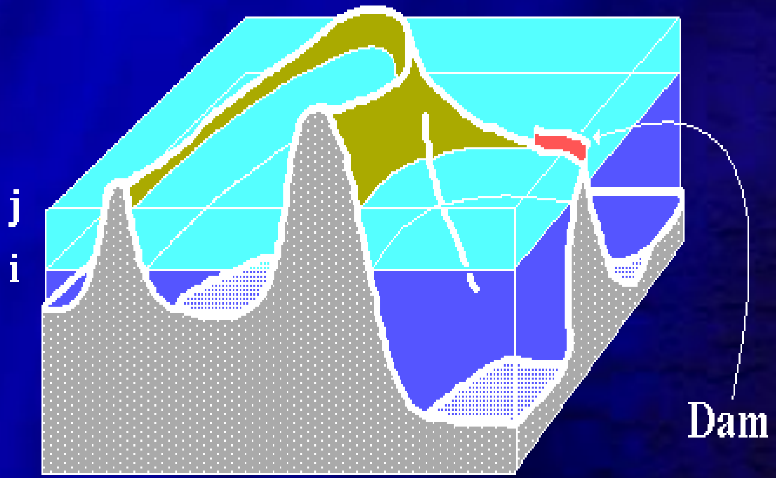
- Dist: geodesic distance
- E: set X

The Watershed Transform

The watershed transform is a powerful segmentation tool.
It is the extension to functions of the SKIZ concept.



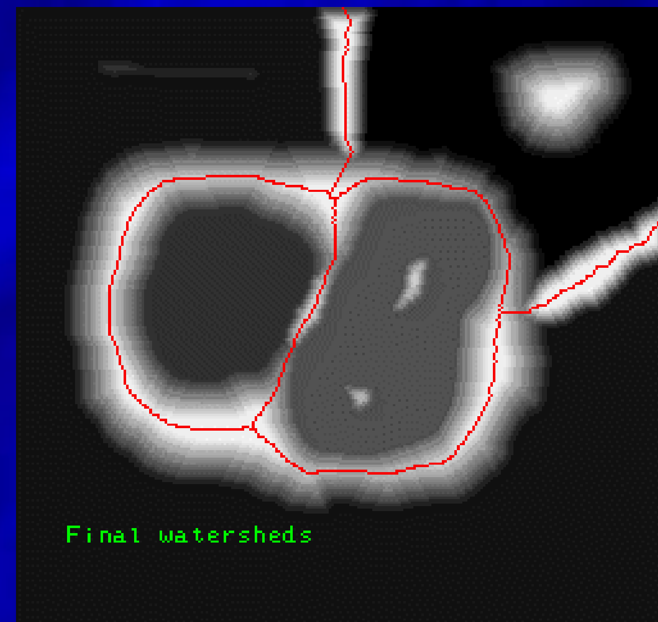




The watershed is the simulation of a flooding process on the topographic surface.

Marker-controlled Watershed

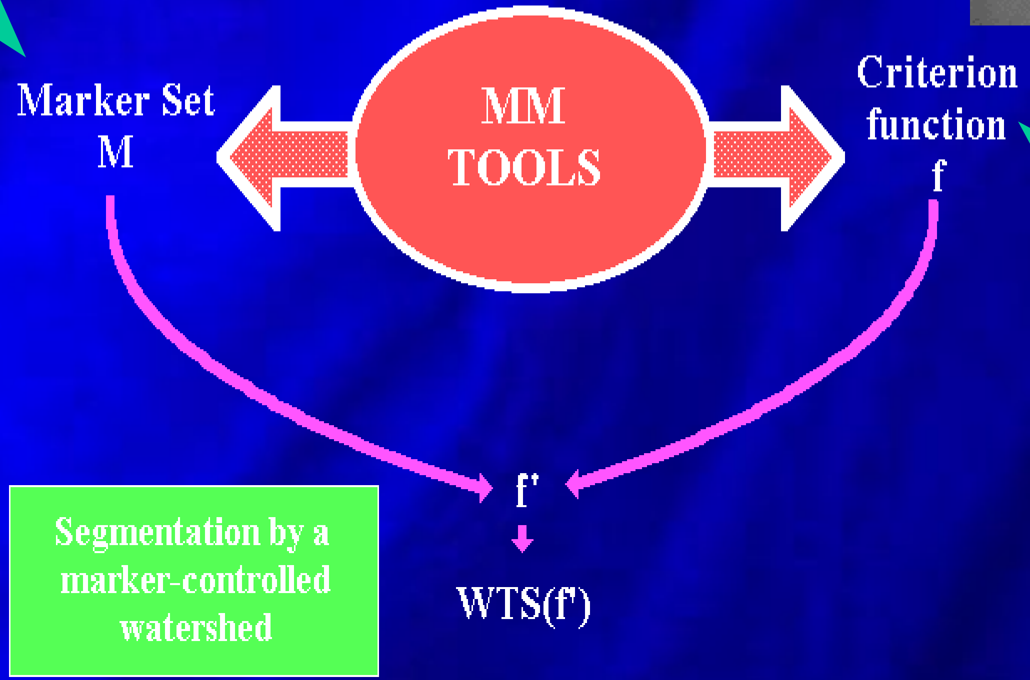
The topographic surface can be flooded from any source (marker) set.



Non marker-controlled watershed

Watershed in Image Segmentation

The watershed transform is given with directions for use...



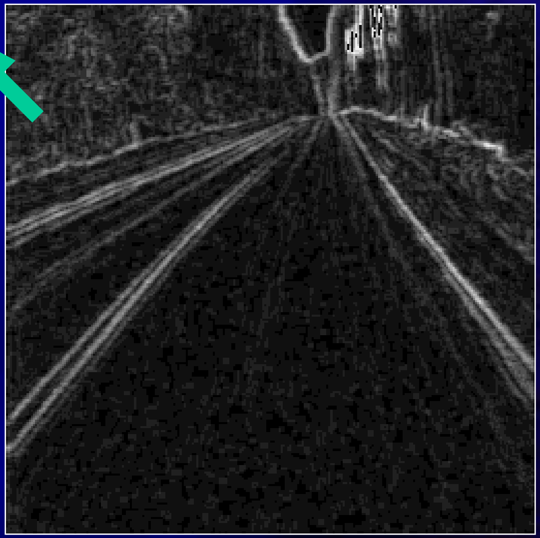
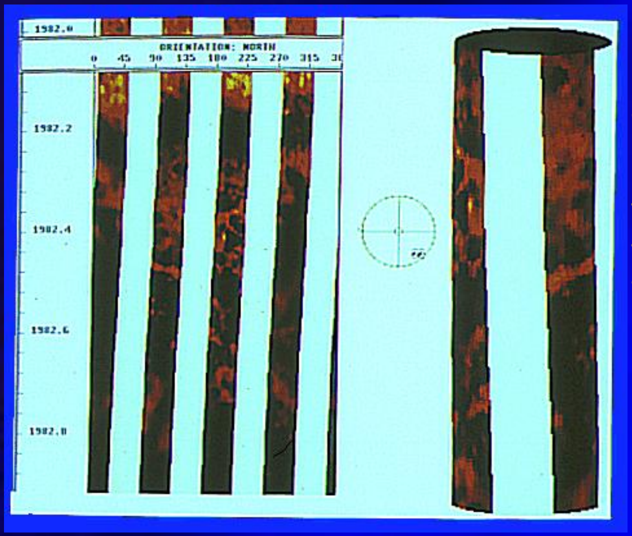


Image Analysis of Electrical Logs



Resistivity measures provided by a FMS tool.

Directional morphology

The image is partitioned according to the directions (azimuths) of the gradient...

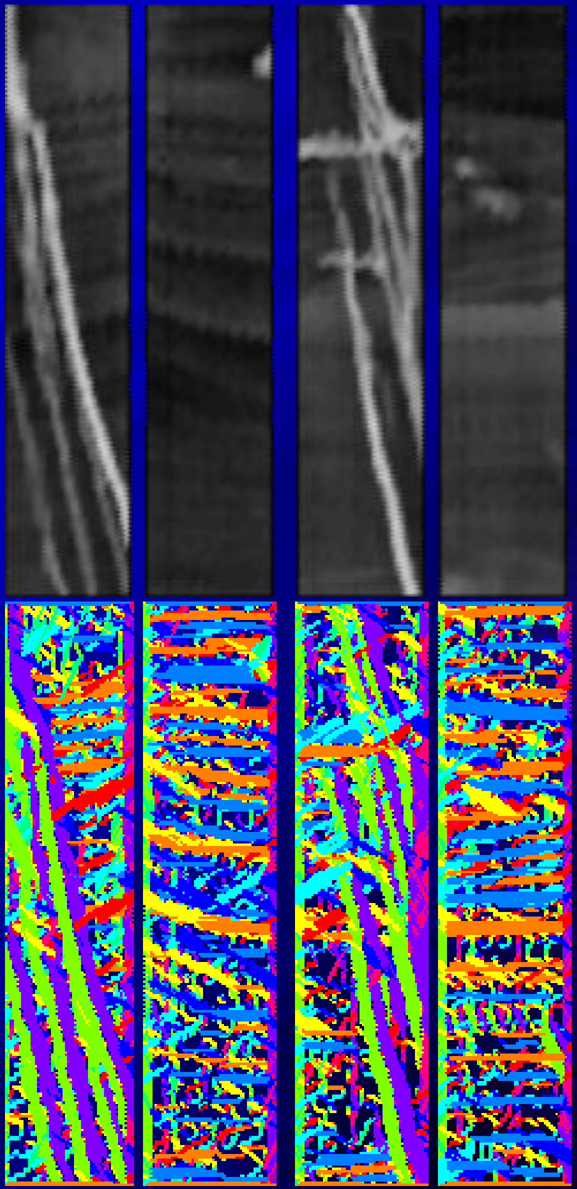
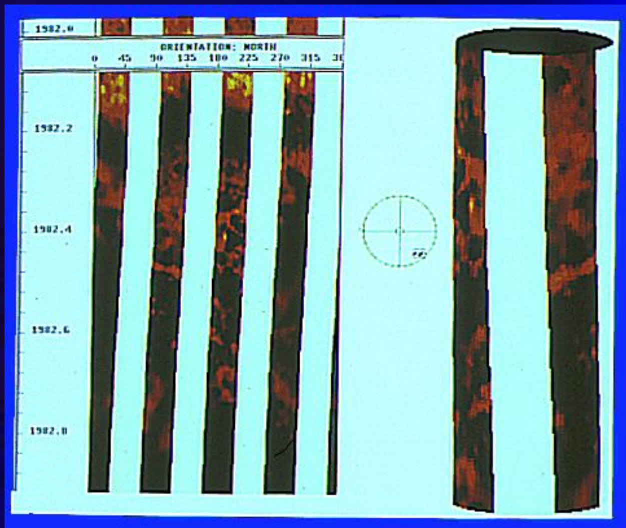
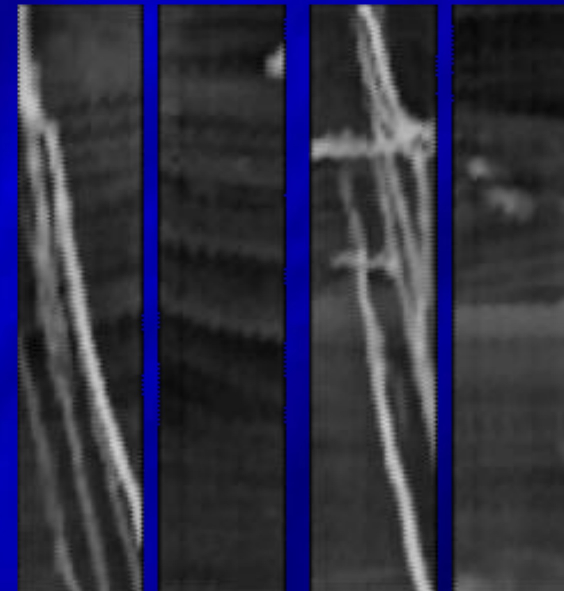


Image Analysis of Electrical Logs



Resistivity measures provided by a FMS tool.



Directional morphology

…allowing the definition of directional distance functions...

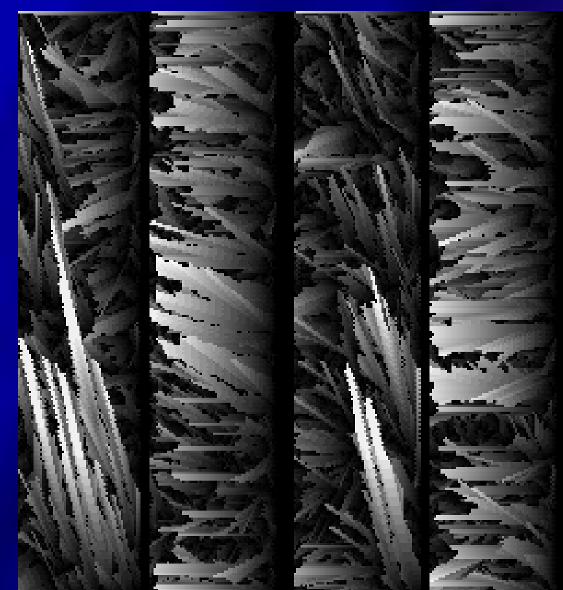
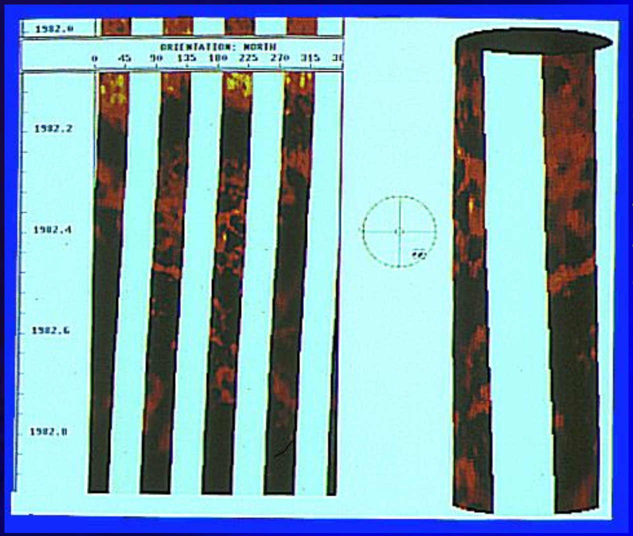
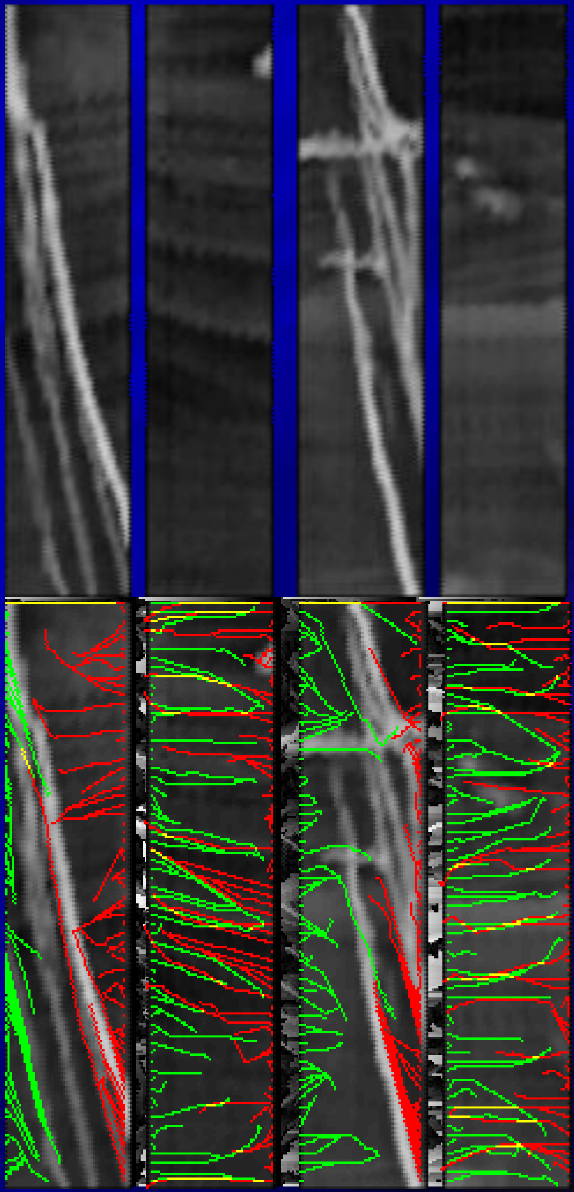


Image Analysis of Electrical Logs



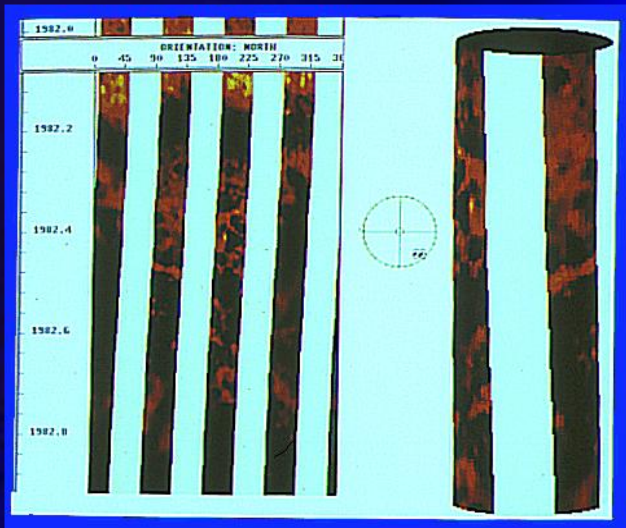
Resistivity measures provided by a FMS tool.



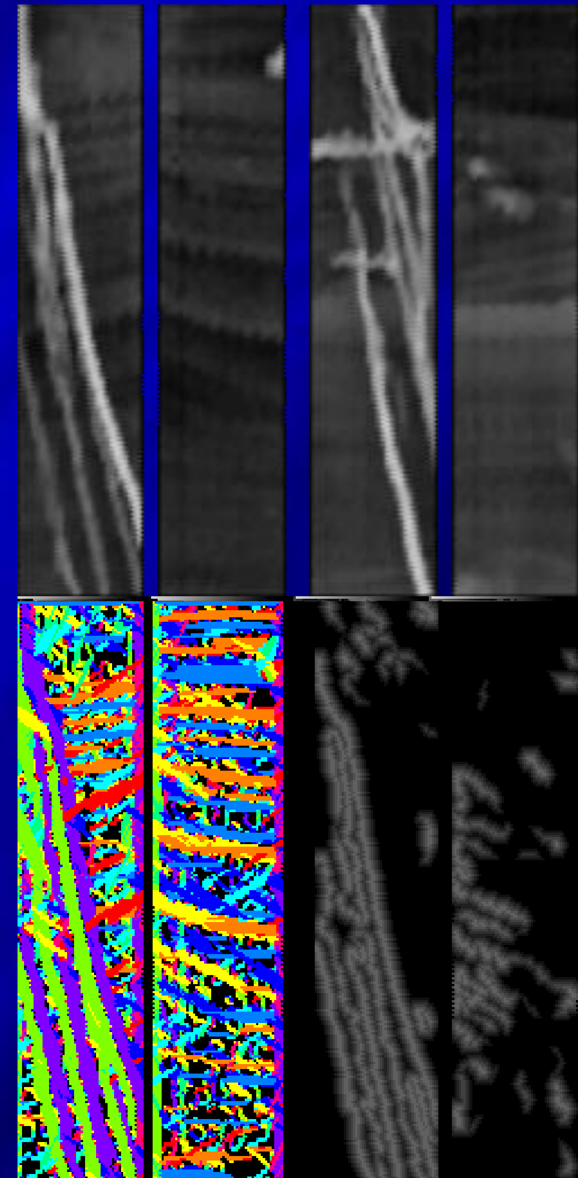
Directional morphology

Geodesic dilations are performed to follow the linear structures...

Image Analysis of Electrical Logs



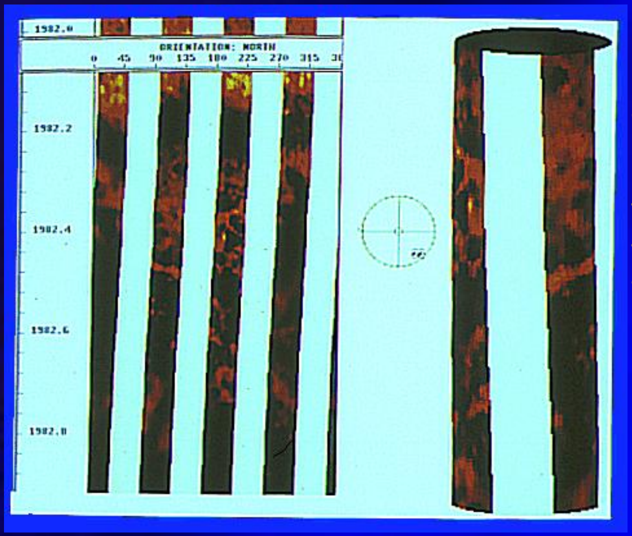
Resistivity measures provided by a FMS tool.



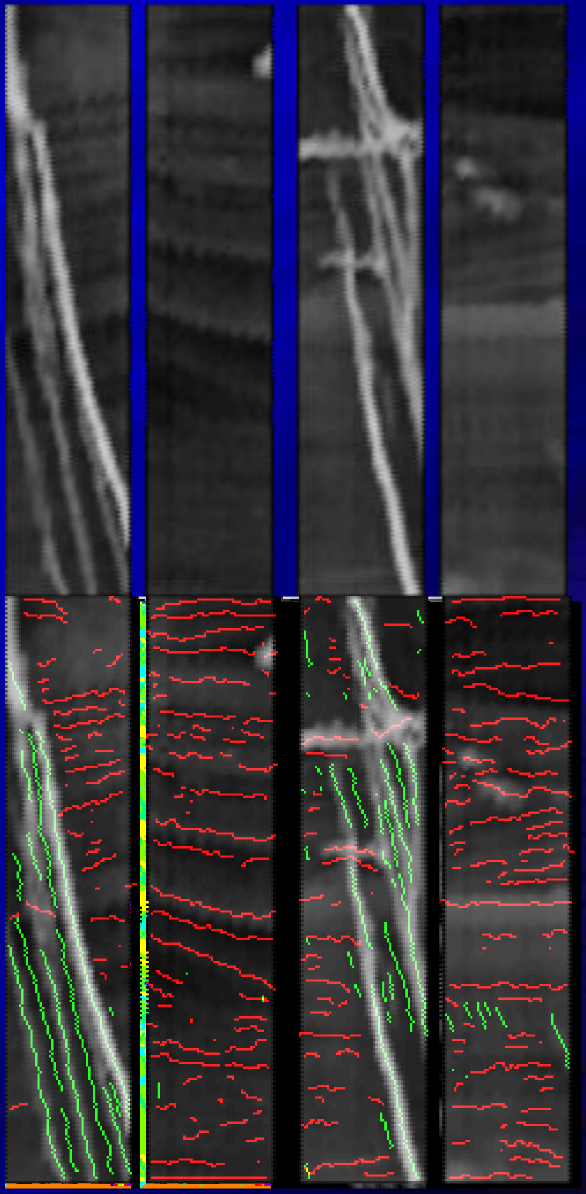
Directional morphology

…other distance functions can be designed from the partition...

Image Analysis of Electrical Logs



Resistivity measures provided by a FMS tool.



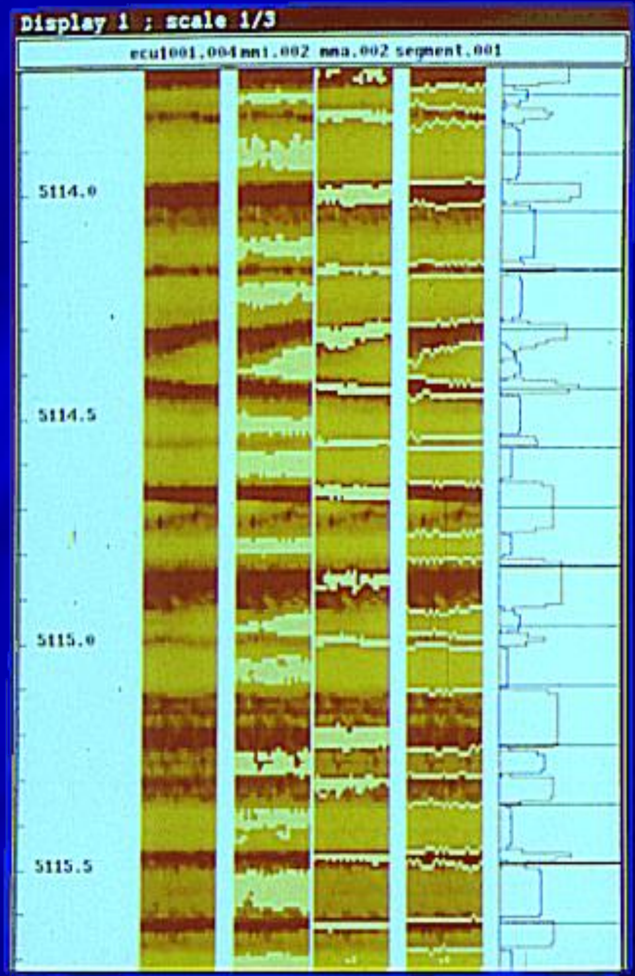
Directional morphology

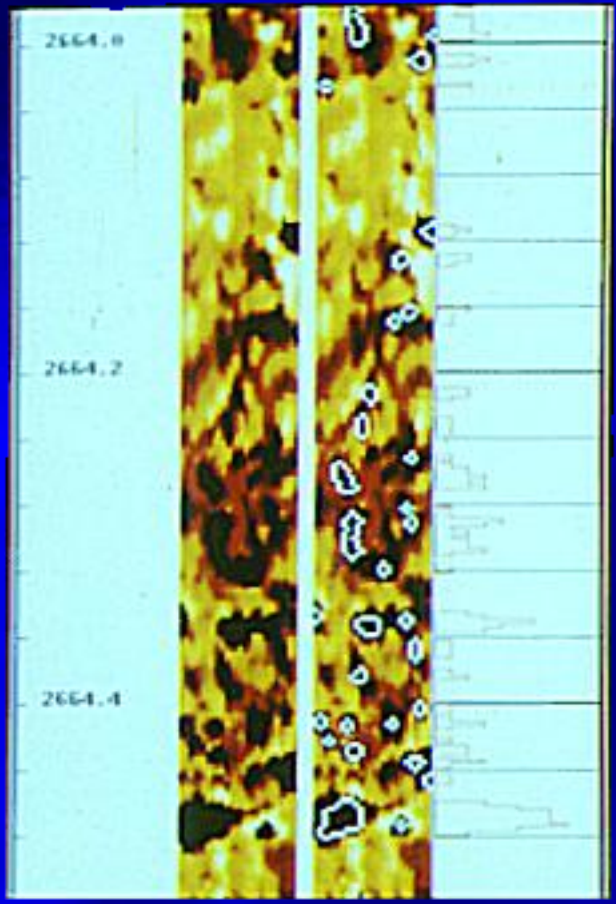
…and used to build (by watersheds) the directional features embedded in the images.

Image Analysis of Electrical Logs (2)

Segmentation of the bedding:

- **Reconstruction of crossing features**
- **Watershed of the gradient**

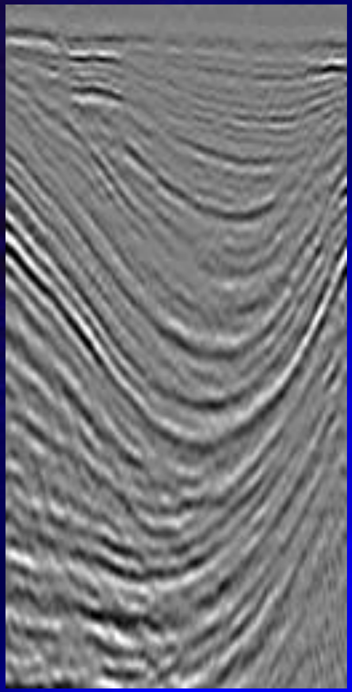




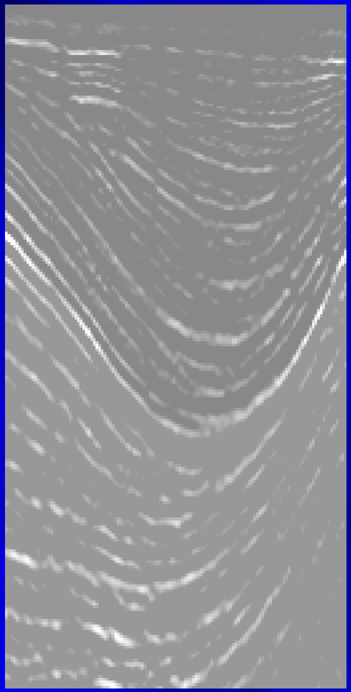
Vugs detection

Seismic Data Processing

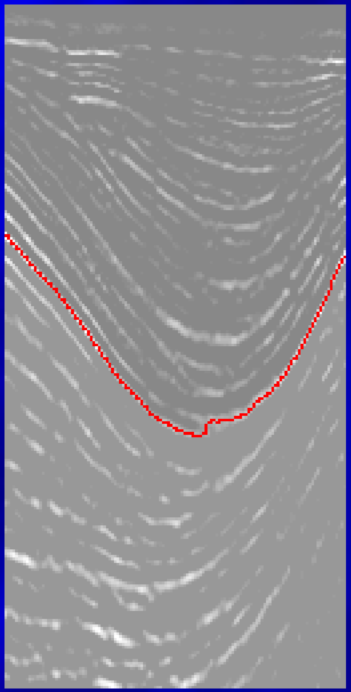
Automatic picking



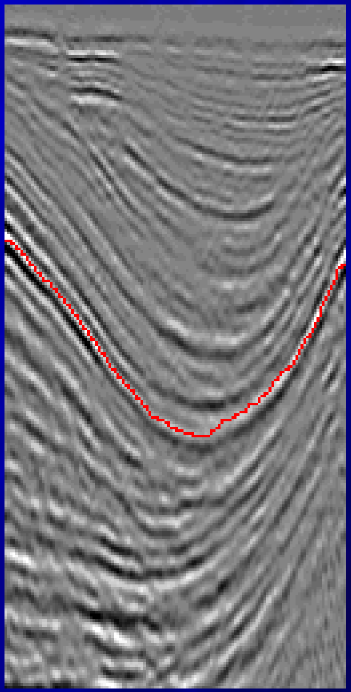
Original image



Closing-reconstruction filter



Watershed transform

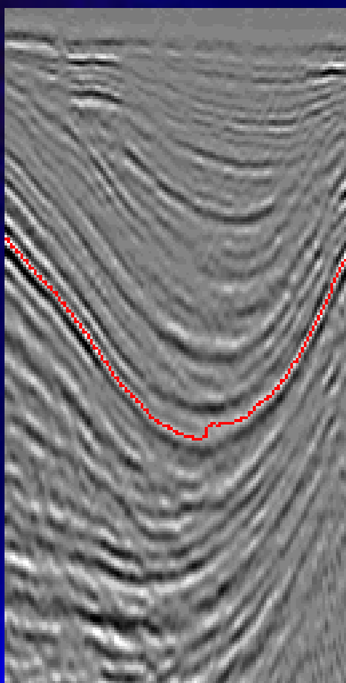


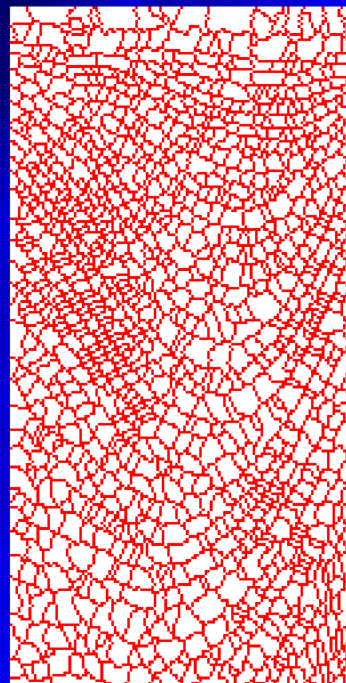
Smoothed profile

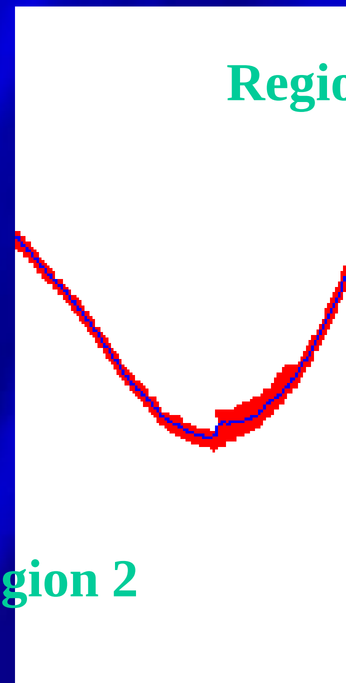
Seismic Data Processing (2)

The process can be iterated → Hierarchic segmentation

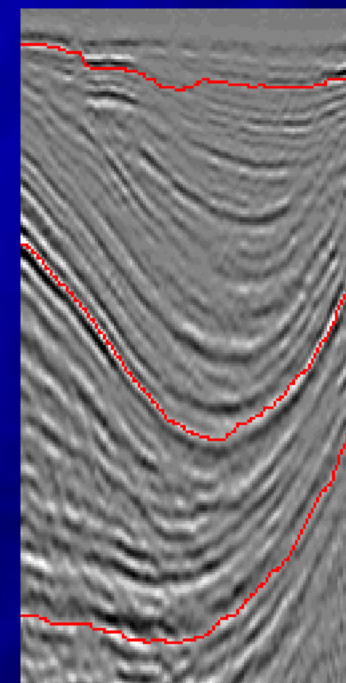
Previous step







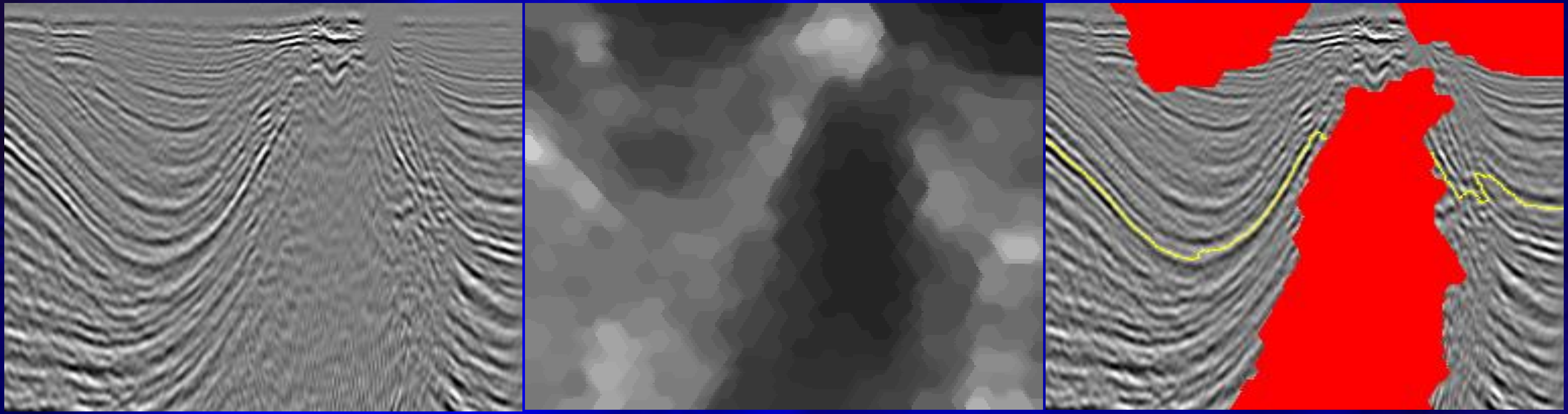
Next result



The initial watershed image is used to build the IZ of the profile. The process is iterated in the two regions separated by the previous profile.

Seismic Data Processing (3)

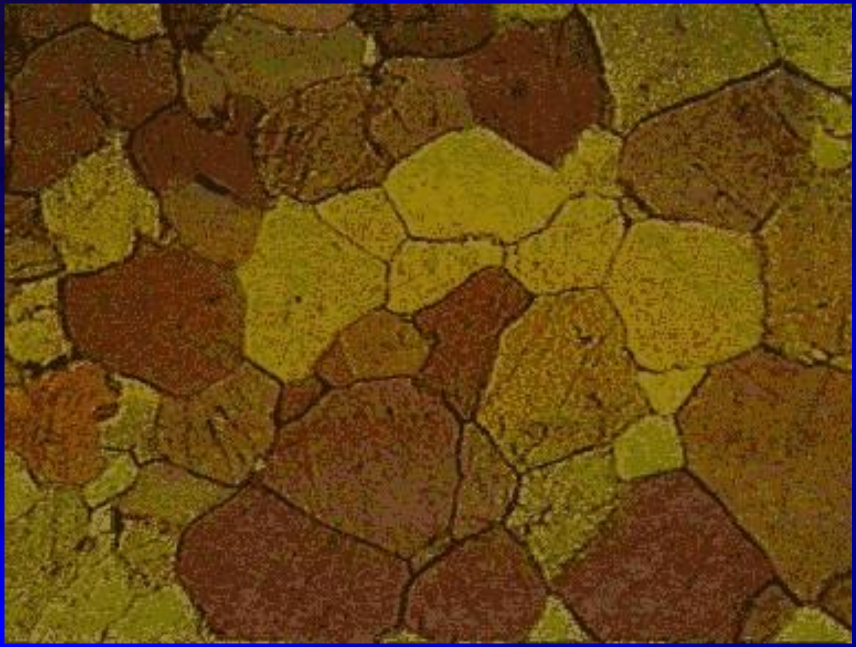
A morphological filtering can be used to segment regions with different textures.



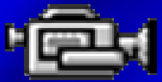
Alternate sequential filters (openings and closings) enhance the different textures in the image. The segmented regions can be used as masks and/or as geodesic spaces.

Colour Segmentation of Petrographic Sections

Segmentation of grains in a petrographic section (sequence recorded with a rotating polarizer)

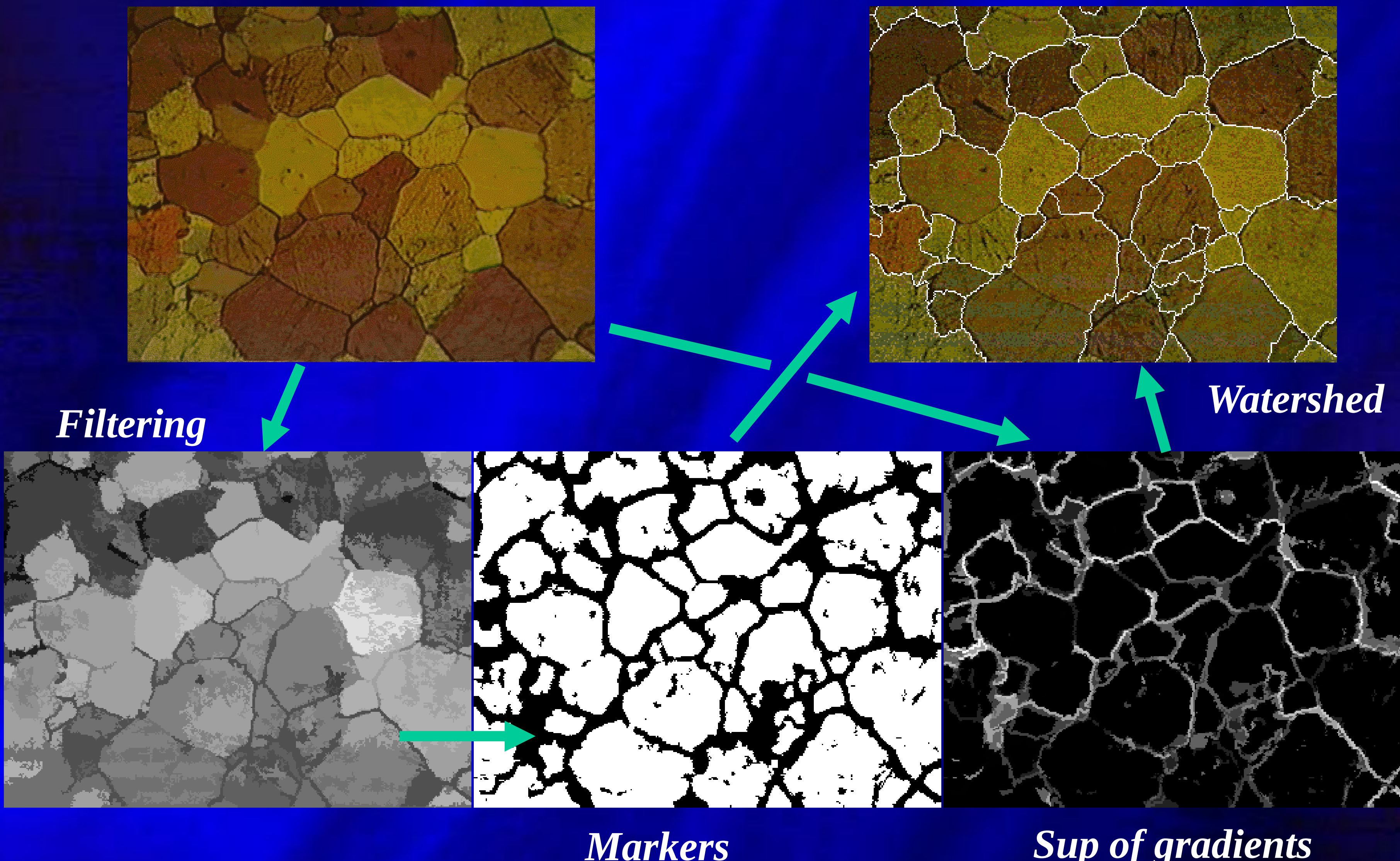


Dolomite section



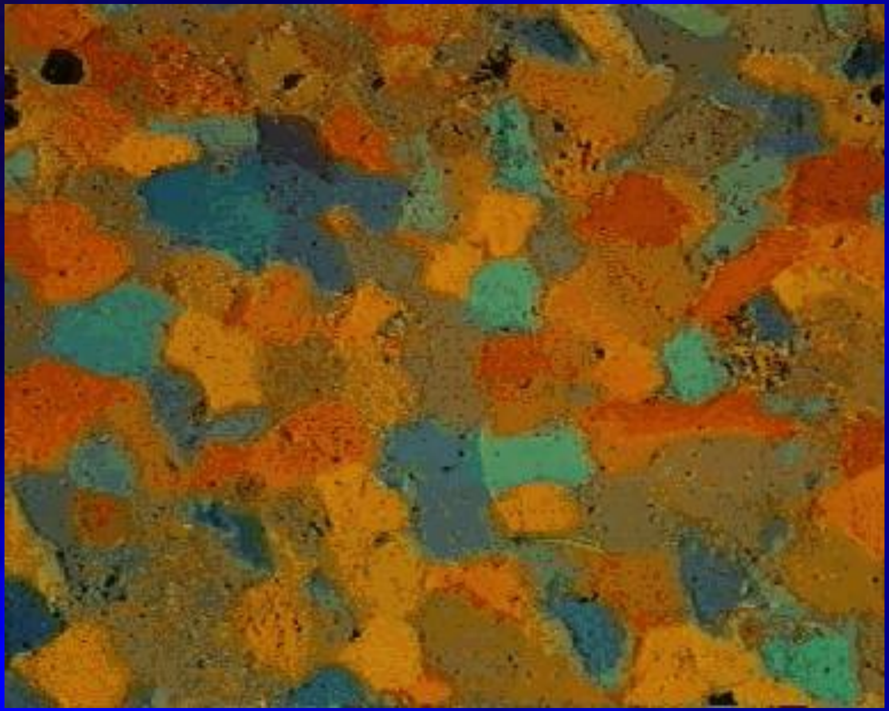
- **Filtering of the sequence by an alternate sequential filter based on reconstruction.**
- **Morphological gradient of each image (each channel).**
- **Marker extraction (significant minima) on each image and superposition.**
- **Sup of the gradient images.**
- **Marker-controlled watershed of the Sup.**

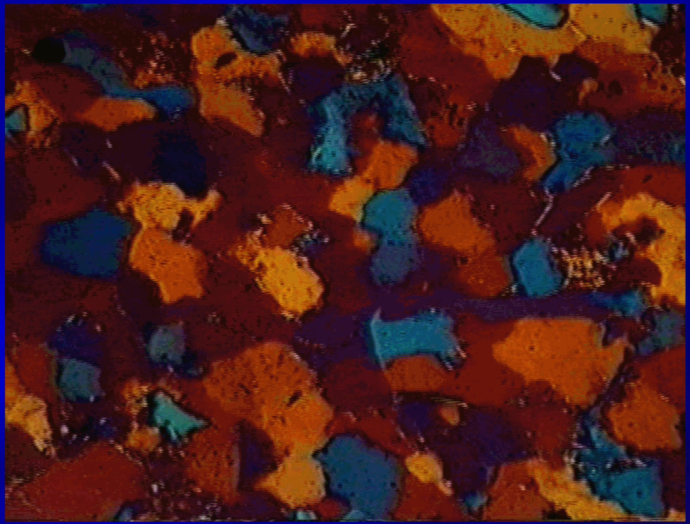
Colour Segmentation of Petrographic Sections (2)



Colour Segmentation of Petrographic Sections (3)

Another example: Quartzite





The Morphological Approach

Mathematical Morphology can be easily handled, through topographic analogies:

- **Most of the transformations can be easily understood.**
- **These transformations act on sets and remain in the image domain (quite interesting to iterate them).**
- **MM can be applied on various image analysis fields: 2D, 3D, colour images, moving sequences, etc.**
- **The morphological toolbox is very important and it is still increasing...**

Morphological tools

- **Hardware (image analysis systems)**
- **Software (various platforms)**