

# FAST IMPLEMENTATION OF WATERFALL BASED ON GRAPHS

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**Abstract** The waterfall algorithm is a contrast-based hierarchical segmentation approach. In this paper we propose an efficient implementation based on the minimum spanning tree of the neighborhood graph. Furthermore, other hierarchies are proposed and compared to the original version of the algorithm.

**Keywords:** Hierarchical image segmentation, watershed, waterfall, minimum spanning tree, graphs.

## Introduction

Segmentation, together with filtering is often the first step of image analysis or image interpretation. The success of the whole chain of treatment relies on the accuracy of the segmentation results. Important efforts have been devoted to segmentation during the last years and it still remains a key topic of research.

The watershed transformation [3, 1] is the paradigm of segmentation of Mathematical Morphology. It has proved to be a powerful tool used in the solution of multiple applications. Its main drawback is the over-segmentation produced. Two approaches are proposed in the literature to overcome this drawback:

- the selection of markers [9], which supposes that the characteristics of the interesting objects are known;
- hierarchical approaches, that are able to rank the importance of each region.

We focus on hierarchical approaches because of their ability to segment generic images.

Several hierarchical approaches can be found in the literature. Grimaud [4] introduced the dynamics of minima that assign to each minimum a measure of its contrast. By thresholding this measure with increasing values, a hierarchy is obtained. Najman and Schmitt [10] showed that the same measure of dynamics may be assigned to a contour and introduced the geodesic saliency of watershed contours. Vachier and Meyer [11] generalized the concept of dynamics with the extinction values and proposed to assign to a minimum other measure than contrast such as area or volume. Volume extinction values result in a well adapted criterion for evaluating the visual significance of regions.

Meyer proposed a graph-based implementation of these hierarchies [7],[8]. Nodes correspond to the catchment basins of the topographic surface. If two catchment basins are neighbors, their corresponding nodes are linked by an edge. The valuation of this edge is the minimum pass point of the gradient along their common frontier. In the following we will refer to this graph as the neighborhood graph. Meyer found that all the information of a hierarchy may be stored in a very condensed structure: the minimum spanning tree (MST). This is due to the fact that the flooding always follows the path of minimum height, the same that chooses the MST of the neighborhood graph. This consideration leads to a very efficient algorithm of hierarchical segmentation [7] and has also been used for interactive segmentation [12].

In [1, 2] Beucher proposed a very interesting hierarchical segmentation approach: the waterfall. Starting from the watershed result, it consists in an iterative algorithm that at each step removes all the watershed contours completely surrounded by higher ones. Typically, less than 10 hierarchical levels are produced by iterating the waterfall algorithm. In [2] each step is implemented by a reconstruction process followed by a new flooding of the resulting image. Another implementation based on graphs is also proposed in [1, 2].

The hierarchies based on extinction values produce a different level for each merging of two regions. This is useful for interactive segmentation approaches because it offers flexibility. The waterfall generates several steps of the hierarchy with an autocalibrated number of regions. This autocalibration may be interesting for segmenting generic images without imposing a given number of regions, which can be a tricky parameter to fix.

In [5] an automatic track detection application is developed using the waterfall algorithm in the initialization step. Several waterfall iterations may be necessary until a region compatible with the track geometry is found. The existing implementation of the waterfall does not allow this application in real time.

In this paper we propose an implementation of the waterfall algorithm based on the MST. It allows to access to different levels of the hierarchy in a very efficient way. Furthermore it allows the possibility to obtain different hierarchies

based on other criteria than the frontier height, used in the original version of the waterfall algorithm.

Section 1 describes the waterfall algorithm introduced by Beucher. The proposed efficient implementation based on graphs is presented in section 2. Other hierarchies are easily introduced in the new framework, as shown in section 3. Finally section 4 concludes.

## 1. Waterfall

Let's consider a partition  $P$ . It can be the outcome of the watershed of the gradient image (as proposed in [2]) or any other partition. The frontiers are valued with a distance between regions (e.g. the minimum pass point of the gradient along the frontier, see figure 1(a)).

The waterfall algorithm removes from a partition all the frontiers completely surrounded by higher frontiers (see figure 1(b)). Thus, the importance of a frontier is measured with respect to its neighborhood. This process can be seen as a watershed applied not to the pixels of an image but to the frontiers of a partition. The iteration of the waterfall algorithm finishes with a partition of only one region.

Figure 2 illustrates the result of the waterfall process applied to a real image. Figure (a) shows the original image, (b) its gradient and (c) the watershed of the gradient. Figure (d), (e) and (f) are the different hierarchy levels produced by the iteration of the waterfall algorithm.

A first implementation of this algorithm based on graphs is proposed in [2]. The proposed graph contains a node for each arc frontier of the input partition and an edge between every pair of arcs (frontiers) delimiting the same catchment basin. (Note that this graph is different from the one we propose in the next section). The algorithm is considered as complex by the author and an image-based algorithm is proposed instead. The image-based algorithm consists in a reconstruction of the image from the watershed lines followed by the watershed transformation. Thus, a reconstruction and a new watershed computation are required to obtain a new level of the hierarchy.

In this paper, we propose an implementation of the waterfall algorithm that only requires a flooding of the image to obtain all levels of the hierarchy. The rest of the process is performed on the MST, that is much more efficient.

## 2. Waterfall based on the Minimum Spanning Tree

Given that the flooding always follows the path of minimum height, the MST obtained from the neighborhood graph, contains all the information required for the flooding process [6]. The MST is a very condensed way to store the information. Thus, it leads to very efficient implementation of hierarchical segmentation approaches.

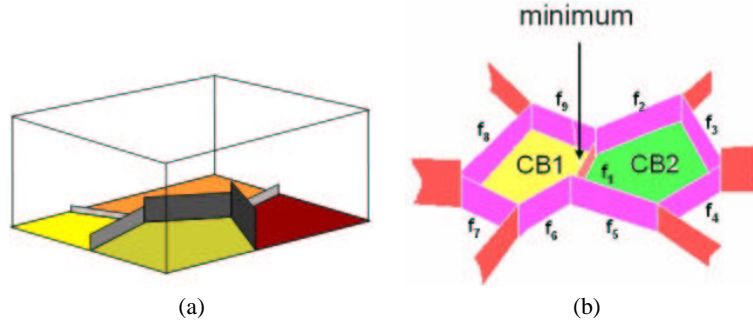


Figure 1. (a) Partition with valuated frontiers and (b) example of frontier : as the value of frontier  $f_1$  is smaller than the values of its neighboring frontiers ( $f_2$  to  $f_9$ ) it will be removed by the waterfall algorithm.

We propose an implementation of the waterfall algorithm based on the MST. The algorithm is performed in two steps:

- Minimum spanning tree (MST) generation
- Waterfall from the MST

### MST generation

We consider as input partition of the waterfall algorithm the result of the watershed. Thus, the gradient image is flooded to obtain the initial partition. The MST is obtained simultaneously to the flooding process [7, 12]. The graph is initialized with a node corresponding to each minimum of the image and without any edge. A lake is associated to each minimum. During the flooding each time that two regions of different lakes meet, an edge is added to the graph, linking both regions and the corresponding lakes are merged. Its valuation is the height of water when regions meet. At the end of the flooding process the graph has become the MST because:

- an edge is added only if regions that meet belong to different lakes (so the graph does not contain cycles),
- at the end of the flooding all the image belong to the same lake (it is a spanning tree)
- the flooding follows the path of minimum height (it is a minimum tree).

Note that the edges of the MST are valuated (not the nodes). We can define a regional minimum of the MST as a connected component of the graph, such as all the values of its edges are equal and it is surrounded by edges with strictly higher values. This definition will be used in the following subsection.

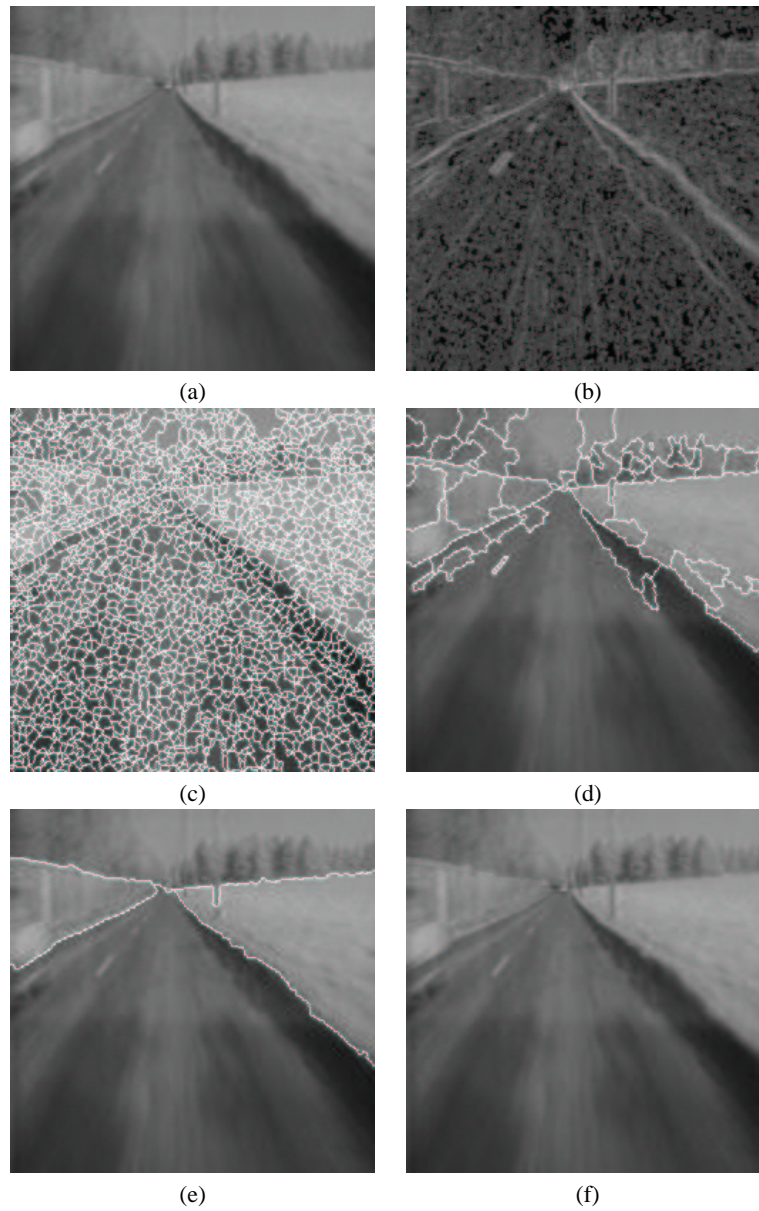


Figure 2. Waterfall iteration; First row: (a) original image and its (b) gradient. Second row: (c) Watershed segmentation and (d) first waterfall result Third row: (e),(f) Two more iterations of the waterfall algorithm

Figure 3 shows an example of partition (a) and its corresponding Minimum Spanning Tree (b).

In the following step we address the waterfall algorithm based on the MST.

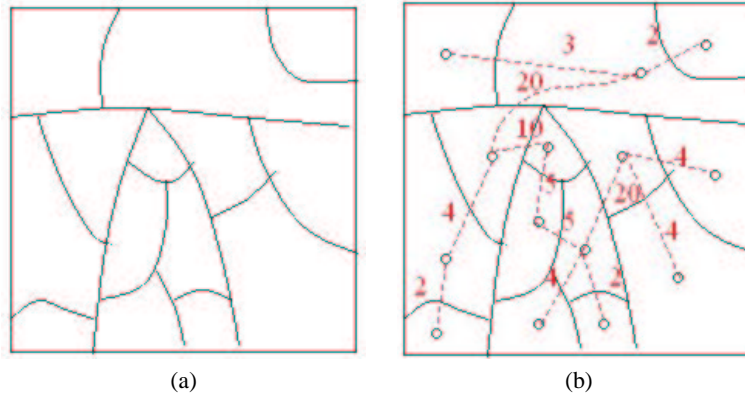


Figure 3. Example of partition (a) and its associated Minimum Spanning Tree (b).

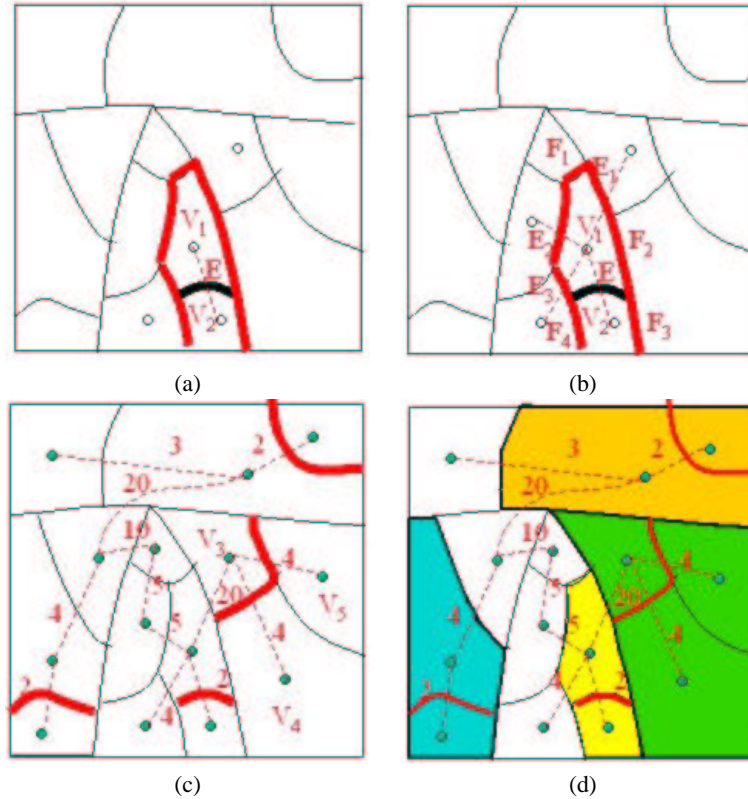
### Waterfall from the MST

The waterfall algorithm removes from a partition the frontiers that are completely surrounded by higher frontiers, leading to a coarser partition. The waterfall can be implemented on the MST. The first step consists in identifying regional minima frontiers.

Let's consider the partition of figure 3(a) and its corresponding MST 3(b). If we take for example edge E linking regions  $V_1$  and  $V_2$  of figure 4(a) we should compare its valuation with the valuation of frontiers surrounding catchment basins  $V_1 \cup V_2$ . These frontiers are drawn in bold line in figure 4(b) (E1, E2, E3, F1, F2, F3 and F4). Edges named with an F do not belong to the MST, so by construction their valuations are higher than valuations of at least one edge named with an E. Therefore, in order to know if E is a regional minimum, it is enough to compare it with E1, E2 and E3. Thus, the MST has all the information required to identify regional minimum edges. In practice, we will compare the valuation of an edge E between  $V_1$  and  $V_2$  with edges of the MST having as one extremity  $V_1$  or  $V_2$ . More generally, we are looking for all edges that belong to a regional minimum of the MST (defined in the previous subsection).

If E is a regional minimum edge, it corresponds to a frontier that should be removed by the waterfall. This is implemented by assigning the same label to both extremities (nodes) of the minimum edge. These labels identify regions in the output partition. Thus, a different label is assigned to each minimum edge. See figure 4(d). Regions in white (those that are not neighbors of a

minimum edge) are not granted any label at this stage. Regions with a label are considered as markers. In order to obtain the final partition, markers are propagated following edges of the MST in increasing order of their valuation (fig 5). This process is a segmentation from markers on the MST [6].



*Figure 4.* Waterfall on the MST. First row : (a) Edge  $E(V_1, V_2)$  and (b) comparison of  $E$  with its neighboring edges. Second row: (c) regional minimum edges. (d) Vertex labelled from minimal edges

If edges of the MST are valued with the lowest pass point along frontiers, this algorithm is equivalent to the algorithm presented in [2].

### 3. Hierarchies with other criteria

The original waterfall algorithm removes edges according to their height. Thus it produces contrast-based hierarchies. The graph implementation of the algorithm, presented in the previous section, allows to easily produce other hierarchies, changing the edge valuation of the Minimum Spanning Tree. For

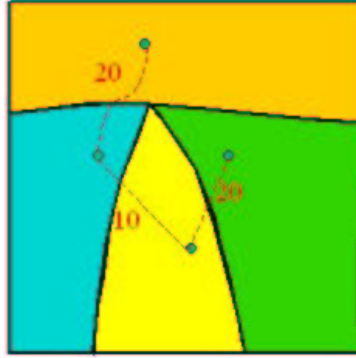


Figure 5. Waterfall result (label propagation from 4(d)).

example, we can value the edges with the volume extinction values [11], a trade-off between the contrast and the size of a region. This is equivalent to applying the waterfall algorithm to an image of saliency of watershed contours, valued with volume extinction values.

Figure 6(a) shows a level of the contrast-based hierarchy with 36 regions and 6(b) shows a partition with 24 regions obtained with the waterfall based on volume. We can see that the volume produces a more significant partition with less regions. For example, we can see that the waterfall does not get the hat, because it has a low contrast. However, the volume preserves it because even if the contrast is low it is big enough to be seen. The volume combines size and contrast trying to obtain good perceptual results. This combination is not optimal yet, because it segments regions in the background that are big but not visible at all.

The volume criterion usually over-segments big homogeneous regions. A last step may reduce this problem, just removing frontiers with contrast under a given (and small) threshold. Doing that at the end of the process is much more reliable because frontiers are longer and their contrast may be better estimated, reducing the effect of noise in a small frontier. Figure shows 7 the result of removing frontiers with contrast under 5 gray levels. The partition preserves the important regions (18) removing the over-segmentation of the background.

#### 4. Conclusion

In this paper we propose an efficient implementation of the waterfall algorithm. It consists in obtaining a Minimum Spanning Tree simultaneously to the flooding of the image. Then, all the process is performed on the graph. All the information required to manipulate the hierarchy is stored in a very condensed structure, leading to very efficient algorithms. Real-time applications such as





Figure 6. Comparison between different hierarchies. (a) Contrast-based Waterfall, 36 regions. (b) Volume based Waterfall, 24 regions.



Figure 7. Elimination of low contrasted frontiers. (a) volume based waterfall (24 regions). (b) From (a) low contrasted frontiers (contrast lower than 5) are removed (18 regions remain)

the one described in [5] may be addressed thanks to this new implementation. This implementation is based on the same data structure as the algorithms of volume extinction values, the MST. Thus, it opens the door to a combination of both approaches. For example, we have presented a waterfall based on volume extinction values and compared the results with the original version of the algorithm.

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