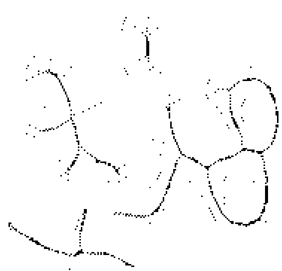
FROM NON CONNECTED TO HOMOTOPIC SKELETONS IN MULTIDIMENSIONAL DIGITAL SPACES

Purpose:

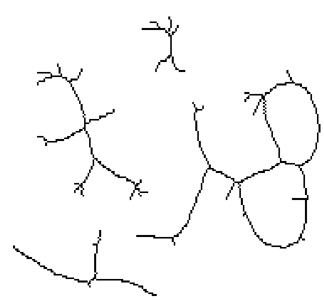
To bridge the gap between the Maximal Balls Skeleton (MBS) and skeletons by thinnings.

Maximal Balls Skeleton



True skeleton
Can be expressed in terms of morphological transforms
Reconstruction and shape descriptor capabilities
Non connected

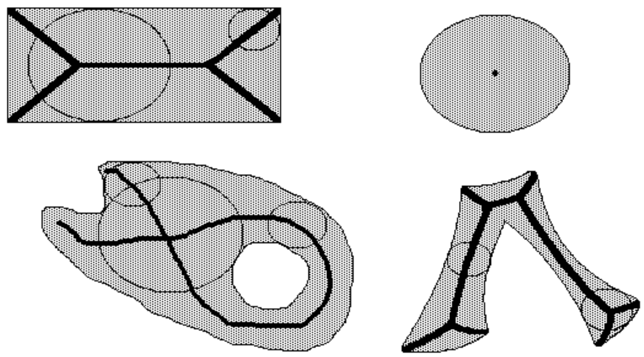
Skeletons by thinnings



Many algorithms (L,M,D thinnings)
No obvious links with the true skeleton
Use sequential thinnings (bias)
Connected (homotopic trans-forms)

Maximal Balls non connected Skeleton

Maximal Balls



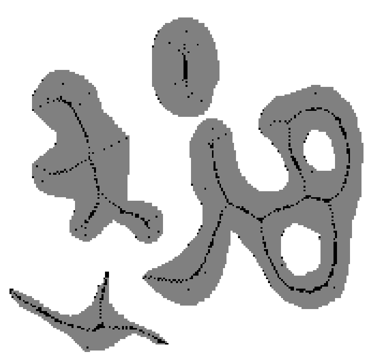
Skeleton of a set X : Set of all the centers of the maximal balls of X.

Skeleton and Morphological Openings

$$S(X) = \bigcup_{i=0}^{\infty} [(X \ominus iB)/(X \ominus iB)_B]$$

Lantuéjoul's Formula

The Maximal Balls Skeleton S(X) is made of the residues by opening of the successive erosions o



This formula does not depend on the dimension of space.

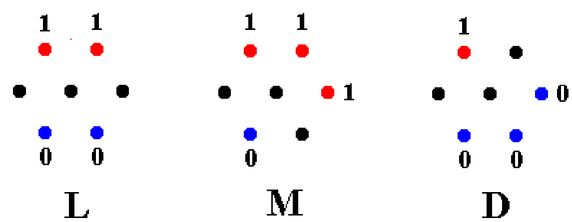
Connected skeletons using Sequential Thinnings

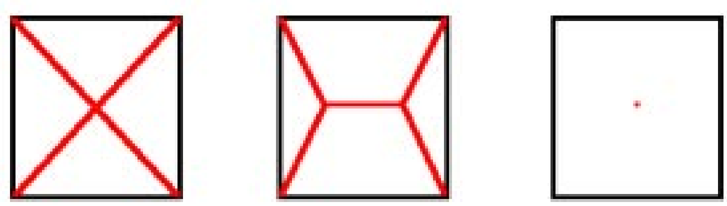
Sequential thinnings

$$(((X \circ T_1) \circ T_2) \circ \ldots\ldots \circ T_n)$$

T_i , successive rotations of an homotopic structuring element.

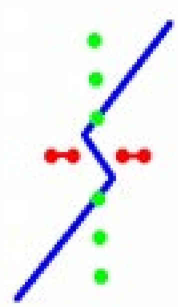
Various thinnings produce various skeletons





L- Skeleton M-Skeleton D-Skeleton

Bias due to rotations



Skeletons and Thinnings (1)

Question 1

Is it possible to express the Maximal Balls Skeleton with non sequential thinnings?

$$S(X) = ((X \circ T) \circ T) \circ \ldots$$

$X \circ T$: **Union thinning** (thinning using a family T of structuring elements)

$$T = \{ T_1, T_2, \ldots , T_i, \ldots \}$$

$$X \circ T = \bigcap_i (X \circ T_i)$$

Question 2

How to exhibit the family T ?

Question 3

How to connect the skeleton by selecting among the family T a sub-family T' preserving homotopy?

Skeleton and Thinnings (2)

Answer 1

Define an iterative transform:

$$X = Z_0$$

$$Z_n = (Z_{n-1} \ominus B) \cup (Z_{n-1} / (Z_{n-1})_B)$$

One can show that :

$$Z_n = (X \ominus nB) \cup S_{n-1}(X)$$

with :

$$S_{n-1}(X) = \bigcup_{i=0}^{n-1} ((X \ominus iB)/(X \ominus iB)_B)$$

$$Z_\infty = S(X) \text{ , Maximal Balls Skeleton}$$

The general form of this iterative thinning is:

$$(Z \ominus B) \cup (Z/Z_B)$$

$$Z \cap [(Z \ominus B)^c \cap Z_B]^c$$

Hit-or-Miss transform:

$$\bigcup_{a,b \in B} [(Z \ominus B_a) \cap (Z^c \ominus L_b)]$$

B_a , translated ball in direction a
L_b , translated point in direction b

The Maximal Balls Skeleton can be written as a succession of union thinnings with:

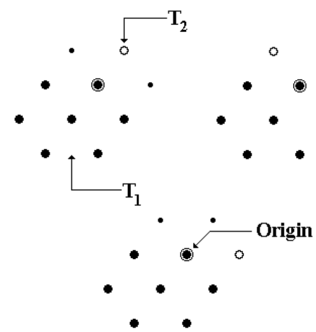
$$T = \{T_{a,b} = (B_a \text{ , } L_b) \text{ a,b} \in \text{ elementary ball}\}$$

Examples of T families

Answer 2

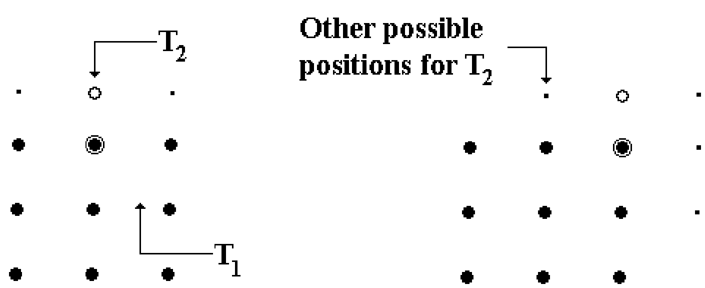
2D hexagonal grid

Elementary ball : Hexagon



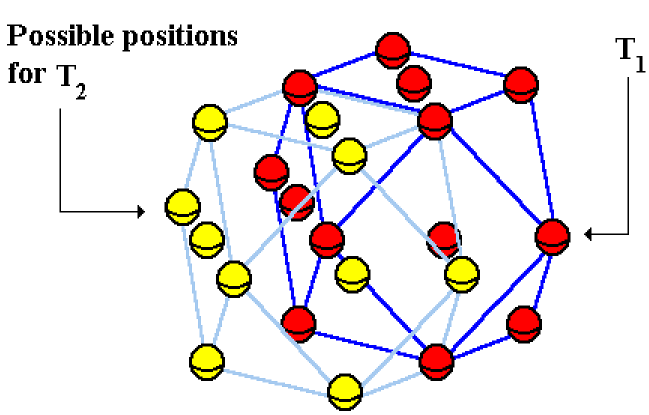
2D square grid

Elementary ball : Square



3D cubic grid

Elementary ball : Cuboctahedron

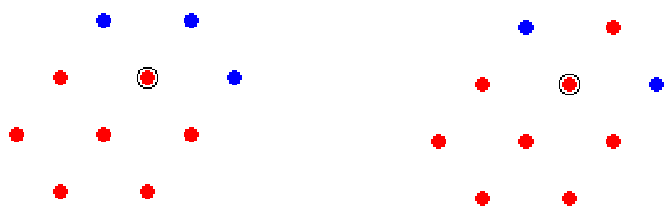


How to connect the previous skeleton?

2 possible approaches

Answer 3

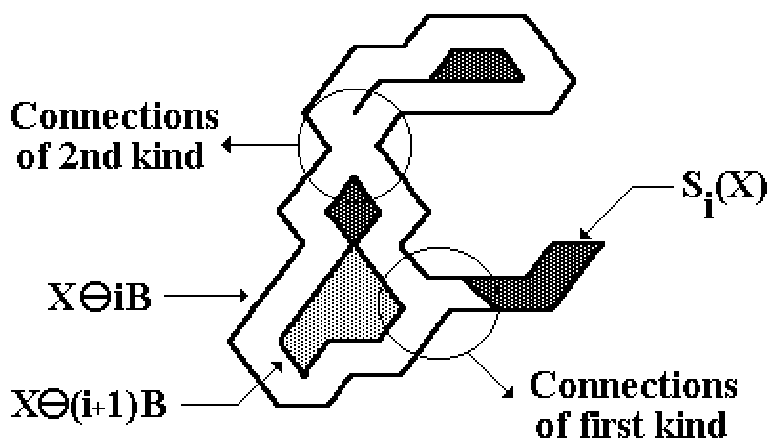
Use T and sort those structuring elements preserving homotopy



Homotopy preserving **Homotopy breaking**

Main drawback: complex (especially in 3D) and long procedure.

Starting from the Maximal Balls Skeleton, define a connection procedure for the residues



Two kinds of connections:

- Connection of the residues at step i with the eroded set of size i+1.
- Connection of the connected components of the eroded set of size i+1, when disconnections happen between step i and i+1.

Connected Skeleton by Union Thinnings

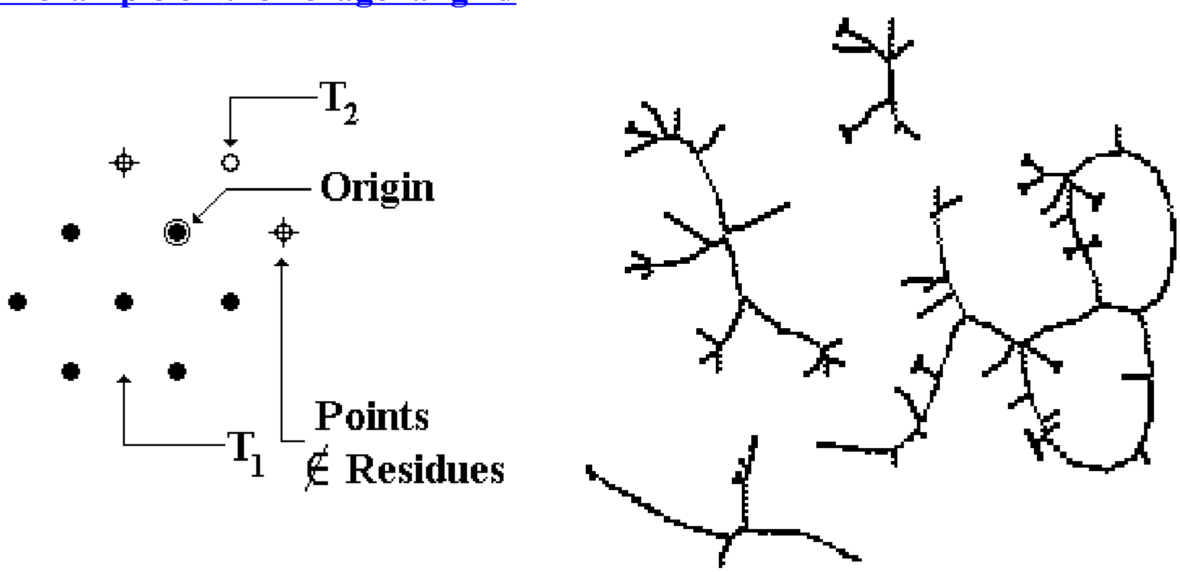
Sub-family $\mathcal{T}' \subset \mathcal{T}$ of structuring elements preserving homotopy

$((X \circ \mathcal{T}') \circ \mathcal{T}') \circ \ldots \circ \mathcal{T}') \circ \ldots . = S_c(X)$

$\mathcal{T}'$ is made of 3-phases structuring elements:

- Points belonging to X_i
- Points belonging to X_i^c
- Points not belonging to the residues of X_i

2D example on the hexagonal grid



Properties of S_c

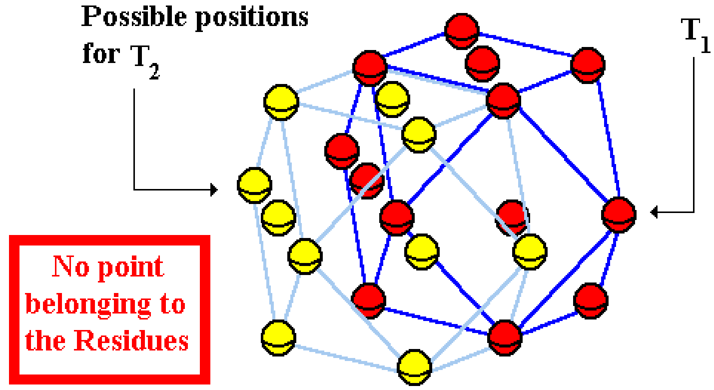
- $S(X) \not\subset S_c(X)$ That means that the minimal set of structuring elements preserving homotopy is not able to produce a "true" connected skeleton.
- If X connected set, then $S(X) \cup S_c(X)$ is connected. Some points of the MBS may have been forgotten by the homotopy thinning, nevertheless these points are adjacent to the connected skeleton.

3D Skeleton

3D connected skeleton on the cubic grid

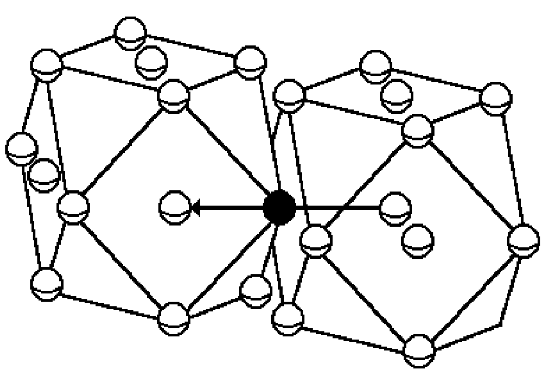
Elementary ball: cuboctahedron

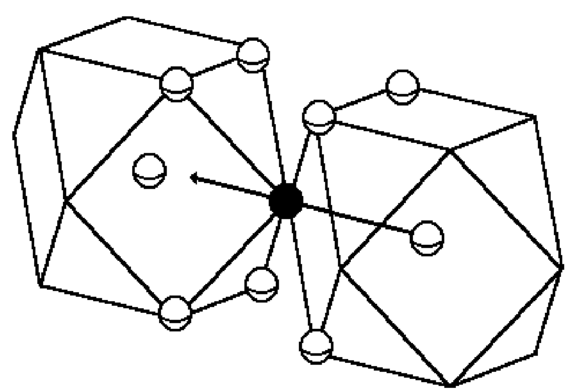
Preserving connections of the first kind



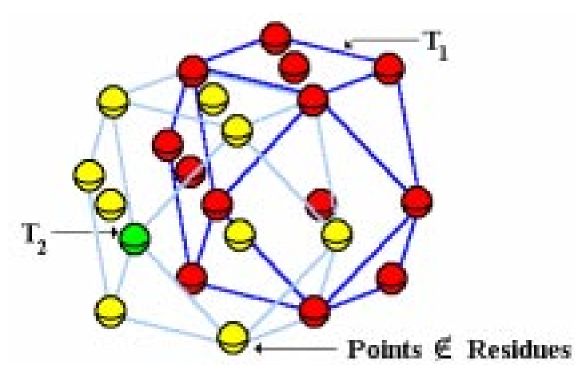
Preserving connections of second kind

Examples of second kind connections





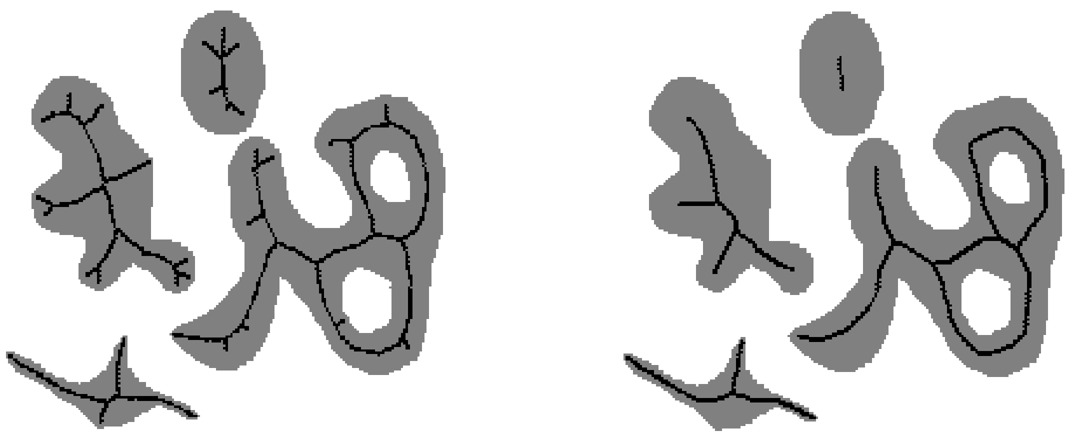
Final family T '



Other applications

Smooth skeletons

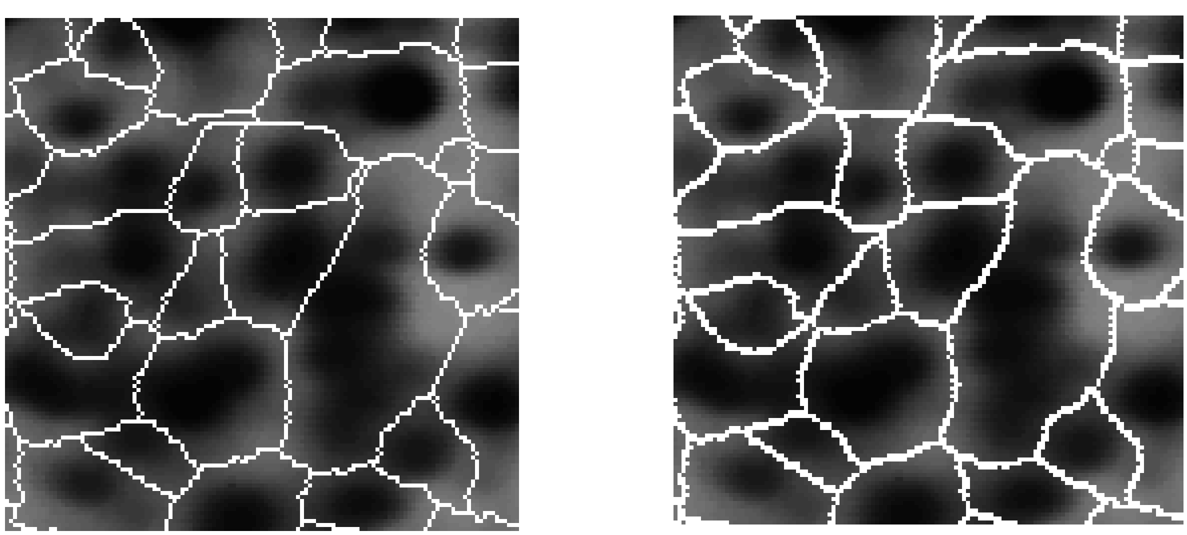
T ' ⟶ T " , Reduced homotopy preserving family



Smooth skeleton when X, regular set (X_B=X)

Geodesic skeletons

Skeletons for functions, watersheds



Comparison between a watershed using thinnings with M structuring elements and a watershed performed with the T ' family.