RECENT ADVANCES IN MATHEMATICAL MORPHOLOGY

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Abstract

This paper aims at presenting some recent advances in mathematical morphology both from the theoretical and the practical point of view. Some new and powerful tools or methodologies will be briefly presented especially for image segmentation. Then, a few algorithms which considerably speed up some image transformations are introduced. Finally, a quick review of new kinds of images which can be processed by mathematical morphology is also given.

Introduction

Although it is difficult to give a definite date of birth of mathematical morphology (abbreviated MM), twenty five years ago, MM started from a very small set of basic transformations applied to binary sets to become a complete methodology of image processing used in various areas. This methodology is based on a wide range of tools, built from the basic ones. Some of these tools may be rather complex but, in most cases, their use remains rather easy because it is not the way they work that matters but how they affect images.

The development of MM is due to the fact that this methodology has always grown up in three directions: first, the theoretical aspect of the MM, second, the practical aspect including software and hardware and third, the application of MM in more and more domains.

Obviously, these developments are not all recent and there is no synchronism between the theoretical advances and the practical ones. For instance, the watershed transform was defined a long time ago but its use has become fruitful only recently, mainly because new and powerful algorithms have been designed. Conversely, many morphological filters were used before a suitable theoretical framework of morphological filtering was established.

In this paper, some of the most recent advances in MM will be presented. Although a complete review is almost impossible, we will try to give the reader a flavour of them and to show the close connections between MM theory, its application to real problems and the available means for solving quickly and efficiently these problems.

I) Theoretical advances

Three aspects of theoretical advances will be discussed: geodesic transformations, morphological filters and the use of MM in image segmentation. As a matter of fact, these domains are not the only ones where some theoretical developments have occured. For instance, we will not present in this paper the new developments of MM in the field of stochastic simulations and we will remain in the deterministic domain. Moreover, a detailed presentation of these notions is out of the scope of this paper. The reader will find further references in the bibliography.

I-1) Geodesic transformations

The basic morphological transformations, erosion and dilation, can be used with structuring elements defined on a non euclidean space. This is the principle of geodesic transformations [10].

To perform geodesic operations, we only need the definition of a geodesic distance. The simplest geodesic distance is the one which is built from a set X. The distance of two points x and y belonging to X is the length of the shortest path (if any) included in X and joining x and y (Figure 1).

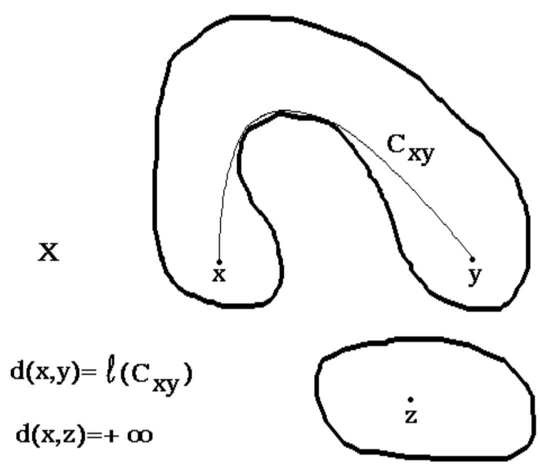


Figure 1. Shortest path and geodesic distance

Two basic transformations can then be defined: the geodesic dilation of a set Y included in X by a geodesic ball of size λ and the geodesic erosion. The geodesic dilation is made of all the points of X which are at a geodesic distance from Y smaller than λ. The geodesic erosion is the dual transformation (Figure 2).

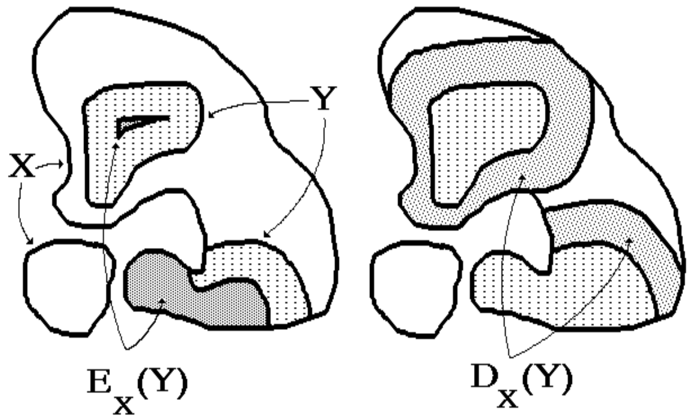


Figure 2. Geodesic erosion and dilation of a set Y in X

A lot of transformations can be derived from the basic ones. Among them, the set reconstruction is a very powerful tool.

Let Y be any set included in X. We can compute the set of all points of X that are at a finite geodesic distance from Y:

$$R_X(Y) = \{x \in X : \exists\ y \in Y,\ d_X(x,y) \text{ finite}\}$$

$R_X(Y)$ is called the X-reconstructed set by the marker set Y. It is made of all the connected components of X that are marked by Y (Figure 3). This transformation can be achieved by iterating elementary geodesic dilations until idempotence (that is until no modification occurs).

Figure 3. Reconstruction (right) of a set from a marker (left)

In the same way, many euclidean transformations as the skeleton by zones of influence may be redefined with the geodesic distance. Suppose now that Y is composed of n connected components Y_i. The geodesic zone of influence $z_X(Y_i)$ of Y_i is the set of points of X at a finite geodesic distance from Y_i and closer to Y_i than to any other Y_j:

$$z_X(Y_i) = \{x \in X : d_X(x,Y_i) \text{ finite and } \forall j \neq i,\ d_X(x,Y_i) < d_X(x,Y_j)\}$$

The boundaries between the various zones of influence give the geodesic skeleton by zones of influence $SKIZ_X$ of Y in X (Figure 4).

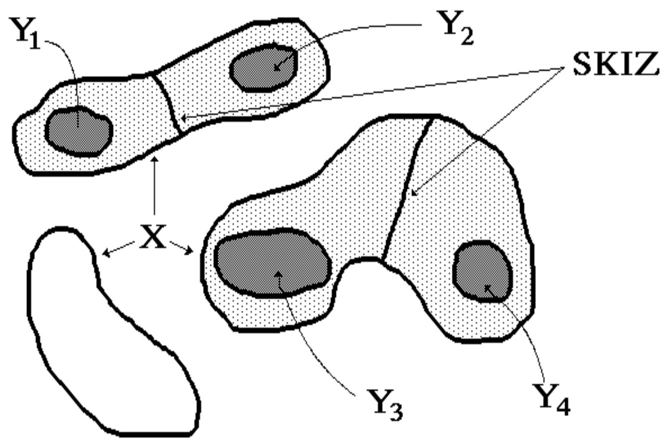


Figure 4. Geodesic skeleron by zones of influence

The extension of geodesic transformations to greytone images is more or less simple. This extension, on the one hand, leads to a very efficient transformation, the greytone reconstruction, and on the other hand, to the notion of watershed.

Let f and g be two greytone images, with $g \leq f$. The reconstruction of f by g is given by successive dilations of g "under" f. It is proved that this reconstruction can be performed by the following iteration:

$$g' = (g \oplus H) \wedge f = \mathrm{Inf}\,(g \oplus H, f)$$
$$g' \rightarrow g$$

until idempotence (Figure 5).

The reconstruction is widely used in MM. It provides a lot of feature extraction methods such as selection of extrema, construction of controlled watersheds or design of filters as described below.

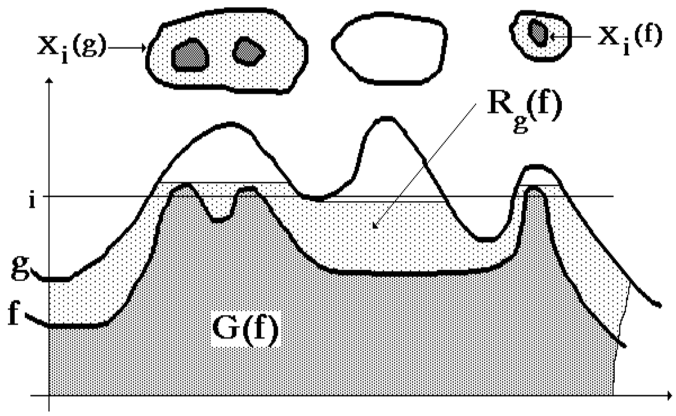


Figure 5. Reconstruction of a function by a marker function

The geodesic distance can also be extended to a general case. This extension simply consists in weighting the vertices joining two adjacent points of a digital binary image. The distance between these points is then equal to the value of the vertex. This generalization is straightforward and produces generalized dilations and generalized SKIZs (Figure 6).

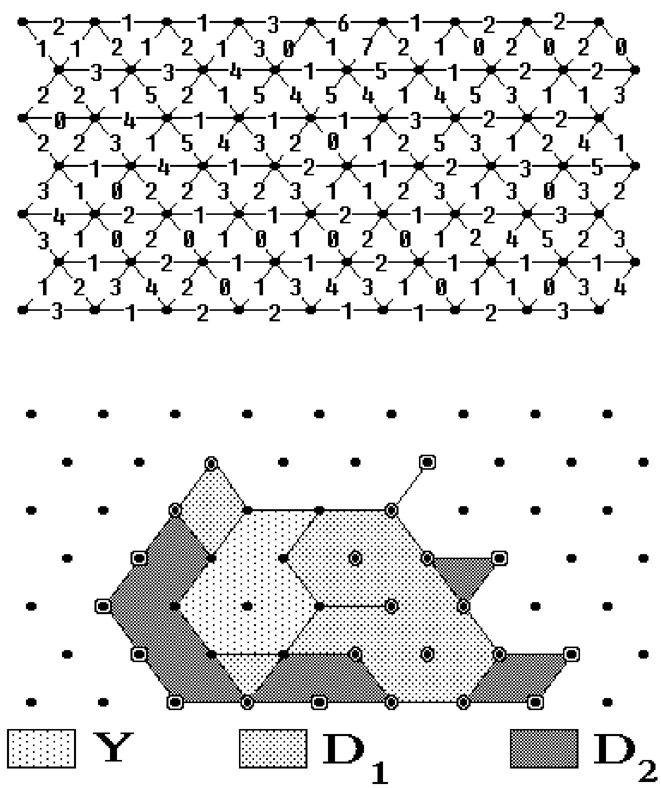


Figure 6. Example of generalized geodesic distance (a) and successives geodesic dilations of a set Y (b)

I-2) Morphological filters

The morphological approach to filtering is completely different from the linear approach which works in the frequency domain. In fact, many morphological transformations are filters. A transform Φ (applied to a set X or a greytone image f) is a morphological filter if and only if Φ is increasing and idempotent. Φ increasing simply means that if a set Y is included in a set X, the resulting filtered set $\Phi(Y)$ will be included in $\Phi(X)$: a morphological filter preserves the original order. The idempotence means that applying twice a filter will have no effect: $\Phi(\Phi(X)) = \Phi(X)$ [11].

As an example, the simplest morphological filters are the opening (erosion followed by a dilation) and its dual transformation, the closing. Their filtering properties are well known through the notion of size distribution. The recent developments of the morphological filtering theory using the mathematical concept of lattice has lead to the establishment of rules for building new filters starting from simpler ones and even from general morphological transforms which are not necessarily filters [18,19].

Among the various filters which can be built by applying these rules, the alternate sequential filters (ASF) are the most useful. They are defined as a sequence of alternate openings and closings of increasing sizes. Let γ_i be an opening of size i and φ_j a closing of size j. An ASF can be built by iterating the following sequences:

$$\gamma_i\varphi_i,\ \varphi_i\gamma_i,\ \gamma_i\varphi_i\gamma_j,\ \varphi_i\gamma_i\varphi_j \text{ with } i<j$$

Other interesting filters can be designed with the reconstruction: they are the erosion-reconstruction opening and the dilation-reconstruction closing. The erosion-reconstruction is a transform γ_λ made of a classical erosion of size λ followed by a geodesic reconstruction of the original set by the eroded one. The dual transformation φ_λ is made of a size λ dilation followed by a dual reconstruction.

These filters have two major advantages. First, these filters separate the influence of the size from the shape of the particles in the sieving process: after a classical opening, the connected components of a set X smaller than the structuring element are suppressed, but the shape of the remaining ones has been smoothed. It is not the case with the erosion-reconstruction. The objects which are not eliminated remain unchanged (Figure 6). Secondly, the erosion-reconstruction and the dilation reconstruction act independently on the particles and on the pores.

When applied to greytone images, these filters are very powerful, especially when they are combined with other MM tools like the watersheds, the extrema detection and so on [9].

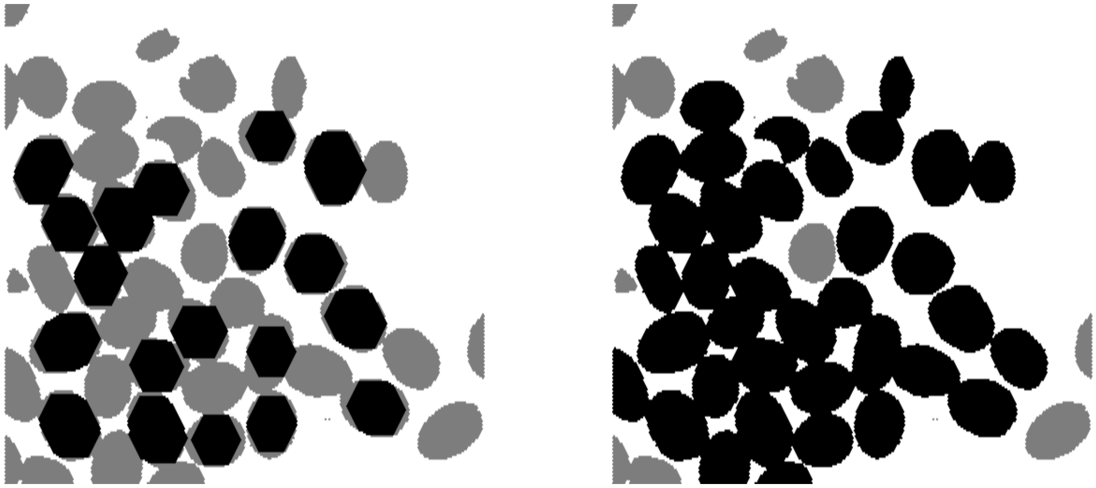


Figure 7. Comparison between the classical opening (left) and the erosion reconstruction opening (right)

I-3) Image segmentation

MM provides tools for image segmentation but, in addition, a methodology, that is the directions for using them. These tools are the watershed transform and the marker-controlled watershed transform.

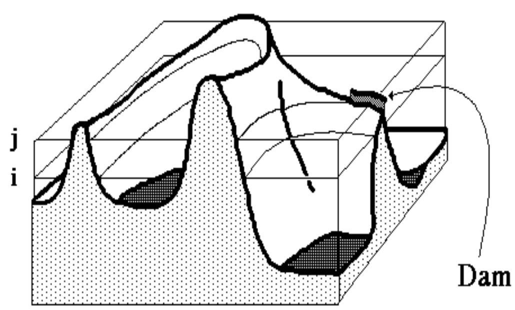


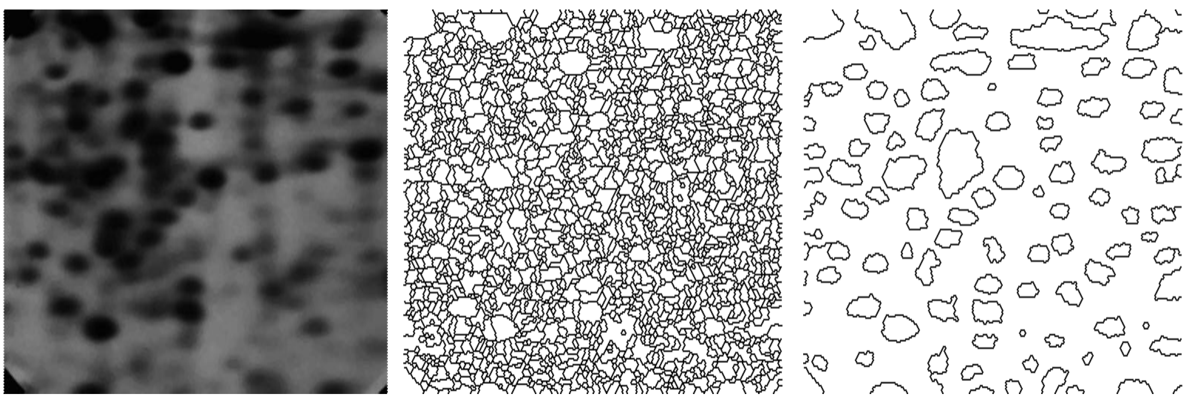
Figure 8. Flooding of the topographic surface and construction of dams

The simplest way to introduce these notions is to consider an image f as a topographic surface S and define the catchment basins of f and the watershed lines by means of a flooding process [1,23]. Imagine that we pierce each minimum of the topographic surface (a minimum can be considered as a sink of the topographic surface), and that we plunge this surface into a lake with a constant vertical speed. The water entering through the holes floods the surface S. During the flooding, two or more floods coming from different minima may merge. We want to avoid this event and we build a dam on the points of the surface S where the floods would merge. At the end of the process, only the dams emerge. These dams define the watershed of the function f. They separate the various catchment basins $CB_i(f)$, each one containing one and only one minimum (Figure 8).

Watershed transformations in picture segmentation are often applied to the morphological gradient image because (at least in theory) the contours of the objects present in an image f correspond to the watershed lines of the gradient image g(f) [14,17]. This gradient is defined as:

$$g(f) = (f \oplus B) - (f \ominus B)$$

where $f \oplus B$ and $f \ominus B$ are respectively elementary dilation and erosion of f.



(a) (b) (c)

Figure 9. Gradient watershed (b) of the original image (a)
Marker controlled watershed (c)

Unfortunately, the real watershed transform of the gradient present many catchment basins produced by small variations in the grey values. This over-segmentation can obviously be reduced by morphological filterings, but a better result is obtained if we mark the patterns to be segmented before performing the watershed transformation of the gradient. We consider again the topographic surface of the gradient image and the flooding process, but, instead of piercing the minima of this surface, we will only make holes through the components of the marker set M. The flooding will invade the surface and produce as many catchment basins as markers comprised in the markers set. Moreover, the watershed lines will occur on the crest lines of this topographic surface which correspond to the contours of the objects (Figure 9).

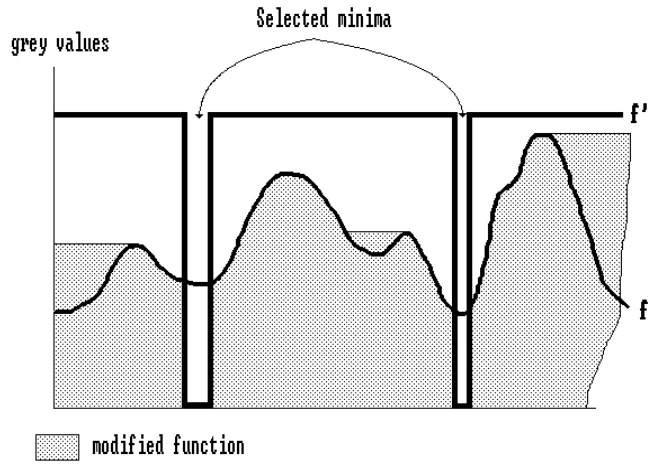


Figure 10. Principle of the homotopy modification of a function f by a set of selected minima

This procedure may be split in two steps. The first one consists in modifying the gradient function g in order to produce a new gradient g'. This new image is very similar to the original one, except that its initial minima have disappeared and have been replaced by the set M. This image modification can be performed by means of a reconstruction of the original gradient image by a marker function (Figure 10). The second step is simply

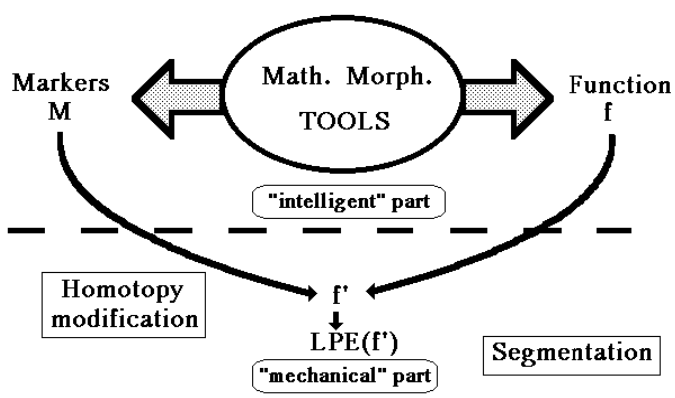


Figure 11. Paradigm of the morphological segmentation methodology

the watershed construction of g'. This approach leads to a general meyhodology of the segmentation consisting in selecting first a markers set M pointing out the objects to be extracted, then a function f quantifying a segmentation criterion (this criterion can be, for instance, the changes in grey values). This function is modified to produce a new function f' having as minima the set of markers M. The segmentation of the initial image is performed by the watershed transform of f'. The segmentation process is therefore divided in two steps: an "intelligent" part whose purpose is the determination of M and f, and a "straightforward" part consisting in the use of the basic morphological tools which are watersheds and image modification (Figure 11). A lot of segmentation problems may be solved according to this general scheme [14].

II) New algorithms and new processors

Designing new morphological tools is helpful as soon as the computation time for achieving these transformations is not too long. Two solutions exist for improving the computation speed: a hardware solution, consisting in using efficient morphological processors, and a software solution, that is finding new and fast algorithms. These two approaches may be closely linked and very often, good software algorithms are sooner or later implemented into hardware.

II-1) New algorithms

Some algorithms already exist which highly increase the effectiveness of some morphological tools. For instance, the recursive algorithms allow to build very quickly the distance function and the geodesic distance function of a set. These functions provide euclidean and geodesic erosions and dilations in a time which is not proportional to their size.

Unfortunately, it is not possible to obtain greytone erosions and dilations by this means because there exists no equivalent of the distance function for a greytone image. However, the recursive algorithms can be used both on binary and greytone images for the reconstruction transformation.

II-1-1) Recursive algorithms

The principle of a recursive algorithm is to transform any point x of an image by using its neighbourhood already transformed points. The computation speed is dramatically increased because of the propagation of the transformation in the image. As an illustration, take the algorithm for the recursive reconstruction of a function f by a marker function g. For any point x_0 of f, the new value of f is given by:

$$f(x_0) = \text{Inf}\ [g(x_0),\ \text{Sup}\ [f(x_0),f(x_1),f(x_2),f(x_3)]]$$

on the hexagonal grid and in the first step. Then, in the second step, the value of f at point x_0 becomes:

$$f(x_0) = \text{Inf}\ [g(x_0),\ \text{Sup}\ [f(x_0),f(x_4),f(x_5),f(x_6)]]$$

x_1 x_2
x_3 x_0 x_6
x_5 x_4

The process is repeated, in the direct and reverse scanning order of the image until idempotence. For most pictures, this idempotence is reached in less than five scans (Figure 12).

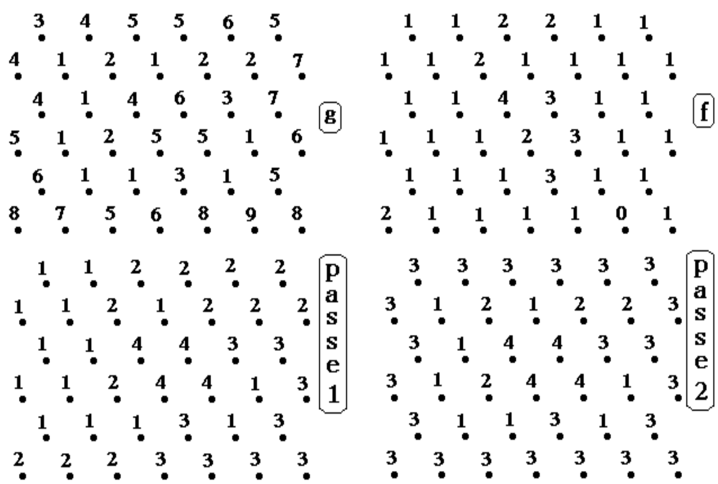


Figure 12. Recursive reconstruction of a function by another function

II-1-2) Speeding up the watershed

Many algorithms have been designed for speeding up the watershed construction. Some of them use the already existing architecture of the morphological processors and simply try to reduce the number of flooded levels by using mathematical anamorphosis. Although they produce a slight loss of information, in most cases, they are of a great help, especially when dealing with scene analysis. Other algorithms producing true watershed and true marker-controlled watershed have also been designed. Among them, one can distinguish between the procedures which simulate the flooding process and the algorithms which try to directly extract the watershed lines. In the first group, the algorithm using ordered queues are very attractive.

During the flooding of a topographic surface, there appears a dual order relation between the pixels (we consider here the flooding with sources placed at the regional minima or the function). It is clear that a point x is flooded before a point y if y is higher than x on the relief. This constitutes the first level of the hierarchy. It is simply the order relation between the grey values. A second order relation occurs on the plateaus. Let X be a plateau at an altitude h. Before X begins to be flooded all neighboring points of X, with a lower altitude than h have been flooded. One supposes that the flooding of the plateau is not instantaneous but progressive. The flood progresses inwards into the plateau with uniform speed. The first neighbors of already flooded points are flooded first. Second neighbors are flooded next, etc. . This introduces a second order relation among points with the same altitude, corresponding to the time when they are reached by the flood. If two points x and y belong to the same plateau X, of height h, x will be reached by the flow before y if the geodesic distance within the plateau X to the points of lower altitude is smaller for x than for y. An ordered queue naturally introduces this hierarchical order relation. Implementing an ordered queue between the pixels of an image leads to reconstruction and watershed algorithms which are very fast because every point is treated one time as clients in a queue. The complete description of the watershed algorithm using an ordered queue would be to long to explain in the scope of this paper. Refer to [4] for further information.

In the second group, one can find algorithms which use a special representation of the image: the arrowing representation [2].

From $f : \mathbb{Z}^2 \rightarrow \mathbb{Z}$, we may define an oriented graph whose vertices are the points of $\mathbb{Z}^2$ and with edges or arrows from x to any adjacent point y iff $f(x) < f(y)$ (Figure 13).

The definition does not allow the arrowing of the plateaus of the topographic surface. This arrowing can be performed by means of geodesic dilations. The operation is called the completion of the arrows graph. Moreover, in order to suppress problems due to the fact that a watershed line is not always of zero thickness, a more complicated procedure called over-completion is used, which leads to a double arrowing for some points. Then, starting from this complete graph (over-completed), we may select some configurations which, locally, correspond to divide lines. These configurations are represented on Figure 14 for the 6-connectivity neighborhood of a point on a hexagonal grid (up to a rotation).

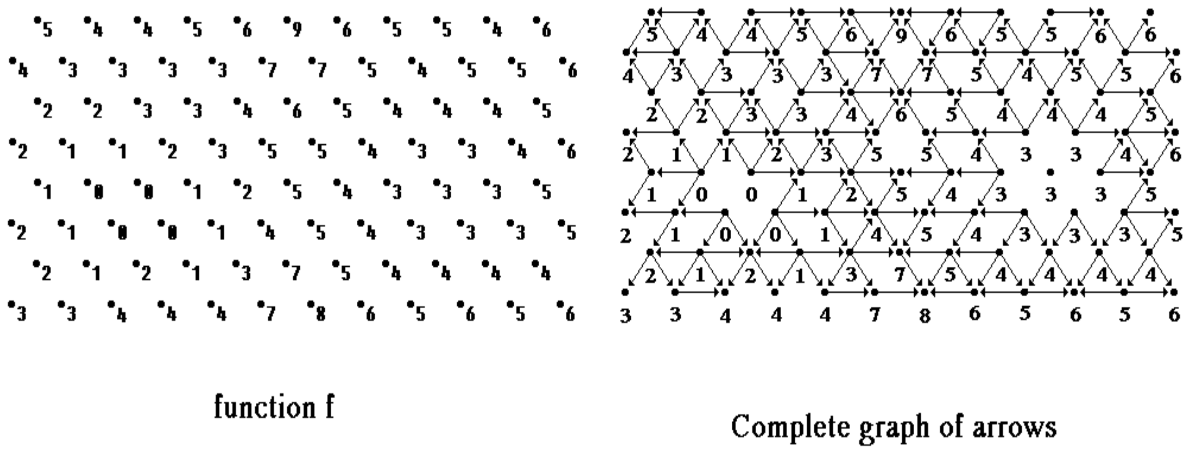


Figure 13. Function f and its complete graph of arrows

Any point receiving arrows from more than one connected component of its neighborhood may be flooded by different lakes. Consequently, this point may belong to a divide line. In a second step, the arrows starting from the selected points must be suppressed. These points, in fact, cannot be flooded, so they cannot propagate the flood. Doing so, we change the arrowing of the neighboring points and consequently the graph of arrows. Provided that the over-completion of this new graph has been made, some new divide points may then appear. The procedure is re-run until no new divide point is selected (Figure 15).

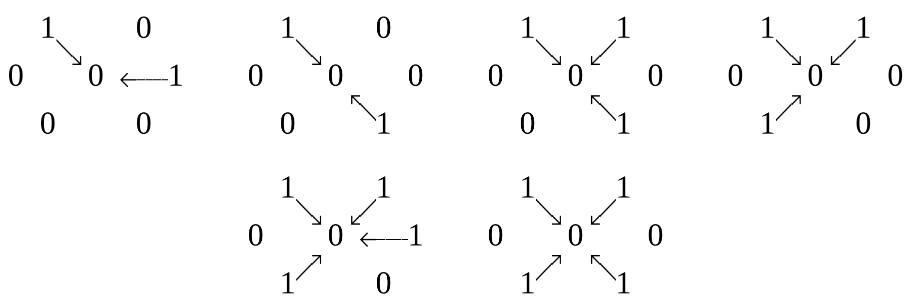
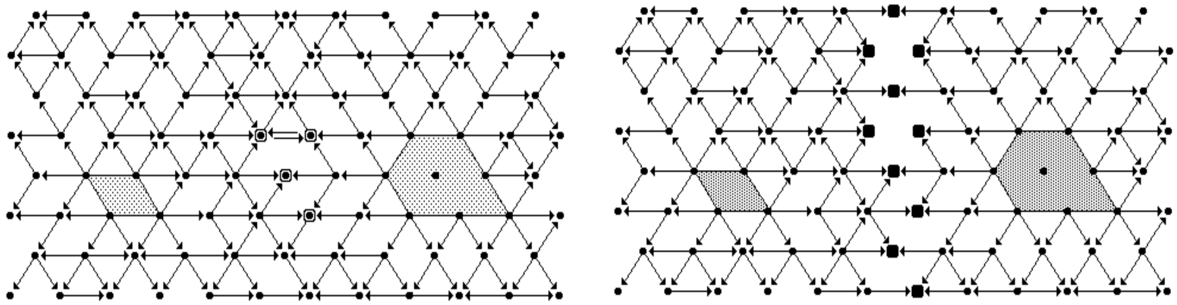


Figure 14. Configurations of arrows corresponding
to possible divide points (hexagonal grid)

This algorithm produces local watershed lines. The true divide lines can be extracted easily; they are the only ones which form closed curves.



Selection of primary points — Final result

Figure 15. Watershed by arrowing: primary divide points (left) final result (right)

II-2) Morphological processors

A classical morphological processor is made of a neighbourhood logic which allows to compute basic morphological transformations with elementary structuring elements. As a matter of fact, the higher the speed of this elementary logic, the higher the overall performances of the whole system. The latest morphological processors obviously include this elementary logic, both for binary and greytone images but also a large set of capabilities in the field of geodesic transformations. The most recent developments in the area of morphological processors have materialized into an ASIC named PIMM1, an acronym for Integrated Mathematical Morphology Processor [15]. This integrated circuit designed at the CMM is a complete morphological processor for greytone and binary images of any size. The neighbourhood logic enables treatments on a square or hexagonal grid. This chip also contains a recursive logic for the fast computation of distance functions and for fast binary reconstructions. It has some capabilities of arrowing and an arithmetic logic allows the use of anamorphosis to reduce the computation time of the watershed.

However, many algorithms are not implemented in this chip, in particular greytone reconstruction and hierarchical queues. Nevertheless, the fact that it can be pipelined allows to design architectures which meet the needs of real-time processing. Some realizations are pending at the CMM, their final goal is to segment macroscopic images in real-time (that is in a few hundred milliseconds).

III) New areas of application

III-1) Greytone morphology

For many people, MM is *par excellence* a methodology for binary 2D images. This is definitely inadequate. Nowadays, MM is mainly used with greytone images because they are typically the kind of images we find in the real world. Moreover the result obtained when dealing with greytone images are often better than those available with binary morphology because the loss of information when transforming pictures is better controlled. At the beginning of MM, two areas of application existed: the material sciences and the biomedical area. In both cases, pictures were mainly microscopic ones and the main purpose of MM was to quantify the structures. Now, new fields of application appear, especially in the macroscopic world. Many image processing problems in scene analysis (Figure 16) and industrial vision have been solved with the help of MM [16,21,23]. With the development of refined sensors in radiology, it is now possible to extract fuzzy features from the

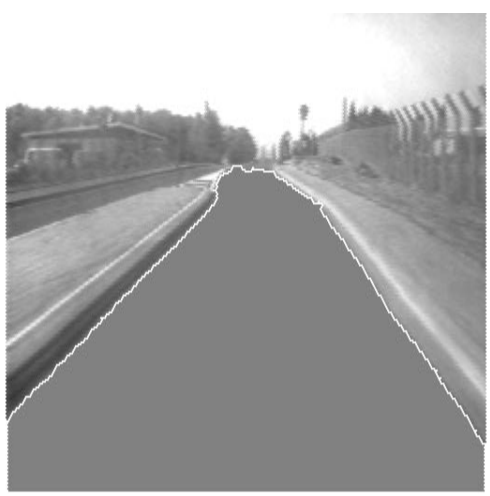


Figure 16. Examples of watershed segmentation of trafic pictures
Lanes segmentation (upper), road segmentation (lower)

images in medical radiology (micro-calcifications for instance in the early screening of cancer) or in non destructive industrial inspection (defect detection in aircraft engines).

In electron microscopy also, MM is helpful [3]. First, many electron microscopes are directly connected to an image analyzer and second, the new tools of MM, particularly in the filtering process, are very efficient (Figure 17).

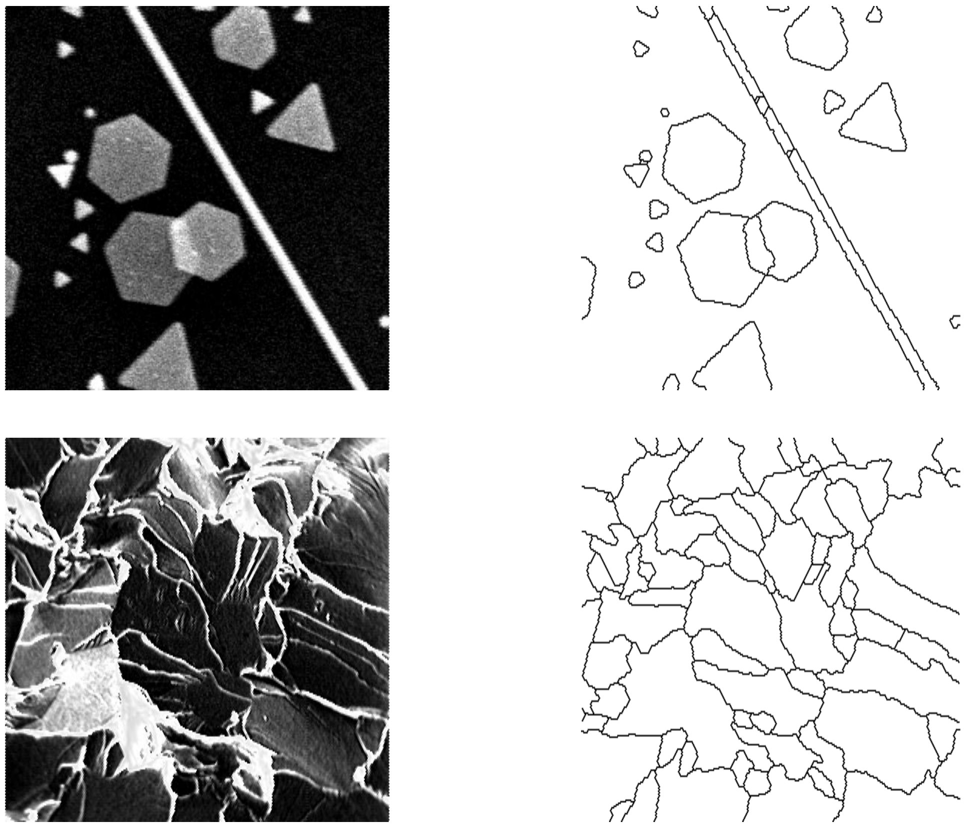


Figure 17. Segmentation of grains in TEM images, two examples

III-2) From 2D to 3D

A major advantage of MM is its straightforward extension from the 2D domain to 3D [7,8,13]. Almost, all the morphological tools which have been designed for 2D pictures can be directly used in 3D. It is the case for basic operations (erosions, dilations, openings, closings and so on) but also, and it is the great advantage of MM, for the more complex ones such picture reconstruction, watersheds, skeletons... . As a consequence, MM provides, in the 3D domain, efficient tools for image quantification and segmentation. Moreover, the extension of MM to 3D greytone images is possible. This capability is widely used to process images delivered by many new sensors in tomography, NMR, confocal microscopy, holography, etc.. (Figure 18). Another interesting application of 3D morphology is given by motion images. A sequence of images can be considered as a 3D picture and processed as such by MM. The results are very interesting because the topological relationships between the different objects in the scene are preserved both in the spatial domain and in the temporal domain.

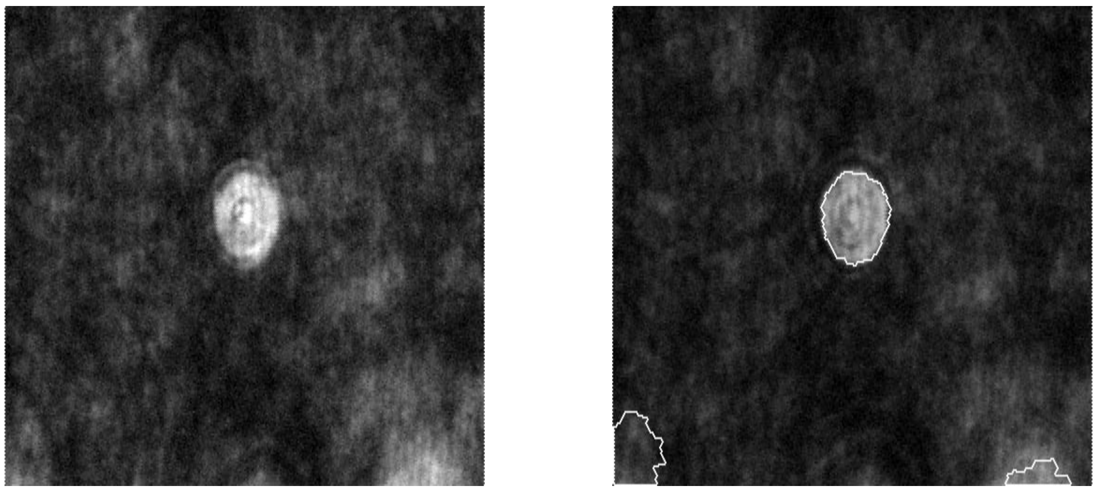


Figure 18. Segmentation of a 3D holographic picture of droplets (section)

III-3) Multi-spectral images

MM can be used with multi-spectral images. The main source for such images is remote sensing and color images. Practical problems may arise when dealing with such images but also theoretical ones. In fact, contrary to greytone images where basic morphological transformations have a physical meaning, it is not the case for color images: what could be the definition and meaning of a color image erosion? The main reason of this difficulty is that it is not possible to define arbitrarily an order relation between the pixels in a color image (there is no underlying lattice). For that reason, when working with multi-spectral images, one must first build this order relationship according to the problem under study. For instance, in a color image, if you are interested in the red objects, you will build, starting from the original image, a new greytone image where the red pixels will be the lightest ones. Then, the whole set of MM tools will be available for analyzing this new images (Figure 19) [12].

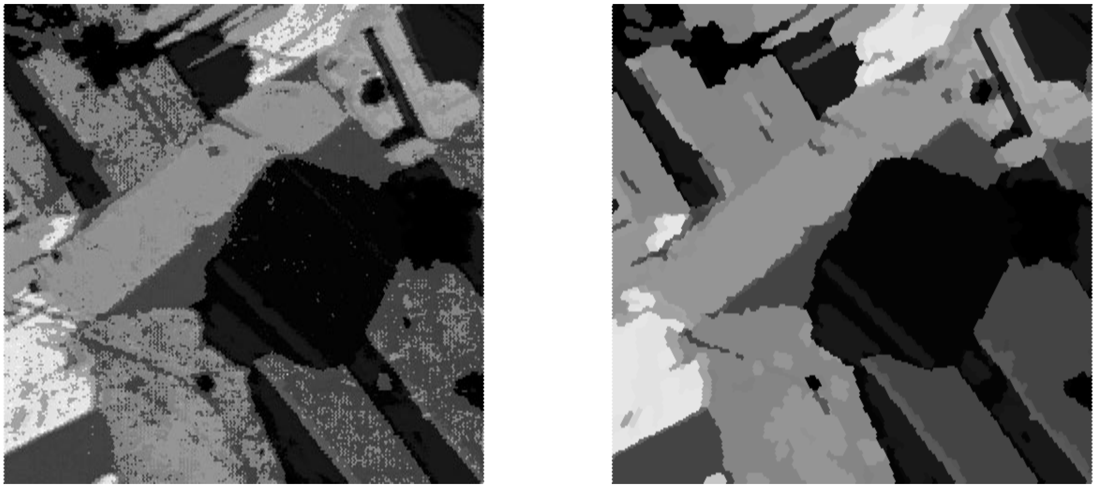


Figure 19. Example of color segmentation: the segmentation (right) allows to simplify the original image (left)

The future of MM

In twenty years, MM, which was considered as an "exotic" technique has become a complete methodology for image processing. It is no longer possible now to work with an image analyzer which is not equipped with morphological tools together with linear image processing tools. The latest developments undoubtedly show the fast emergence of real-time processors used in many new fields where they are indispensable: scene analysis, robotics control, video image compression and restoration, image communication, etc.. [5]. If the increase of computation speed allows to use more and more complex tools, the major problem which may arise for the end-user is to learn how to use these tools. Solving any image application by MM needs to concatenate thousands of elementary operations and it is not a simple task to catch in the MM toolbox the most efficient operators. For that reason, it is of primary importance to provide with the fast processors efficient programming languages. Image analysis in general, and MM in particular, are areas where the available language for translating your ideas in terms of a program must be well matched. Many efforts are made in this field, especially in the direction of "threaded" and object oriented languages. Moreover, in order to give the end-user morphologist a quick and easy know-how, techniques of artificial intelligence are presently developed to help him to select among the various available tools those which can be useful to solve his problem [6,20]. It is a fact that MM transformations are well adapted to this approach. The marker-controlled segmentation for instance is a good example of a methodology where AI can be used.

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