

The morphological-affine object deformation

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Abstract

The combination of morphological interpolation and affine transformation is presented. The proposed approach unites the advantages of both methods: the displacement is performed by using affine transformation, and the shape deformation by morphological interpolation. It allows the transformation of one binary set into another in semi-automatic or fully-automatic way.

athematical morphology, interpolation, affine transformation, objects deformation

1 Introduction

The current paper describes the combination of morphological interpolation and affine transformation and its application to the deformation of binary image objects. The morphological interpolation (section 2) allows one to *deform* the object's shape in a very elegant and robust way. The disadvantage is that the result of interpolation of distant sets is either not possible to obtain or not very realistic, depending on the method applied. On the other hand the affine transformation (section 3) is an ideal solution for *displacements* like translation and rotation. The change of shape however, could be performed to a very restricted extent: only by rescaling and shearing, which does not allow the modification of a shape into another (as morphological interpolation does).

The approach presented in the current paper unites the advantages of both methods: the displacement is performed by using an affine transformation and the shape deformation by a morphological interpolation. The method proposed (section 4) consists of three major steps. At the beginning an affine transformation is applied to each of the sets in order to place them in the central position. In the next step, the morphological interpolation is performed. Finally, in the third step, the interpolated set is moved to its final position by once again using an affine transformation. Contrary to morphological interpolation, the affine transformation requires some input parameters. The methods of calculation of

these parameters are described in section 5. Section 6 contains the results and conclusions.

2 Morphological interpolation of sets

Morphological interpolation is a recent subject. The main papers on it were presented during the last two ISMM symposiums: in 1996 by F.Meyer [4, 5], and in 1998 by J.Serra [6, 7] and S.Beucher [1, 2]. The interpolation method applied here is based on the interpolation of the intermediary sets between two sets with non-empty intersection. Two approaches to this issue have been proposed. The first one is based on the median set [1, 2, 6, 7], the second one - on the geodesic distance functions [4, 5]. The median set of two binary sets with non-empty intersection is an influence zone of the intersection of both in the union of them. It could also be expressed by using the basic morphological operators: dilation and erosion as follows [6, 7]:

$$M(P, Q) = IZ_{(P \cup Q)}(P \cap Q) = \cup\{((P \cap Q) \oplus \lambda B) \cap ((P \cup Q) \ominus \lambda B), \lambda \geq 0\} \quad (1)$$

where P, Q ($P \cap Q \neq \emptyset$) are the initial sets and $M(P, Q)$ is the median set. The interpolated set at a given level could be obtained by the successive generation of medians ([1, 2]). This approach, however, is not very fast. A faster solution based on geodesic distance functions, which produces the same results, has been proposed in [4, 5]. It allows one to obtain the interpolated set at a given level by a simple thresholding of two interpolation functions, namely $int_{P \cap Q}^P$, which interpolates between $P \cap Q$ and P ; and $int_{P \cap Q}^Q$, which interpolates between $P \cap Q$ and Q . The interpolated set at level $0 \leq k \leq 1$, is the union of two cross-sections of the interpolation function:

$$Z_k = Thr_k(int_{P \cap Q}^P) \cup Thr_{(1-k)}(int_{P \cap Q}^Q) \quad (2)$$

where Thr_k operator represents the thresholding at level k , and Z_k is the interpolated set at that level. The interpolation function $int_{P \cap Q}^P$ is obtained as a combination of two geodesic distance functions: d_1 and d_2 . The first one is the distance from $P \cap Q$ to P and is obtained by the successive dilations of $P \cap Q$ within mask P . The function d_2 represents the geodesic distance function from \overline{P} to $\overline{(P \cap Q)}$, obtained by successive geodesic dilations of \overline{P} with mask $\overline{(P \cap Q)}$. Finally, the interpolation function $int_{P \cap Q}^P$ equals $\frac{d_1}{d_1 + d_2}$. Function $int_{P \cap Q}^Q$ is obtained in a similar way.

3 Affine transformation

An affine transformation allows one to *translate*, *rotate*, *rescale* and *shear* an image. In the case described in the current paper, this operation is applied to the transformation of binary sets. We match two sets as well as possible, leaving the more precise change of shape to the morphological interpolation.

3.1 General form

The transformation can be explained in terms of the general transformation matrix [8, ?] which in the case of an affine transformation can be expressed as follows:

$$[x, y, 1] = [u, v, 1] \cdot A; A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \quad (3)$$

The affine transformation considered here includes three kinds of operations: translation, rotation and scaling¹; the transformation matrices of which are, respectively, the following:

$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}; R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}; S(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The equations above describe the *forward* mapping [8, ?]. It means that we calculate the coordinates of the final image for every pixel from the initial one. In order to obtain an appropriate matrix for the *inverse* mapping (which characterizes the calculation of the coordinates on the initial image for every pixel of the final one), the matrix A from Eq. 3 should be inverted². In general, if we consider the affine transformation consisting of n basic operations: translations, rotations, scalings or shearings :

$$A = A_1 \cdot A_2 \cdot \dots \cdot A_n \quad (5)$$

the inverse transformation matrix A' will be equal to:

$$A' = A^{-1} = A_n^{-1} \cdot \dots \cdot A_2^{-1} \cdot A_1^{-1} \quad (6)$$

The inverse matrices of the three basic transformations introduced above (defined as in Eq. 4) are respectively:

$$T^{-1}(t_x, t_y) = T(-t_x, -t_y); R^{-1}(\theta) = R(-\theta); S^{-1}(s_x, s_y) = S\left(\frac{1}{s_x}, \frac{1}{s_y}\right) \quad (7)$$

4 Combination of morphological interpolation and affine transformation

The proposed approach combines both methods described above. In the first step an affine transformation is performed on each of the two sets under study; this results in locating both (modified) sets in a central position. It is shown in Fig. 1(b) (initial sets: (a) and (c)). This first transformation consists of translation, rotation and scaling. In the second step the morphological interpolation using the interpolation function is performed. Finally the morphologically interpolated set is affine-transformed to its final position(s) (Fig. 1(d,e,f)). In

¹The fourth kind of the affine transformation: shearing is not taken into consideration.

²The inverse matrix describes also an affine transformation

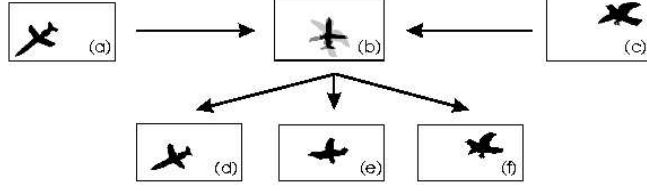


Figure 1: generation of the interpolated set: (a),(c)-initial sets, (b)-central position, (d,e,f)-interpolated sets.

fact, the transformation to the final position could be done directly from both initial sets, and the morphological interpolation could be performed there without transforming them into the central position. But the proposed solution is faster with regard to the speed of the calculations of the morphologically interpolated set. The interpolation functions are generated only once, and the final interpolated sets at different levels are obtained by the appropriate thresholding and one affine transformation to the final position for each level. The particular steps of treatment are described in the following sections.

4.1 Putting both sets in the central position

The first step of treatment is performed separately for two sets P and Q . In order to perform the affine transformation the following auxilliary data sets must be associated with them: the *middle point* of the set (for the set P : (x_{P0}, y_{P0}) , and (x_{Q0}, y_{Q0}) for Q), and the *proper angle* (α_P and α_Q). The first data is used twice: firstly for the translation to the central position, and secondly as a center of the rotation. The second data - the proper angle, describes the orientation of the set and is the angle between the x -axis and the line indicating the characteristic direction of the set. There exists also the third input data, which depend on the relation between both sets: the *scaling coefficients* s_x, s_y . They describe the proportion ratio between the sets. We will consider now all these data as known a priori. How to obtain them is described in the next section. The transformation which moves the initial sets P to the central position is the following:

$$A_P = T(-x_{P0}, -y_{P0}) \cdot R(-\alpha_P) \cdot T(x_C, y_C) \quad (8)$$

It rotates the set around the point (x_{P0}, y_{P0}) and translates it in such a way that the middle point is placed in the central point (x_C, y_C) . The appropriate transformation of the set Q contains one additional step, rescaling:

$$A_Q = T(-x_{Q0}, -y_{Q0}) \cdot R(\alpha_Q) \cdot S(s_x, s_y) \cdot T(x_C, y_C) \quad (9)$$

The only difference with the former transformation is that between the rotation and the translation, the set Q is rescaled by using the coefficients s_x, s_y ³. Both equations shown above tell us what has to be done with the initial set in order to obtain the transformed one - they define the forward mapping. In this case for each point from the initial image (containing the initial set), its new coordinates on the final one (which contains the set in the central position) are calculated. This approach, however, has one important disadvantage. Due to the numerical inaccuracies, it can happen that not all the points on the final image are filled by the values of the appropriate points from the initial one and some holes are present. In order to avoid it, the inverse mapping is applied. In this case, according to the equations 6 and 7, the transformation matrices are respectively the following:

$$\begin{aligned} A_P^{-1} &= T(-x_C, -y_C) \cdot R(\alpha_P) \cdot T(x_{P0}, y_{P0}) \\ A_Q^{-1} &= T(-x_C, -y_C) \cdot R(-\alpha_Q) \cdot S(\frac{1}{s_x}, \frac{1}{s_y}) \cdot T(x_{Q0}, y_{Q0}) \end{aligned} \quad (10)$$

By using both equations indicated above one transform both sets to the central position (see Fig. 2(b)). Now the distance functions: $int_{P' \cap Q'}$ and $int_{P' \cap Q'}$ (where P' and Q' represent the transformed sets) are produced. They allow one to later obtain the morphologically interpolated set at given level. All the steps described in this section are performed only once, even if the interpolated sets on different levels have to be produced.

4.2 Interpolated set at given level

Let $0 \leq k \leq 1$ be the level on which the interpolated set is calculated (for $k = 0$ it is equal to P and for $k = 1$, to Q). The interpolation functions calculated in the previous step are thresholded according to Eq. 2. In order to place it in the final position the affine transformation is applied once again. This time, however, new transformation parameters are calculated (all of them depend on k): final middle point $(x_F(k), y_F(k))$, rotation angle $\beta(k)$ and scaling coefficient $s'_x(k), s'_y(k)$. The appropriate matrix of the affine transformation is (in case of forward mapping):

$$A_{int}(k) = T(-x_C, -y_C) \cdot S(s'_x(k), s'_y(k)) \cdot R(-\beta(k)) \cdot T(x_F(k), y_F(k)) \quad (11)$$

The transformation parameters are calculated by using the following equations:

$$s'_x(k) = \frac{1}{(1 + k(s_x - 1))}; \quad s'_y(k) = \frac{1}{(1 + k(s_y - 1))} \quad (12)$$

$$\beta(k) = \alpha_P + k \cdot (\alpha_Q - \alpha_P) \quad (13)$$

$$x_F(k) = x_{P0} + k \cdot (x_{Q0} - x_{P0}); \quad y_F(k) = y_{P0} + k \cdot (y_{Q0} - y_{P0}) \quad (14)$$

As in the previous case, instead of the forward mapping, the reverse one is applied, the matrix of which is the following:

$$A_{int}^{-1}(k) = T(-x_F(k), -y_F(k)) \cdot R(-\beta(k)) \cdot S(\frac{1}{s'_x(k)}, \frac{1}{s'_y(k)}) \cdot T(x_C, y_C) \quad (15)$$

³In order to match both sets, only one of them - in our case Q - has to be rescaled.

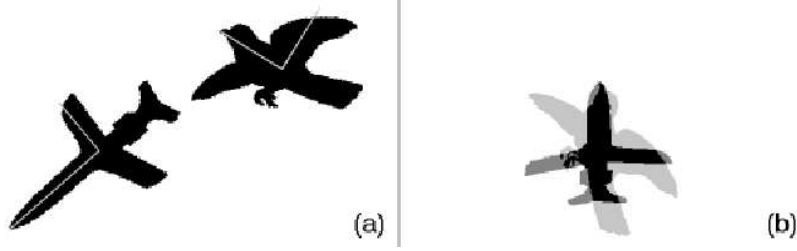


Figure 2: on the left: two initial sets (with control points), on the right: transformed set in the central position

5 Calculation of the parameters of the affine transformation

For calculating the parameters of the affine transformation four methods have been developed - one semiautomatic and three fully automatic ones.

5.1 Semiautomatic by the triplet of points

In the current method a triplet of control points is associated with each of the two sets. The affine transformation can transform any tripple of non collinear points into any other in the image space by using the combination of all four base transformations. In the current paper we neglect shearing, which reduces the triplets which can be transformed. Let $\{p_1 = (x_{P1}, y_{P1}), p_0 = (x_{P0}, y_{P0}), p_2 = (x_{P2}, y_{P2})\}$ and $\{q_1 = (x_{Q1}, y_{Q1}), q_0 = (x_{Q0}, y_{Q0}), q_2 = (x_{Q2}, y_{Q2})\}$ be the triplets of control points of respectively the sets P and Q . The necessary condition is that they must be the vertices of a right-angled triangle, which can be expressed by using the following equation (the right angle is indicated by a pixel with index 0): $x_0(x_0 - x_1 - x_2) + y_0(y_0 - y_1 - y_2) = y_1y_2 - x_1x_2$. Example sets and the triplets are shown on Fig. 2. The appropriate points are the ends of the gray straight lines.

Both sets are transformed using the affine transformation in such a way that their appropriate triplets of points superimpose (see Fig. 2(b)). The middle points are already known and they are equal to p_0 for the set P and q_0 for the set Q . The proper angle α_P ⁴ is defined as an angle between the line passing through the points p_0 and p_1 and the x -axis of the base coordinate system:

$$\alpha = \text{sgn}(y_{P1} - y_{P0}) \cdot \arccos \left(\frac{x_{P1} - x_{P0}}{\sqrt{(x_{P1} - x_{P0})^2 + (y_{P1} - y_{P0})^2}} \right) :: -\frac{\pi}{2} < \alpha \leq \frac{\pi}{2} \quad (16)$$

Scaling coefficients are calculated by consdering the lengths of segments p_1p_0 ,

⁴When the equation considers only one set, it means that the appropriate equation for the second one is to be obtained in exactly the same way.

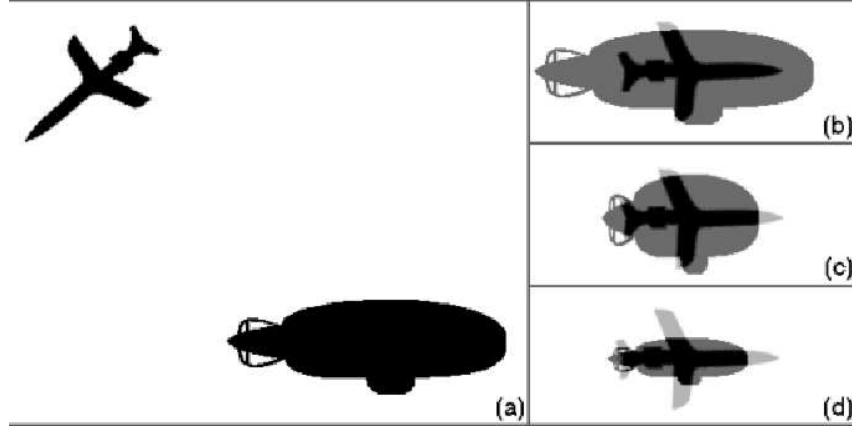


Figure 3: The initial sets (a) and the automatic matching obtained by using different criteria: (b)-without rescaling, (c)-by the smallest recangle matching, (d)-with the aeral criterion.

p_2p_0 and their relationship to the lengths of segments q_1q_0 , q_2q_0 :

$$s_x = \sqrt{\frac{(x_{P1} - x_{P0})^2 + (y_{P1} - y_{P0})^2}{(x_{Q1} - x_{Q0})^2 + (y_{Q1} - y_{Q0})^2}} :: s_y = \sqrt{\frac{(x_{P2} - x_{P0})^2 + (y_{P2} - y_{P0})^2}{(x_{Q2} - x_{Q0})^2 + (y_{Q2} - y_{Q0})^2}} \quad (17)$$

5.2 Automatic without rescaling

In the automaic method, at first the middle point is calculated as a *center of gravity* of the set. In order to obtain the proper angle α , one measures the intercepts $h_{\lambda,x}$ of the set P at every point $x \in P$ and for every angle λ . Let l be the maximum lengths of these intercepts:

$$l = \sup \{h_{\lambda,x}(P), x \in P, \lambda \in [0, \pi]\} \quad (18)$$

then λ is the direction associated with l - the proper angle of P^5 .

The scaling coefficients are not calculated here($s_x = s_y = 1$). The change of size is performed only by means of the morphological interpolation.

5.3 Automatic by smallest rectangle matching

The middle point and the proper angle is calculated as in the previous method. In order to obtain the scaling coefficients the size of the smallest rectangle

⁵ Of course a set can have more proper angles λ - not only one.

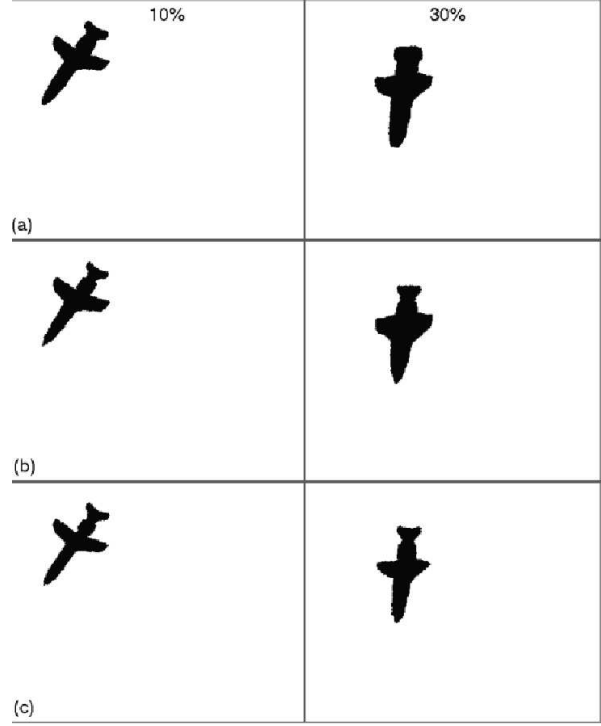


Figure 4: The sequence generated by using the automatic calculation of the parameters (a)-without rescaling, (b)-by the smallest rectangle matching, (c)-with areal criterion.

containing the set is considered. The coefficients are then calculated by using the extreme values of the coordinates of the points belonging to the set:

$$s_x = \frac{x_{Pmax} - x_{Pmin}}{x_{Qmax} - x_{Qmin}} :: s_y = \frac{y_{Pmax} - y_{Pmin}}{y_{Qmax} - y_{Qmin}} \quad (19)$$

where indices *max* and *min* stand for the extremal coordinate values along the appropriate axes. These values can be computed either before or after performing the rotation - in the result shown in the next section they are calculated after the rotation.

5.4 Automatic by using the areal criterion

In this approach, the middle point and the proper angle are calculated in the same way as in the former ones. The difference lies in the calculation of the scaling coefficients. They are calculated according to the relation between the

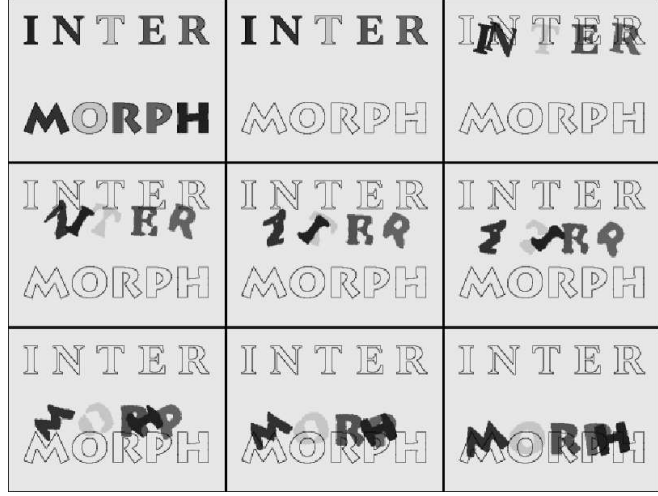


Figure 5: An animation obtained automatically by using the affine-morphological deformation.

areas of both sets - the areal criterion:

$$s_x = s_y = \sqrt{\frac{area(P)}{area(Q)}} \quad (20)$$

where $area(P)$ and $area(Q)$ represent the area of respectively P and Q .

6 Results and Conclusions

The results of the automatic methods are presented in Fig. 4. The automatically computed parameters are the following: $x_{P0} = 133$, $y_{P0} = 184$, $x_{Q0} = 353$, $y_{Q0} = 273$, $\alpha_P = -140^\circ$ and $\alpha_Q = 0^\circ$. The result of the first method (a) shows that the absence of the rescaling leaves the entire change of size to the morphological interpolation, the results of which is not as precise as in the combination with the rescaling. Next two methods contain the rescaling, the automatically computed parameters of which are following for (b): $s_x = 0.56$, $s_y = 1.05$; for (c): $s_x = s_y = 0.5$. If we compare both methods, the second one (c), with the areal criterion produces finer interpolated sets (because the transformed sets in the central position are smaller). The initial sets and the matched sets in the central position are shown in Fig. 3.

The proposed method deals with objects. The objects, however, are always a part of a particular image. They are represented as connected components. If one considers images, one cannot avoid the question of noise-sensitivity. In case of noisy image one have to filter the noise before the interpolation starts. Input images for the interpolation must be noise-free and contain only the objects.

The method of set deformation proposed in the current paper combines the advantages of the affine transformation and the morphological interpolation. It allows one to transform one binary set into another one in a semi-automatic or fully-automatic way. The shape of the interpolated sets looks natural and the transition is performed smoothly. It can be applied in different areas of image processing. One of the possible application areas is the animation. It could be applied to animate the titles, graphics, or other objects on the image. The example of the animation obtained by using the proposed method is presented on Fig. 5. The first word 'INTER' is transformed into another one: 'MORPH'. Each frame of the animation has been obtained as a superposition of the interpolations of single letters. Each letter was transformed by using the automatic method consisting of translation and rotation (rescaling has not been performed). In order to improve the smoothness of the final animated frame, an additional morphological filtering (closing of size 1) has been applied.

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