ISMM 2000 ,
Xeros Center
Palo-Alto, June
2000

The Birth of Mathematical Morphology

Georges Matheron and Jean Serra

Context

- Before considering how Mathematical Morphology originated in 1964, we will describe briefly the backgrounds of its two founders at the beginning of that year,
- even though they were hardly aware of the theoretical turn they would take a few months later.

First Protagonist

- Georges Matheron was a mining engineer from the Corps des Mines, 34 years old, working with the B.R.G.M. (*i.e. geological survey)* , in Paris.
- At the end of 1963, he had just achieved the doctrinal body of his theory concerning the estimation of deposits, called Geostatistics, and started in Algeria ten years before.

First Protagonist (II)

- It was time for a break and for investigating new horizons beyond the mining world.

- His publication of reports paused at that time, decreasing from fifty in ten years to none in ten months. He wouldn't return to geostatistics until January 1969, five years later, via universal kriging.

Second Protagonist

- Jean Serra was a civil engineer from the Ecole des Mines de Nancy. He was 24 years old and was completing a bachelor's degree in philosophy focus on Gestalt psychology.

- One year before, he had been hired by IRSID as a PhD student for a detailed inventory of Lorraine iron deposit by geostatistics .

Lorraine Iron Orebody

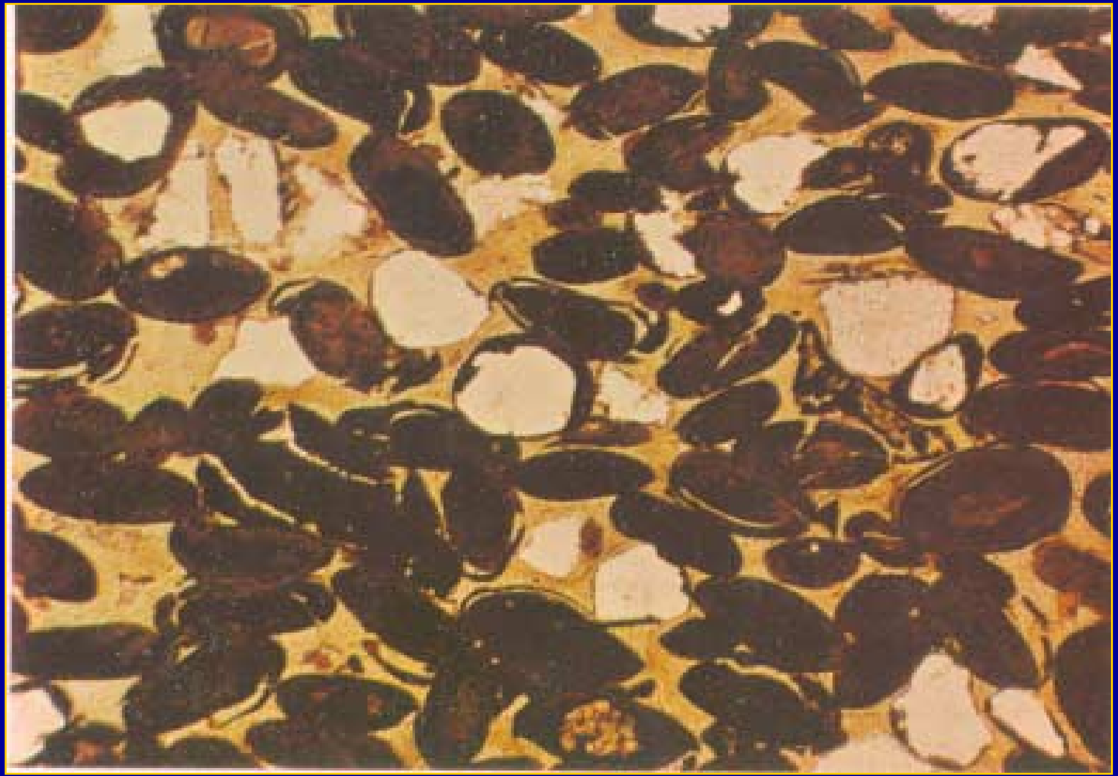
- Three different scales were to be considered. First of all, best profitable areas had to be estimated within the 600 Km of the deposit.

- But the panels to be kriged at the 100m scale were affected by a strong “nugget effect” acting at the metric scale, due to local migrations of calcite.

2nd scale : Calcite migration

Calcite migration provokes a bias in the drillings grades which had to be quantified.

3rd scale : Petrography (100 μm)



- *in brown, olites of limonite (Φ : 100μm to 300μm) ;*
- *in green, chlorite cement with (poor) iron ;*
- *in white, quartz and calcite.*

Petrography (II)

- For the enrichment of the ore, which had become essential, the grade of iron was less important than its petrographic distribution between oolites and chlorite cement .
- Therefore quantifying meant estimating the proportions of the petrographic phases and measuring their sizes.

Nested Phenomena

- In parallel with these economic objectives, J. S. also hoped to link the three scales through a point variogram, which would serve as a vital lead for this huge structural zoom and would cover the space from the micron to the hundred kilometer scale.

Chronology : Spring 1964

- Quantitative study of the above thin section by J. S. by means of a slide projector. The thin section was projected screen equipped with a square grid.
- The phases found in each of the 6.000 grid vertices were read manually. Direct and cross variograms were calculated in the 4 basic directions on a 1410 IBM computer.

Chronology : Oct. 1964

- G.M. report, entitled *''Fonctions aléatoires du type tout ou rien''*, in which he attempts to model petrography in random terms, through the classical approach of the spatial law, based on a finite number of points.

- First sketch of the Boolean model, calculation of its covariance.

Chronology : Dec. 1964

- J.S. report, entitled *« l'analyse pétrographique quantitative »*.
- This report, which detailed the results of petrographic surveys, introduced the two concepts of

a structuring element, called ''spot'',

and of the hit or miss transformation.

Chronology : Dec. 1964 (II)

- J.S. could not link variograms to the desired size properties. Hence he proposed to start from shapes which would be given a priori - the structuring elements - and to make them interact with the objects under study.
- According to their geometry (circle, straight line, pair of points), they would reveal the various structural features of the objects .

Chronology : Dec. 1964 (III)

- G.M. report, entitled *''Etude théorique des granulométries''*, starts from this first study to introduce number of new ideas ;
- J. S. begings the construction of a devoted device, the "texture Analyser", with J.C. Klein.

These two texts and the device form the very start of Mathematical Morphology.

Chronology : Dec. 1964 (IV)

- Limiting Hit-or-Miss mapping to one-phase structuring elements, G.M. described its algebraic properties and finds the analytical representation for erosion and dilation ;
- He re-formulated the Boolean model in a convenient framework, which forecast the theory of random sets, and uses the erosion formalism to find the functional Q(B).

Chronology : Dec. 1964 (V)

In addition, in the same report, G. M.

- introduced the concepts of the adjunct opening and closing,
- found their main algebraic properties,
- and interpreted openings according to discs of increasing radius as size descriptors : the granulometries .

Chronology : July 1965

- Registration of the first patent of the texture analyser, inventor : J. S., company : IRSID.

- This ended the first year of Mathematical Morphology :

  - G.M. report was the basis for “éléments pour une théorie des milieux poreux” (1967);

  - J.S. report was published in the "bullein du BRGM” in 1966 .

Texture Analyser Patent (I)

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RÉPUBLIQUE FRANÇAISE

MINISTÈRE DE L'INDUSTRIE

SERVICE
de la PROPRIÉTÉ INDUSTRIELLE

BREVET D'INVENTION

P.V. n° 23.273 — N° 1.449.059

Classification internationale : G 01 n

Dispositif d'analyse de textures.

INSTITUT DE RECHERCHES DE LA SIDÉRURGIE FRANÇAISE (I.R.S.I.D.), résidant en France (Seine-et-Oise).

Demandé le 2 juillet 1965, à 14h 53m, à Paris.
Délivré par arrêté du 4 juillet 1966.
(*Bulletin officiel de la Propriété industrielle*, n° 33 de 1966.)

(*Brevet d'invention dont la délivrance a été ajournée en exécution de l'article 11, § 7, de la loi du 5 juillet 1844 modifiée par la loi du 7 avril 1902.*)

La présente invention, due à M. Jean Serra, dans le cadre des travaux qui lui ont été confiés par le demandeur et qui font suite aux travaux de M. G. Matheron, concerne un dispositif pour l'analyse statistique automatique de la répartition géométrique de qualités distinctes réparties dans pour totaliser séparément le nombre de concordances ou de discordances des valeurs mises en mémoire, pour des mesures séparées par diverses distances dans ledit milieu.

Dans une variante d'exécution, l'invention est caractérisée en ce qu'elle comprend en combinai-

Texture Analyser Patent (II)

United States Patent Office | **3,449,586** | **Patented June 10, 1969**

1

3,449,586
AUTOMATIC SCANNING DEVICE FOR ANALYZING TEXTURES
Jean Serra, Stations d'Essais-Irsid, Maizieres-les-Metz, Moselle, France
Filed June 30, 1966, Ser. No. 561,932
Claims priority, application France, July 2, 1965, 23,273
Int. Cl. G01n *21/30, 21/00;* **G01b** *11/28*
U.S. Cl. 250—219 **10 Claims**

ABSTRACT OF THE DISCLOSURE

An automatic scanning device analyzes the texture of a heterogeneous medium with means which detects a quality in one zone of the medium and converts the detected quality into an electric signal, means for displacing the zone within the medium and for taking uniformly spaced measurements throughout the medium, a memory system wherein each signal is stored, logical selection means for comparing the stored signals, and counting means for integrating separately the number of concordances and discordances of the stored signals.

This invention relates to a device for the automatic statistical analysis of the geometrical distribution of distinct qualities which are distributed in a heterogeneous

2

sively the oldest value contained in the said storage means; logical selection means for comparing the stored values *k* by *k* after each measurement; and counting means for integrating throughout the course of the analysis the total number of concordances or discordances of the values contained in the storage means and corresponding to an arrangement of *k* zones which are located with respect to each other at constant multiple distances of the order 1 to *n* of the analysis pitch.

In a preferred embodiment which is applicable to the analysis of texture of the images of a heterogeneous medium, the invention is more especially characterized by the combination of the following elements: at least one photoelectric receiver having a spectral sensitivity which is adapted to the color which is sought on the image and fitted with a suitable optical device which delivers an electric signal representing one zone of the image; scanning means for displacing the measurement zone over the surface of the image to be analyzed along successive lines; at least one storage shift register corresponding to each photoelectric receiver, each register comprising *n* binary storage elements; at least one digital counter associated with a logic circuit which determines the concordance or discordance between the signals contained in *k* storage elements to which it is connected; a selection matrix for connecting the said logic circuit to the selected storage elements; and a control circuit which is synchronized with the image-scanning means so as to produce at regular intervals the displacement by one element of the data con-

Chronology : 1965

- Surprisingly, the first year, which was so rich in fruitful exchanges, was followed by two years of almost complete silence on the subject. G. M. turned to hydrodynamics (eight studies written during that period)

The main reason for this situation was the development of the texture analyser that took the entire year of 1965

Chronology : 1966

- The whole year of 1966 was devoted to the inventory of "good" structuring elements and to establishing logical relations between them.
- the machine allowed the construction of every possible structuring element based on oone or two straight lines, whcih was a small technical revolution.

Chronology : 1967 (I)

- Looking back on the year 1967, it appears that G.M. and J.S. were both busy finishing previous work, and preparing the creation of C.M.M., even if they did not suspect it yet.

- The national service in a military research center allowing some free time, J. S. ended his mining studies, with a PhD and two publications respectively.

Chronology : 1967 (II)

- Still in 1967, G. M., after he had chosen to take some distance from Geostatistics, left B.R.G.M. and wrote the first version of “Random sets and Integral Geometry”.

- Also, he gathered his morphological papers and some texts on hydrodynamics in the book *’Eléments pour une théorie des milieux poreux’*

Chronology : End of the Begining

- On April 20, 1968, The Ecole des Mines created the "“**C**entre de **M**orphologie **M**athématique", and appointed G.M. director.
- Despite the imposing size of the building, the center was reduced to two people: G.M. and J. S. , The later had just been hired as "Maître de Recherches" by the Ecole des Mines, and appointed to C.M.M.

From 1968

- From 1968, some previous results were to be developed (***Boolean model, hit or miss transformation***), others less (***star, radii of curvature***), and the original machine evolved rapidly to other proptotypes and commercial devices (***Leitz, Allen Bradeley***) ;

…But a specific approach was drawn.

Comments

- Chronology sharpens the events, makes more precise their apparitions, but says nothing about their reasons.

- And also , some questions arise , which are still open...

Questions

- What happened during the three years we have just skipped through, and above all during the first one ?

- What kind of impulse did these two men give by working together with practically no intellectual connection with the people around them ?

- Why did this methodology gradually gain in influence instead of another ? Is it due to a better ability to render the concepts of the time, and integrate them into a unified approach ?

- Or is it just the result of the evolution in computer science that enabled the synchronized development of Math. Morphology as a sort of by-product ?

First Outside : the Stereologists

- G. M. had rediscovered the set addition defined by H. Minkowski (1903) and studied in detail by H. Hadwiger in 1957.
- This oriented G.M. and J.S. towards Integral Geometry and the int. Society for stereology

The Stereologists (II)

- When the new ideas were presented before the Stereologists, in 1971 , they were received with rather mixed feelings.

- The postulate that one can study a set only by associating it with numbers was so deeply rooted in their minds, that they were just unable to imagine other ways of thinking.

The Stereologists (III)

- The physical intuition of morphology that perceiving a set, or an image, means to transform it, either qualitatively or quantitatively, was too far removed from theirs.

- And yet the new approach used to open number of questions : dilation was to be a more robust operation than erosion, although no experiment could verify it ;
- Also, one could wonder whether such a theory would apply to gray level images, or uniquely to binary ones. Etc..

But the stereologists did not pay attention to theses aspects…

Are ideas just the fruits of technological evolution ?

From this point of view, the evolution of Mathematical Morphology over the past thirty years is instructive :

- Two-dimensional operators were discovered four years before television technology enabled their implementation (1969);

-the memory of the size of an image, with its very peculiar loop functioning, occurred in 1973, simultaneously with a growing interest for binary iterative algorithms (ultimate erosion, skeleton, thinning).

-Five years later, the numerical memory opened the door to gray tone image processing , precisely at the time when morphological filtering and the watershed algorithm were invented.

-More recently, the morphological theory for connection, that governs automatic segmentation of images and sequences, was developed during the nineties, just when the multimedia applications arose.

What a lot of coincidences !

How an idea was born ? (I)

- Even if we are not always aware of it, we all "surf" on successive technological waves. However, this this subconscious pressure applies uniformly over all the 1500 image processing laboratories in the world.

- Why then strong ideas do crystallize in a few places only, and at some privileged periods ?

How an idea was born ? (II)

- Take the case of the notion of a granulometry

- *The first question was* : how to know the size of oolites and of chlorite cement in Lorraine iron ores (at least one mean volume parameter, and more if possible) ?

How an idea was born ? (III)

- *First answer* : The results of the first point variograms led J.S. to the following

tel type de grains particulier, ou les pores. Considérons un spot circulaire de diamètre x qui balaye la lame. Le spot étant centré au point courant x, nous dirons :

$f_\lambda(x) = 0$ si aucun point du cercle n'appartient à G

$f_\lambda(x) = 1$ si au moins un point du cercle appartient à G,

de même :

$f_{-\lambda}(x) = 0$ si au moins un point du cercle appartient à P

$f_{-\lambda}(x) = 1$ si tous les points du cercle appartiennent à G

Ce contour de base circulaire n'a rien d'obligatoire. Nous pensons en particulier que la même transformation définie pour un segment de droite de direction donnée et de largeur λ pourrait être féconde.

Dans le cas particulier où il n'y a qu'une seule espèce de grains G et des pores P, les valeurs moyennes des transformées f_λ quand λ varie représentent vraissemblablement une certaine vision de la granulométrie, vision dans laquelle les notions de grain, ou de milieu intergrains relèveraient d'un même concept (transformées $\pm\lambda$). De même, pour certaines formes de contour de base (segment de

How an idea was born ? (IV)

- J.S. was wrong when considering dilations or distance functions as granulometric tools, but he did feel the need for convex structuring elements.
- He formulated the question differently in terms of set Æ set transformations (and not of sets Æ numbers) which was new, and introduced the paradigm for this type of operations.

How an idea was born ? (V)

- *Second step* : G.M. revived the idea of transforming sets and showed that dilations using disks should be combined with their dual operations to obtain a size result

II.- OUVERTURE ET FERMETURE ASSOCIEES A UNE TRANSFORMATION DE SERRA.

Un ensemble B étant supposé choisi, à tout ensemble A, nous pouvons faire correspondre les ensembles suivants :

44) $$\begin{cases} A_{\omega} = \mathscr{S}_{\check{B}} \, \mathscr{S}'_{B} \, A = (A \ominus \check{B}) \oplus B \\ A_f = \mathscr{S}'_{\check{B}} \, \mathscr{S}_{B} \, A = (A \oplus \check{B}) \ominus B \end{cases}$$

Nous dirons que A_{ω} est l'ouverture de A selon B et A_f la fermeture de A selon B, les opérateurs d'ouverture et de fermeture étant $\mathscr{S}_{\check{B}} \mathscr{S}'_{B}$ et $\mathscr{S}'_{\check{B}} \mathscr{S}_{B}$ respectivement.

Montrons qu'il s'agit effectivement d'une ouverture et d'une fermeture algébrique.

Plaçons-nous dans le cas où l'ensemble A est mesurable, et soit $S = S(o)$ sa mesure. Désignons par :

$S(\lambda)$ la mesure de A_{f_λ} (fermeture de A selon B_λ)

$S(-\lambda)$ la mesure de A_{ω_λ} (ouverture de A selon B_λ).

$S(\lambda)$ est <u>une fonction non décroissante de λ.</u>

En effet, soit $0 < \lambda \leq \mu$. D'après (50), B_μ est ouvert selon B_λ, et, d'après la conséquence 4, on a les inclusions :

$$A_{f_\lambda} \subset A_{f_\mu}$$

$$A_{\omega_\lambda} \supset A_{\omega_\mu}$$

Par suite, on a aussi :

$$\begin{cases} S(\lambda) \leq S(\mu) \\ S(-\lambda) \geq S(-\mu) \end{cases}$$

Cette propriété montre que $S(\lambda)$ possède les caractéristiques d'une courbe granulométrique cumulée. Elle en possède aussi la signification physique, comme nous allons le voir, et c'est pourquoi nous lui donnerons le nom de <u>courbe granulométrique généralisée</u> de la régionalisation en tout ou rien A.

How an idea was born ? (VI)

- Once again a good insight emerged out of a questionable mathematical statement : the function S_λ was certainly not decreasing, but the same was true already for dilations according to disks.

How an idea was born ? (VII)

- The strength of G.M. approach actually came from the introduction of idempotent operators. Indeed, this property was proved for opening a few pages before, but its author did not venture yet to substitute it (with two other axioms) as constitutive of the physical concept of granulometry.

Dialectics

- Thus, ideas first need time to take shape, then to expand and link together.

- Platonists are not always aware of this point. Those who try to construct a mathematical structure are sometimes lured by the strong impression that ***"it does exist"*** somewhere, outside the sensible world.

- But because they interpret this impression from a retrospective, therefore static, point of view, they tend to forget that they feel that ***"it does exist"*** because ***"it does resist"*** (a feeling that is not so Platonic..)

- In this story, experimental facts are never used to verify ready-made formulas. They appear on the contrary as a support to help an idea under construction to find its structure and shape.

Internal reports (I)

Georges Matheron

- Fonctions aléatoires du type tout ou rien, October 1964
- Etude théorique des granulométries, December 1964
- Axiomatique des milieux poreux aléatoires, March 1965
- Pour une théorie des structures aléatoires, Aug.-Sept. 1967

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Jean Serra

- Contribution à l'analyse pétrographique quantitative, December 1964
- La transformation en tout ou rien. February 1965
- L'Analyse des Textures par la géométrie aléatoire, May 1965
- Morphologie mathématique et genèse des concrétions carbonatées des minerais de fer de Lorraine, April 1967

Publications (I)

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- Random sets theory and its applications in stereology. - 3rd Int. Congress Stereology, Berne, 28-31 Aug. 1971. *J. of Microscopy*,Vol. 95, Pt 1, Feb. 1972, 15-23.
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- Stereology and structuring elements. - 3rd Int. Congress Stereology, Berne, 28-31 Aug. 1971 J. of Microscopy, Vol.95(1), 1972, 93-103.