EDGE EXTRACTION BY STATISTICAL DEPENDENCE ANALYSIS: APPLICATION TO MULTI-ANGULAR WORLDVIEW-2 SERIES

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ABSTRACT

Edges are crucial descriptors for image analysis and rely mostly on local spectral distances. In this paper, local spectral changes are assimilated to the local statistical dependences which are measured by the local mutual information. This metric is shown to be invariant to unknown bijective transforms which makes it a good candidate for analyzing consistently satellite images which are affected by uncontrolled spectral distortions. Such an edge property is assessed with WorldView-2 multi-angular sequence of images, where spectral distortions arise with far nadir view angles.

***Index Terms*—** Local mutual information; dependence; transform invariance; edges.

1. INTRODUCTION

Edges are crucial descriptors in an image since they represent significant local intensity changes. Many remote sensing applications, such as image registration, image segmentation, make use of the edges as a preprocessing stage. Most of the edge detection methods take into consideration local intensity differences which highly depend on the signal dynamic.

The new generation of agile sensor produces sequences of multi-angular very high resolution images. These images were shown to suffer from atmospheric spectral distortions as the view angle moves away from nadir [1]. These spectral distortions are difficult to model and have a direct impact on the local intensity changes.

In this paper, we propose a new measure of local spectral changes based on the analysis of statistical dependences. The local mutual information [2] is used as a measure of the statistical dependence between neighbor pixels [3]. The metric is proven to be invariant to any bijective transform applied independently on each pixel [4], and this property is of main interest for handling unknown bijective transforms, such as atmospheric spectral distortions.

The stability of the local mutual information is demonstrated on a sequence of 5 multi-angular multi-spectral WorldView-2 images representing a subpart of the city Rio, Brazil. The results emphasize the higher robustness of this measure in comparison to the Euclidean distance or the spectral angle.

The paper is organized as follows. Section 2 includes the definition of the local mutual information and a demonstration of its invariance property. Experiments are conducted in Sec.3. Finally, conclusion is drawn in Sec.4.

2. METHOD DESCRIPTION

2.1. Local Mutual Information

Given an image I defined on a grid Ω, neighbor pixels are denoted by $p \sim q, p, q \in \Omega$ and their values are denoted by $I(p)$ and $I(q)$. Let us assume that all neighbor values $(I(p), I(q)) \mid p \sim q$ are distributed with an identical distribution $p_{X,Y}$ of two random variables X, Y taking their values in the image value space. Let denote a realization of the couple (X, Y) by (x, y), such that the marginal distributions are given by: $p_X(x) = \int p_{X,Y}(x, y)\mathrm{d}y$ and $p_Y(y) = \int p_{X,Y}(x, y)\mathrm{d}x$. Thus, the local mutual information is expressed by [2]:

$$\mathrm{i}(x, y) = \log \frac{p_{X,Y}(x, y)}{p_X(x)p_Y(y)}. \quad (1)$$

Then, the local statistical dependence between neighbor pixel is simply given by $\mathrm{i}(I(p), I(q)) \;\; |p \sim q$. Organizing these measurements with respect to the direction of the pixel links, two images representing the horizontal dependences E_H and the vertical dependences E_V are obtained. These images are comparable to horizontal and vertical edge maps obtained by computing the Euclidean distance $\|I(p) - I(q)\|$ or the spectral angle $\alpha(I(p), I(q))$ [5] such that $p \sim q$.

2.2. Invariance Property

Let f a bijective and differentiable function defined between two random variables X and Z, such that their realizations are deterministically linked by $z = f(x)$. The local mutual information defined between the realizations of (X, Y) is not impacted by the transformation of X into Z, such that $\mathrm{i}(z, y) = \mathrm{i}(f(x), y) = \mathrm{i}(x, y)$. In other words, estimating the local mutual information from (X, Y) or from (Z, Y) does not depend on the any bijective function linking Z to X. This property is simply obtained by applying the rule a variable change (see also [4]):

$$\begin{aligned} \mathrm{i}(f(x), y) &= \log \frac{p_{Z|y}(z \mid y)}{p_Z(z)}, &(2)\\ &= \log \frac{p_{X|y}(f^{-1}(z) \mid y) |\det \frac{\mathrm{d}x}{\mathrm{d}z}|}{p_X(f^{-1}(z)) |\det \frac{\mathrm{d}x}{\mathrm{d}z}|}, &(3)\\ &= \log \frac{p_{X|y}(x \mid y)}{p_X(x)}, &(4)\\ &= \mathrm{i}(x, y). &(5)\end{aligned}$$

By symmetry the property holds for the second variable. Assuming a bijective and differentiable function g acting on Y, we obtain $\mathrm{i}(x, y) = \mathrm{i}(f(x), y) = \mathrm{i}(f(x), g(y))$. Thanks to this invariance property, if the pixel values of an image I are transformed into an image J by an unknown and differentiable function f, the dependence in between adjacent pixels remains unchanged: $\mathrm{i}(I(p), I(q)) = \mathrm{i}(f(I(p)), f(I(q))) = \mathrm{i}(J(p), J(q))$. Thus, the resulting horizontal E_H and vertical dependences E_V remain unchanged by the spectral transformation.

2.3. Probability Distribution Estimation

The estimation of the joint probability distribution is crucial in providing an accurate measurement of the dependence between neighbor pixels. In a first approximation, the joint probability distribution of $(X, Y) = (I(p), I(q)), p \sim q$ can be assumed to be Gaussian $\mathcal{N}(\mu, \Sigma)$, where $\mu = [\mu_X, \mu_Y]$ and $\Sigma = [\Sigma_X, \Sigma_{XY}; \Sigma_{YX}, \Sigma_Y]$. In that setting, X and Y are distributed by $\mathcal{N}(\mu_X, \Sigma_X)$ and $\mathcal{N}(\mu_Y, \Sigma_Y)$, respectively. Then, the local mutual information under Gaussian assumption is denoted by i^G and admits the following analytical formulation [6]:

$$\begin{aligned} &2\,\mathrm{i}^G(x, y) = \\ &([x, y] - \mu)^T \left(\Sigma_i^{-1} - \Sigma^{-1}\right) ([x, y] - \mu) - \log \frac{\det \Sigma}{\det \Sigma_i} \quad (6)\end{aligned}$$

where $\Sigma_i = [\Sigma_X, 0; 0, \Sigma_Y]$ is the variance matrix of the equivalent independent variables, meaning that the correlation matrix is null $\Sigma_{X,Y} = 0$. As complementary information, this function is well-known in finance modeling and is the derivative of Gaussian copula [7], and its application in credit modeling is considered to be one of the reason behind the 2008-2009 financial crisis [8].

When the Gaussian assumption is not fulfilled, the joint probability can be estimated by other methods. The statistical depth function was first proposed in [9] to perform multispectral classification and further developed in [4] in order to approximate joint probability distributions. The resulting local mutual information is denoted by i^d. We show in the next section that the local mutual information between neighbor pixels following i^d is more stable than that obtained using i^G.

Table 1. The 5 acquisition configurations $\{\theta_1, \ldots, \theta_5\}$ are summarized by the satellite azimuth and elevation angles, resulting in a decreasing sequence of the in off-nadir view angle.

	θ_1	θ_2	θ_3	θ_4	θ_5
Time	13:09:22	13:09:53	13:10:43	13:11:58	13:12:39
Azimuth	12.5 °	12.2 °	9.4 °	193.7 °	193.2 °
Elevation	44.7 °	56.0 °	81.3 °	59.8 °	44.6 °
Off-nadir	39.2 °	29.8 °	7.6 °	-26.7 °	-39.5 °

3. EXPERIMENTS

A set of WorldView-2 multi-sequence images is considered for this experiment. This data set is composed of 5 Ortho Ready Standard WorldView-2 multi-angular acquisitions, including a 16 bit panchromatic image and a multispectral 8-band image. These images represent a subpart of the city of Rio in Brazil, and were acquired on January 10, 2010. The 5 acquisition angles are summarized in Table 1. The corresponding image sequence is depicted in Figs. 1(a)–(e), by displaying the red, green, and blue color compositions.

The horizontal and vertical dependences are computed using the 8 bands, before being merged by a maximum operator: $E = \max(E_H, E_V)$. The joint distributions are estimated by the depth function for each view angle, enabling the computation of the local dependences. The resulting local dependences are denoted by E^d and are depicted in Fig. 1(f)–(j). Those obtained using i^G (Eq. 6) instead of i^d are denoted by E^G and are displayed in Fig. 1(k)–(o). Moreover the local horizontal and vertical edges obtained from the L1 norm or the spectral angle are also fused by the maximum operator. They are denoted $E^{\|\|_1}$ and E^{α}, respectively. The resulting edge images are displayed in Fig. 1(p)–(t) and Fig. 1(u)–(y) respectively.

The robustness of the various local intensity change measures is assessed by computing the normalized correlation in between the edge images, while ignoring the problem of parallax and temporal changes (few seconds between successive images, see Table 1). Table 2 summarizes the results, where the proposed local dependence is shown to be less impacted by the viewing angle, thus the atmospheric spectral distortions. This analysis shows that the local dependence can be an interesting alternative in representing edges, while it does not require any radiometric correction. It also reveals that the spectral angle dissimilarity measure leads to more stable edges than the L1 norm.

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Fig. 1. (a)-(e) The input multi-angular sequence of multi-spectral images with a size of $768 \times 768 \times 8$ and a resolution of 1.85m (true color composition), see acquisitions parameters in Table 1. Credit DigitalGlobe 2010. (f)-(j) The edge maps obtained with local mutual information with joint probability approximated by the depth function. (k)-(o) The edge maps obtained with local mutual information assuming a Gaussian joint probability. (p)-(t) The edge maps obtained with the Euclidean spectral distance between neighbor pixels. (u)-(y) The edge maps obtained with the spectral angle between neighbor pixels.

4. CONCLUSION

In this paper, a new measure of local spectral change was proposed as an edge descriptor. The local mutual information was used to measure the local dependence between adjacent pixels. Furthermore, this metric was proven to be invariant to any bijective transform applied independently on each pixel. Such a property is of main interest for handling signals or images which are transformed by unknown transforms. The invariance property was experimentally demonstrated with WorldView-2 multi-angular, where spectral distortions impact the usual edge measurements based on Euclidean norm or spectral angle. The results emphasized that the local mutual information provides the most stable edge measurements across view angles. Further experiments demonstrating that constrained connectivity segmentations [10] using the local mutual information for defining the dissimilarity between adjacent pixels rather than the Euclidean norm are more stable for varying acquisition angles are detailed in [4].

5. REFERENCES

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Table 2. Normalized correlation obtained between the edge images computed from the multi-angular sequence. The highest normalized correlation coefficients among the 4 edge measures are highlighted in bold.

E^d	θ_1	θ_2	θ_3	θ_4	θ_5
θ_1	1.00	**0.86**	**0.65**	**0.63**	**0.61**
θ_2	**0.86**	1.00	**0.74**	**0.65**	**0.62**
θ_3	**0.65**	**0.74**	1.00	**0.64**	**0.57**
θ_4	**0.63**	**0.65**	**0.64**	1.00	**0.74**
θ_5	**0.61**	**0.62**	**0.57**	**0.74**	1.00
E^G	θ_1	θ_2	θ_3	θ_4	θ_5
θ_1	1.00	0.57	0.33	0.31	0.30
θ_2	0.57	1.00	0.43	0.36	0.31
θ_3	0.33	0.43	1.00	0.37	0.30
θ_4	0.31	0.36	0.37	1.00	0.51
θ_5	0.30	0.31	0.30	0.51	1.00
$E^{\|\|\|\|_1}$	θ_1	θ_2	θ_3	θ_4	θ_5
θ_1	1.00	0.64	0.43	0.38	0.35
θ_2	0.64	1.00	0.52	0.42	0.37
θ_3	0.43	0.52	1.00	0.47	0.39
θ_4	0.38	0.42	0.47	1.00	0.57
θ_5	0.35	0.37	0.39	0.57	1.00
E^α	θ_1	θ_2	θ_3	θ_4	θ_5
θ_1	1.00	0.71	0.48	0.41	0.38
θ_2	0.71	1.00	0.57	0.45	0.41
θ_3	0.48	0.57	1.00	0.50	0.42
θ_4	0.41	0.45	0.50	1.00	0.64
θ_5	0.38	0.41	0.42	0.64	1.00